

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 4

MA 1140 : Linear Algebra

Max Marks: 50

Each question carries 5 marks

1. If A and B are similar matrices, then show that A and B have the same eigenvalues with the same geometric multiplicities. How about the algebraic multiplicities? [5]
2. Find the eigenvalues of $\begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$ and find the geometric multiplicities of each of the eigenvalue. [5]
3. Diagonalize the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, if possible. [5]
4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix with $a, b, c, d \in \mathbb{R}$.
 - (a) Find a condition on the entries of A such that eigenvalues will be real. [1]
 - (b) Given that the algebraic multiplicity of an eigenvalue of A is 2, then find that eigenvalue and also a condition for it. [2]
 - (c) If $a = 1, b = 0$, then find a d such that we will get only one eigenvalue and also find that eigenvalue, whose algebraic multiplicity is 2. [2]
5. Calculate A^{100} for the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & -3 & 5 \\ -1 & -2 & 4 \end{bmatrix}$. [5]
6. Let $P(x) \in \mathbb{R}[x]$ be a polynomial. Prove that, if v is an eigenvector of T with eigenvalue λ , then v is an eigenvector of $P(T)$ with eigenvalue $P(\lambda)$. [5]
7. If A and B are similar matrices, then
 - (a) Do A and B have the same determinant, same characteristic polynomial and eigenvalues? Answer them mathematically. [3]
 - (b) Let $f(x)$ be a polynomial in $\mathbb{R}[x]$. Does $f(A)$ similar to $f(B)$? and A, B have the same minimal polynomial? [2]Explain your answers clearly in both cases.
8. (a) Prove the Cayley-Hamilton Theorem for diagonalisable matrices. [3]
(b) Write A^{-1} as a polynomial function of A using Cayley-Hamilton Theorem, [2]
where $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

9. (a) Find whether the set of vectors $V = \{(1, 2, -1, 1), (3, 1, 4, -1), (0, 1, -1, -3)\}$ are orthogonal or not. If so, construct an orthonormal set based starting with this set. Is the set V linearly independent? [2]
- (b) Determine the distance from the point $(3, -1, 2)$ of $\mathbb{R}^3$ to the subspace consisting vectors of the form $\{(x, -3y, 2x) : x, y \in \mathbb{R}\}$. (Hint: Compute the orthogonal projection of $(3, -1, 2)$ on the subspace $\{(x, -3y, 2x) : x, y \in \mathbb{R}\}$.) [3]
10. Suppose $\{x_1, x_2, x_3\}$ is a basis for a subspace W of $\mathbb{R}^4$. Using Gram-Schmidt process:
- (a) Find an orthogonal basis for W where $x_1 = (1, 2, 3, 0), x_2 = (1, 2, 0, 0), x_3 = (1, 0, 0, 1)$. [2]
- (b) Find an orthonormal basis for W where $x_1 = (1, 1, 1, 1), x_2 = (-1, 4, 4, 1), x_3 = (4, -2, 2, 0)$. [3]