

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 1

MA 4010 : Real Analysis

Max Marks: 35

1. (a) Produce a set A and a relation R such that it satisfies exactly two conditions of an equivalence relation but not the third one. [3]
(b) Produce a set A and a relation R such that it satisfies exactly one condition of an equivalence relation but not the rest. [3]
2. Let $\{A_i\}_{i \in I}$ be a partition of A . Can we find an equivalence relation R on A such that A_i 's are exactly the equivalence classes of A with respect to R ? If so, produce an R . [2]
3. Given a non-empty set $E \subseteq \mathbb{R}$, write down the definitions of a lower bound of a set E and infimum of a set E . [2]
4. Let (X, d) is a metric space and E be a subset of X . If E^c be closed in X , then show that E is an open set in X . [2]
5. Write down the converse statement of, in a metric space X , a closed subset of a compact set is also compact. [2]
6. In a metric space (X, d) , a compact set need not be bounded. Is this statement true? justify your answer. [2]
7. Show that an arbitrary union of closed sets need not be closed. [3]
8. Let S be an ordered set with lub property. Let $B \subseteq S$ be a non-empty set which is bounded below. Prove that infimum of B exists in S . [3]
9. For any $n \in \mathbb{N}$, we let $E_n = \left(1 - \frac{1}{n}, 1 + \frac{1}{n}\right)$. Show that $\bigcap_{n=1}^{\infty} E_n = \{1\}$. [3]
10. Let (X, τ_X) be any topological space and A be a subset of X . Show that A is dense in X if and only if A intersects every non-empty open set in X . [3]
11. Take any finite set A in $\mathbb{R}$. Show that the induced topology on A from $\mathbb{R}$ is in fact the discrete topology. [3]
12. Given $\underline{x} = (x_1, \dots, x_n), \underline{y} = (y_1, \dots, y_n) \in \mathbb{R}^n$, define $d_i : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^+$ as [4]

$$d_1(\underline{x}, \underline{y}) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \quad \& \quad d_2(\underline{x}, \underline{y}) = \sum_{i=1}^n |x_i - y_i|.$$

Out of d_1 & d_2 , which one of them is a metric on $\mathbb{R}^n$?