

1) For system to be singular, coefficient matrix should determinant should be zero.

$$\begin{vmatrix} 1 & 4 & -2 \\ 1 & 7 & -6 \\ 0 & 3 & -9 \end{vmatrix} = 0$$

Expanding along 3rd row

$$0 - 3(-6 + 2) - 9(7 - 4) = 0$$

$$-3 \times -4 - 3 \times 9 = 0$$

$$\boxed{9 = 4}$$

2)

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{array} \right]$$

$$R_2 \rightarrow R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{array} \right]$$

$$R_3 \rightarrow R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\Rightarrow a=0, b=0; c=1$$

3.)

$$A = \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{pmatrix}$$

$\times$

$$\begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix} = U$$

$$\text{Inverse of } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} \text{ is } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} = L$$

$$A = LU$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{pmatrix}$$

$$4) \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 2 & 6 & 13 \end{pmatrix} \quad \hookrightarrow E_1$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 2 & 6 & 13 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{pmatrix} \quad \hookrightarrow E_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix} = U \quad \hookrightarrow E_3$$

$$\cancel{E_1} \cancel{E_2} \cancel{E_3} \cdot E_3 E_2 E_1 A = U$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} U$$

$$A = LU$$

$$\varepsilon_3 \varepsilon_2 \varepsilon_1 A = U$$

$$\varepsilon_3^T \varepsilon_3 \varepsilon_2 \varepsilon_1 A = \varepsilon_3^T U$$

$$\varepsilon_2 \varepsilon_1 A = \varepsilon_3^T U$$

$$\varepsilon_2 \varepsilon_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\varepsilon_1 A = \varepsilon_2^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\varepsilon_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\dot{\varepsilon}_1 A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = \varepsilon_1^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = LU$$

$$(LU)(x) = B$$

$$\text{let } Ux = y$$

$$Ly = B$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 13 \\ 4 \end{pmatrix}$$

$$\Rightarrow y_1 = 3$$

$$3y_1 + y_2 = 13 \Rightarrow y_2 = 4$$

$$2y_1 + y_2 + y_3 = 4 \Rightarrow y_3 = -6$$

$$\text{Now } Ux = y$$

$$\begin{pmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix}$$

$$3x_3 = -6$$

$$\Rightarrow x_3 = -2$$

$$2x_2 + 2x_3 = 4 \Rightarrow x_2 = 4$$

$$x_1 + 2x_2 + 4x_3 = 3$$

$$x_1 = 3$$

$$\therefore x_1 = 3, x_2 = 4, x_3 = -2$$

5)

$$Ax = b$$

$$A = \begin{bmatrix} 2 & -2 & 2 \\ 2 & 2 & 2 \\ 2 & 4 & 8 \end{bmatrix}; B = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

$$|A| = 48$$

$|A| \neq 0$; So A^{-1} exists

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

$$A^{-1} = \begin{pmatrix} 1/6 & 1/2 & -1/6 \\ -1/4 & 1/4 & 0 \\ 1/12 & -1/4 & 1/6 \end{pmatrix}$$

$$x = A^{-1}b$$

$$x = \begin{pmatrix} 1/6 & 1/2 & -1/6 \\ -1/4 & 1/4 & 0 \\ 1/12 & -1/4 & 1/6 \end{pmatrix} \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 8/3 \\ 1 \\ -2/3 \end{pmatrix}$$

$$\textcircled{6} \quad V = \{(x, y) : x, y \in \mathbb{R}\}$$

$$(a, b) \oplus (x, y) = (a+x+1, b+y+1)$$

$$\alpha \odot (x, y) = (\alpha x, \alpha y). \quad \alpha, a, b \in \mathbb{R}.$$

i) if $(x, y), (a, b) \in V$.

$$(x, y) \oplus (a, b) = (x+a+1, y+b+1) \in V.$$

Property is satisfied.

ii) associativity: $((x, y) \oplus (a, b)) \oplus (p, q) = (x+a+1, y+b+1) \oplus (p, q)$
 $= (x+a+p+2, y+b+q+2)$

$$(x, y) \oplus ((a, b) \oplus (p, q)) = (x, y) \oplus (a+p+1, b+q+1)$$

Property is satisfied

$$= (x+a+p+2, y+b+q+2)$$

iii) additive identity

$$\exists (-1, -1) \in V \text{ such that } (x, y) \oplus (-1, -1) = (x, y).$$

Property is satisfied.

iv) additive inverse: (satisfied)

$$\exists (-x-1, -y-1) \in V \text{ such that } (x, y) \oplus (-x-1, -y-1) = (0, 0).$$

v) commutative property

$$(x, y) \oplus (p, q) = (x+p+1, y+q+1)$$

$$(p, q) \oplus (x, y) = (p+x+1, q+y+1)$$

} Both are equal.

Property satisfied.

vi) $\alpha \odot (x, y) = (\alpha x, \alpha y) \in V \Rightarrow$ property satisfied.

$$\text{vii) } \alpha \odot ((x, y) \oplus (a, b)) = \alpha \odot (x+a+1, y+b+1)$$

$$= (\alpha x + \alpha a + \alpha, \alpha y + \alpha b + \alpha)$$

$$\alpha \odot (x, y) \oplus \alpha \odot (a, b) = (\alpha x, \alpha y) \oplus (\alpha a, \alpha b) \\ = (\alpha x + \alpha a + 1, \alpha y + \alpha b + 1)$$

clearly $\alpha \odot ((x, y) \oplus (a, b)) \neq \alpha \odot (x, y) \oplus \alpha \odot (a, b)$

$$\text{viii) } (\alpha + \beta) \odot (x, y) = ((\alpha + \beta)x, (\alpha + \beta)y) \quad \alpha, \beta \in \mathbb{R}, (x, y) \in V. \\ = (\alpha x + \beta x, \alpha y + \beta y)$$

$$\text{But } \alpha \odot (x, y) + \beta \odot (x, y) = (\alpha x, \alpha y) + (\beta x, \beta y) \\ = (\alpha x + \beta x + 1, \alpha y + \beta y + 1)$$

clearly $(\alpha + \beta) \odot (x, y) \neq \alpha \odot (x, y) + \beta \odot (x, y).$

$$\text{ix) } (\alpha \beta) \odot (x, y) = (\alpha \beta x, \alpha \beta y)$$

$$\alpha(\beta \odot (x, y)) = \alpha(\beta x, \beta y) = (\alpha \beta x, \alpha \beta y)$$

Property satisfied

$$\text{x) } \exists \alpha = 1 \in \mathbb{R} \text{ such that } 1 \odot (x, y) = (x, y) \in V$$

Property satisfied.

Hence 'V' is not a vector space.

To be a vector space, it should satisfy the properties

$$\alpha \odot ((x, y) \oplus (a, b)) = \alpha \odot (x, y) \oplus \alpha \odot (a, b)$$

$$\text{and } (\alpha + \beta) \odot (x, y) = \alpha \odot (x, y) \oplus \beta \odot (x, y)$$

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Given

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 1 & 1 & -1 & -2 \\ 1 & -1 & -1 & 2 \\ 1 & -2 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -2 & 0 & 1 & 0 & 0 \\ 1 & -1 & -1 & 2 & 0 & 0 & 1 & 0 \\ 1 & -2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$R_2 \rightarrow R_2 - R_1 \quad \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 & -1 & 1 & 0 & 0 \\ 1 & -1 & -1 & 2 & 0 & 0 & 1 & 0 \\ 1 & -2 & 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_3 \rightarrow R_3 - R_1 \\ R_4 \rightarrow R_4 - R_1 \end{array} \quad \left[\begin{array}{cccc|cccc} 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -2 & -3 & -1 & 1 & 0 & 0 \\ 0 & -3 & -2 & 1 & -1 & 0 & 1 & 0 \\ 0 & -4 & 0 & -2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_1 \rightarrow R_1 + 2R_2 \quad \left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -5 & -1 & 2 & 0 & 0 \\ 0 & -1 & -2 & -3 & -1 & 1 & 0 & 0 \\ 0 & -3 & -2 & 1 & -1 & 0 & 1 & 0 \\ 0 & -4 & 0 & -2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 3R_2 \quad \left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -5 & -1 & 2 & 0 & 0 \\ 0 & -1 & -2 & -3 & -1 & 1 & 0 & 0 \\ 0 & 0 & 4 & 10 & 2 & -3 & 1 & 0 \\ 0 & -4 & 0 & -2 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_4 \rightarrow R_4 - 4R_2 \\ \& R_2 \rightarrow -R_2 \end{array} \quad \left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -5 & -1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & -1 & 0 & 0 \\ 0 & 0 & 4 & 10 & 2 & -3 & 1 & 0 \\ 0 & 0 & 8 & 10 & 3 & -4 & 0 & 1 \end{array} \right]$$

$$\begin{array}{l} R_4 \rightarrow R_4 - 2R_3 \\ \& R_3 \rightarrow \frac{R_3}{4} \end{array} \quad \left[\begin{array}{cccc|cccc} 1 & 0 & -3 & -5 & -1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 3 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 10/4 & 2/4 & -3/4 & 1/4 & 0 \\ 0 & 0 & 0 & -10 & -5/2 & 5/2 & -1/2 & 1 \end{array} \right]$$

$$R_1 \rightarrow R_1 + 3R_3: \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 5/2 & 1/2 & -1/4 & 3/4 & 0 \\ 0 & 1 & 2 & 3 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 5/2 & 1/2 & -3/4 & 1/4 & 0 \\ 0 & 0 & 0 & -10 & -1 & 2 & -2 & 1 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_3 \text{ \& } R_4 \rightarrow R_4 / (-10) \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 5/2 & 1/2 & -1/4 & 3/4 & 0 \\ 0 & 1 & 0 & -2 & 0 & 1/2 & -1/2 & 0 \\ 0 & 0 & 1 & 5/2 & 1/2 & -3/4 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & 1/10 & -1/5 & 1/5 & -1/10 \end{array} \right]$$

$$\begin{aligned} R_1 &\rightarrow R_1 - \frac{5}{2}R_4 \\ \& R_2 &\rightarrow R_2 + 2R_4 \\ R_3 &\rightarrow R_3 - \frac{5}{2}R_4 \end{aligned} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1 & 0 & 0 & 1/5 & 1/10 & -1/10 & -1/5 \\ 0 & 0 & 1 & 0 & 1/4 & -1/4 & -1/4 & 1/4 \\ 0 & 0 & 0 & 1 & 1/10 & -1/5 & 1/5 & -1/10 \end{array} \right]$$

$\Rightarrow$ Inverse of the matrix is:

$$\begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/5 & 1/10 & -1/10 & -1/5 \\ 1/4 & -1/4 & -1/4 & 1/4 \\ 1/10 & -1/5 & 1/5 & -1/10 \end{bmatrix}$$

⑧ i) $v_1 = (1, 4, -3)$, $v_2 = (1, 0, 1)$, $v_3 = (1, 1, 0)$, $v_4 = (3, 1, 2)$

Subspace generated by v_2, v_3, v_4 is: $a_2v_2 + a_3v_3 + a_4v_4$

$$\Rightarrow a_2(1, 0, 1) + a_3(1, 1, 0) + a_4(3, 1, 2) = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\text{Let } \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & 4 \\ 1 & 0 & 2 & -3 \end{array} \right] \Rightarrow R_3 \rightarrow R_3 - R_1: \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & -1 & -1 & -4 \end{array} \right]$$

$$R_3 \rightarrow R_3 + R_2: \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\cancel{R_2} \rightarrow R_2 \quad a_2 + a_3 + 3a_4 = 1$$

$$a_3 + a_4 = 4.$$

It has infinite solutions.

Hence, v_1 lies in subspace generated by v_2, v_3, v_4 .

$$(ii) \quad v_1 = (1, 1, 2), v_2 = (0, 1, 0), v_3 = (3, 5, 6), v_4 = (1, 2, 1)$$

subspace generated by v_2, v_3, v_4 : $a_2 v_2 + a_3 v_3 + a_4 v_4$

$$\Rightarrow a_2 (0, 1, 0) + a_3 (3, 5, 6) + a_4 (1, 2, 1) = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 5 & 2 \\ 0 & 6 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

$$\text{let } \begin{bmatrix} 0 & 3 & 1 \\ 1 & 5 & 2 \\ 0 & 6 & 1 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \\ a_4 \end{bmatrix} = v_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 5 & 2 & 1 \\ 0 & 6 & 1 & 2 \end{array} \right] \Rightarrow R_3 \rightarrow R_3 - 2R_1: \left[\begin{array}{ccc|c} 0 & 3 & 1 & 1 \\ 1 & 5 & 2 & 1 \\ 0 & 0 & -1 & 0 \end{array} \right]$$

$$\Rightarrow 3a_3 + a_4 = 1$$

$$a_2 + 5a_3 + 2a_4 = 1$$

$$-a_4 = 0.$$

$$\Rightarrow 3a_3 = 1 \Rightarrow a_3 = 1/3.$$

$$a_2 + 5a_3 + 0 = 1 \Rightarrow a_2 = 1 - 5/3 = -2/3.$$

$$\text{For } a_2 = -2/3, a_3 = 1/3, a_4 = 0, \text{ we get } a_2 v_2 + a_3 v_3 + a_4 v_4 = v_1.$$

Hence, v_1 lies in the subspace generated by v_2, v_3, v_4 .

$$\textcircled{9} \quad \left\{ \underset{v_1}{(3, -2, 4, 5)}, \underset{v_2}{(0, 0, 2, 8)}, \underset{v_3}{(0, 1, 2, 5)}, \underset{v_4}{(0, -2, -3, -4)} \right\}$$

$$\text{let } a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0.$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 0 & 0 \\ -2 & 0 & 1 & -2 \\ 4 & 2 & 2 & -3 \\ 5 & 8 & 5 & -4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & 0 \\ -2 & 0 & 1 & -2 & 0 \\ 4 & 2 & 2 & -3 & 0 \\ 5 & 8 & 5 & -4 & 0 \end{array} \right]$$

$$\begin{array}{l} R_2 \rightarrow 2R_2 - R_4 \\ R_3 \rightarrow 4R_3 - 3R_4 \end{array} : \left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & 0 \\ -9 & -8 & -3 & 0 & 0 \\ 1 & -16 & -7 & 0 & 0 \\ 5 & 8 & 5 & -4 & 0 \end{array} \right]$$

$$R_2 \rightarrow 7R_2 - 3R_3 : \left[\begin{array}{cccc|c} 3 & 0 & 0 & 0 & 0 \\ -66 & -8 & 0 & 0 & 0 \\ 1 & -16 & -7 & 0 & 0 \\ 5 & 8 & 5 & -4 & 0 \end{array} \right]$$

$$\begin{aligned} 3a_1 &= 0; \quad -66a_1 - 8a_2 = 0, \quad a_1 - 16a_2 - 7a_3 = 0; \quad 5a_1 + 8a_2 + 5a_3 + (-4)a_4 = 0 \\ \Rightarrow a_1 &= 0 & \Rightarrow a_2 &= 0 & \Rightarrow a_3 &= 0 & \Rightarrow a_4 &= 0. \end{aligned}$$

$$\exists a_1 = a_2 = a_3 = a_4 = 0 \text{ such that } a_1 v_1 + a_2 v_2 + a_3 v_3 + a_4 v_4 = 0.$$

Hence, $\{v_1, v_2, v_3, v_4\}$ is linearly independent.

$\Rightarrow \{v_1, v_2, v_3, v_4\}$ is not linearly dependent.

⑩.

$$f(x) = x^2 + 3x - 2$$

$$g(x) = x + 7$$

$$h(x) = 2x^2 - 5x + 1$$

} $\Rightarrow$ can be written as

$$f = (1, 3, -2)$$

$$g = (0, 1, 7)$$

$$h = (2, -5, 1)$$

$$\text{let } c_1 f + c_2 g + c_3 h = 0.$$

$$\Rightarrow c_1(1, 3, -2) + c_2(0, 1, 7) + c_3(2, -5, 1) = (0, 0, 0)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & -5 \\ -2 & 7 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} R_2 \rightarrow R_2 - 3R_1 \\ R_3 \rightarrow R_3 + 2R_1 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -11 & 0 \\ 0 & 7 & 5 & 0 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 7R_2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -11 & 0 \\ 0 & 0 & 82 & 0 \end{array} \right]$$

$$\Rightarrow \text{or } c_1 + 2c_3 = 0$$

$$c_2 - 11c_3 = 0.$$

$$c_3 = 0.$$

$$\Rightarrow c_2 = 0 \text{ \& } c_1 = 0.$$

$$c_1 = c_2 = c_3 = 0.$$

$$\Rightarrow \exists c_1 = c_2 = c_3 = 0 \text{ such that } c_1 f + c_2 g + c_3 h = 0.$$

$$\Rightarrow f, g, h \text{ are linearly independent}$$

$$\Rightarrow \text{The set of functions } f(x), g(x), h(x) \text{ is linearly independent}$$