

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 1

MA 1140 : Linear Algebra

Max Marks: 50

Each question carries 5 marks

1. (a) For which values of m , the given system of equations will have non-trivial solution? [2]

$$m^2x + (m+3)y + (m+2)z = 0$$

$$mx + (m+1)y + mz = 0$$

$$x + y = 0$$

- (b) Find values of n and b for which following system will have : a) one solution, b) infinite solutions, c) no solution. [3]

$$3x + 2y = 4$$

$$(n+2)x + ny = b$$

2. Find the determinant of matrix A using Gaussian elimination. [5]

$$A = \begin{bmatrix} 3 & 2 & 4 & 2 \\ 2 & 0 & 2 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

3. Find the LU -decomposition of the following matrix : [5]

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{bmatrix}$$

4. Find the solution of $X = [x_1, x_2, x_3]^t$ of the matrix, using LU -decomposition. [5]

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}.$$

5. Solve $Ax = B$ using inverse of A , where [5]

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 8 & 4 & 2 \\ 10 & 9 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 5 & 4 & 1 \end{bmatrix}.$$

6. Find the inverse of matrix A , where [5]

$$A = \begin{bmatrix} 13 & 6 & 3 & 5 \\ 7 & 4 & 8 & 9 \\ 10 & 14 & 11 & 12 \\ 15 & 16 & 17 & 18 \end{bmatrix}$$

7. Let $D = \{s | s \in \mathbb{R}, s > 0\}$. If addition and scalar multiplication are defined as follows : [5]

- Addition : For $s \in D, t \in D$, define $s \oplus t = st$
- Multiplication : For $u \in \mathbb{R}, s \in D$, define $u \odot s = s^u$

Is D with these operations a vector space or not? Justify your answers.

8. (a) Let V and W be the subspaces of $\mathbb{R}^2$ spanned by $(1, 1)$ and $(1, 2)$, respectively. Find vectors $v \in V$ and $w \in W$ so $v + w = ((-1)^{x_1} * x_2, ((-1)^{x_3} * x_4))$. Instructions for determining x_i , where $i = 1, 2, 3, 4$: $abcde$ are last five digits of your current IITH roll number (changed roll numbers for branch change students). Consider a to be 0 if only last 4 characters of your roll number are digits. [3]

$$x_1 = a + b + c + d + e$$

$$x_2 = x_1 + 1$$

$$x_3 = d * 10 + e$$

$$x_4 = e * 10 + d$$

- (b) Let V be a vector space over the field $\mathbb{R}$ of real numbers. Prove that V is not equal to the union of a finite number of proper subspaces. [2]
9. (a) Consider vectors $v_1, v_2, v_3, \dots, v_n$ are linearly dependent in a vector space V . Suppose a set, say S , consists of vectors in V which also include $v_1, v_2, v_3, \dots, v_n$. Determine whether S is linearly independent or not. [2]
- (b) Determine whether the following set of functions is linearly independent or not : [3]

$$f(x) = -x^2 + 2x - 5, g(x) = 5x - 1, h(x) = 2x^2 + 7.$$

10. Determine if the set S of vectors in $\mathbb{R}^4$ are linearly independent or not?, where $S = (2, 1, 0, 1), (1, 0, 1, 1), (4, 1, 2, -1), (1, 0, 1, -1)$. [5]