

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 2

MA 1140 : Linear Algebra

Max Marks: 50

Each question carries 5 marks

1. (a) Let $S = \{(2, 3, 0), (1, -1, 2), (p, 5, 6)\}$ be a subset of $\mathbb{R}^3$. For which value of $p \in \mathbb{R}$, the span of S is not $\mathbb{R}^3$. [2]
(b) Does $S = (0, 1, 1), (1, 0, 1), (1, 1, 0)$ spans $\mathbb{R}^3$ or not? If yes, then write vector $(2, 4, 8)$ as a span of S . [3]
2. Find the row space of the matrix [5]

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 \\ 2 & -6 & 9 & -1 \\ 4 & -12 & 18 & -2 \\ -1 & 3 & -4 & 2 \end{bmatrix}.$$

3. Find the column space of the matrix [5]

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 2 & -2 & 1 \end{bmatrix}.$$

4. Let A and B be two $n \times n$ matrices such that $AB = 0$. Show that
(a) the column space of B is contained in the null space of A . [2]
(b) the row space of A is contained in the left null space of B . [3]
5. Find a basis and also mention dimension for the following vector spaces:
(a) $V_1 =$ All the diagonal matrices in $\text{Mat}_{4 \times 4}(\mathbb{R})$. [2]
(b) $V_2 =$ All vectors in $\mathbb{R}^4$ with sum of first and third coordinates is twice of fourth and sum of all coordinates is zero [3]
6. Find the dimension of the following :
(a) The space of the cubic polynomials $p(x) \in \mathbb{R}[X]$ such that $p(7) = 0$. [1]
(b) The space of the cubic polynomials $p(x) \in \mathbb{R}[X]$ such that $p(7) = 0$ and $p(5) = 0$. [2]
(c) The span of the set $\{\cos^2 \theta, \sin^2 \theta, \sin 2\theta, \cos 2\theta\}$ in the space of all real valued functions of one real variable. [2]
7. (a) Find the null space of the matrix [2]

$$A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}.$$

- (b) If A is a $m \times n$ matrix of rank r . Which of the subspaces, null space, column space and row space, are the same for :

i. A and $\begin{bmatrix} A \\ A \end{bmatrix}$ [2]

ii. $\begin{bmatrix} A \\ A \end{bmatrix}$ and $\begin{bmatrix} A & A \\ A & A \end{bmatrix}$. [1]

Explain all in details.

8. (a) Find the rank of matrix A using Gaussian elimination and state whether the row vectors are linearly independent. [3]

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}.$$

- (b) Find the rank of matrix B using determinants (using minors) and state whether the row vectors are linearly independent. [2]

$$B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}.$$

9. Consider two matrices A and B such that AB exists. How the following are related to each other? Explain clearly.

(a) Rank of AB and Rank of A . [3]

(b) Rank of AB and Rank of B . [2]

10. Find the rank, nullity of A and verify rank-nullity theorem for A , where [5]

$$A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}.$$