

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 3

MA 4070 : Elements of Group and Rings

Marks: 25

1. If there is a homomorphism from $\frac{\mathbb{Z}}{m\mathbb{Z}}$ onto $\frac{\mathbb{Z}}{n\mathbb{Z}}$, then what is the divisibility relation between m and n ? [1]
2. Let $G = (\mathbb{C}^*, \cdot)$ and $N = \{z \in \mathbb{C} : |z| = 1\}$. Show that $\frac{G}{N}$ is isomorphic to $(\mathbb{R}^+, \cdot)$. [2]
3. Assume that $\frac{\mathbb{Z}}{m\mathbb{Z}}$ and $\frac{\mathbb{Z}}{n\mathbb{Z}}$ are both homomorphic images of a finite group G . If $(m, n) = 1$ and $\#G \leq mn$, then what can you say about $\#G$? [2]
4. Let G be a group. Show that, the map $\varphi : G \rightarrow G$ defined by $\varphi(g) = g^{-1}$ is a homomorphism iff G is an abelian group. [2]
5. Find the number of homomorphisms from $\frac{\mathbb{Z}}{m\mathbb{Z}}$ onto $\frac{\mathbb{Z}}{n\mathbb{Z}}$. [2]
6. Calculate the number of homomorphisms from $\frac{\mathbb{Z}}{m\mathbb{Z}}$ onto $\frac{\mathbb{Z}}{n\mathbb{Z}}$ & $\#\text{Hom}(\frac{\mathbb{Z}}{m\mathbb{Z}}, \frac{\mathbb{Z}}{n\mathbb{Z}})$. [4]
7. Consider the additive quotient group $\mathbb{Q}/\mathbb{Z}$.
 - (a) Show that every coset of $\mathbb{Z}$ in $\mathbb{Q}$ contains exactly **one** representative $q \in \mathbb{Q} \cap [0, 1)$. [2]
 - (b) Show that every element of $\mathbb{Q}/\mathbb{Z}$ has finite order. Can you write down one observation which is very specific to this example? [2]
8. Suppose that there is a homomorphism from a finite group G onto $\mathbb{Z}/15\mathbb{Z}$. Prove that G has normal subgroups of index 3 and 5. [4]
9. If a group G is generated by $\langle u, v \rangle$ with relations $u^4 = v^3 = 1, uv = v^2u^2$, then show that G has to be trivial. (Bonus: A cute proof will get > marks and a chocolate). [4]