

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 5

MA 4010 : Real Analysis

Max Marks: 30

1. For what values of a_i $0 \leq i \leq 3$, the function $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ is differentiable at 0. [2]
2. Let f, g be differentiable function $\mathbb{R}$ and suppose that $f(0) = g(0)$ and $f'(x) \leq g'(x)$ for all $x \geq 0$. Show that $f(x) \leq g(x)$ for all $x \geq 0$. [2]
3. Suppose f is defined and differentiable for every $x > 0$, and $f'(x) \rightarrow 0$ as $x \rightarrow \infty$. Put $g(x) = f(x+1) - f(x)$, then prove that $g(x) \rightarrow 0$ as $x \rightarrow \infty$. [2]
4. Find out the points where f_i is not differentiable and write a proper reason.
 - (a) $f_2(x) = |x+5|^3$. [1]
 - (b) $f_3(x) = \sin |x-4| + (|x|+1)^2$. [1]
 - (c) [1]

$$f_4(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2x + x^2 & \text{if } x \geq 0. \end{cases}$$

5. Fix $a > 0$, the function f is defined by [3]

$$f(x) = \begin{cases} e^{-\frac{a^2}{a^2-x^2}} & \text{if } |x| < a, \\ 0 & \text{if } |x| \geq a. \end{cases}$$

Prove that f is infinitely many times differentiable.

6. Let $f(x)$ be defined for all real x , and suppose that [3]

$$|f(x) - f(y)| \leq (x - y)^k,$$

where $k > 1$. Then prove that f is constant function. If $k \leq 1$, what can you conclude about continuity and differentiable of f ?

7. Suppose $f : [a, b] \rightarrow \mathbb{R}$ is a differentiable function and f' is bounded then prove that f is uniformly continuous. Suppose f' is not bounded then what can you say about uniform continuity of f ? [3]
8. Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a differentiable function. Prove or disprove the following statements.
 - (a) If f' has infinitely many zeros, then f also have infinitely many zeros. [1]
 - (b) If f has infinitely many zeros, then f' have infinitely many zeros. [1]
 - (c) If f is constant on some subset $A \subset \mathbb{R}$ which is infinite. Then f' having infinitely many zeros. [1]
9. Give an example of a uniformly continuous function on $[0, 1]$ that is differentiable on $(0, 1)$ but whose derivative is not bounded on $(0, 1)$. [3]
10. Suppose f is twice differentiable on $(0, \infty)$, f'' is bounded of $(0, \infty)$, and $f(x) \rightarrow 0$ as $x \rightarrow \infty$. Then prove that $f'(x) \rightarrow 0$ as $x \rightarrow \infty$. [3]
11. Suppose f is a real function on $\mathbb{R}$. Prove that if f is differentiable and $f'(t) \neq 1$ for every real t , prove that f has at most one fixed point. [3]