

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 4

MA 1140 : Linear Algebra

Max Marks: 50

Each question carries 5 marks

1. Suppose $x_1 = (1, 2, 3, 0), x_2 = (1, 2, 0, 0), x_3 = (1, 0, 0, 1)$ is a basis for a subspace W of $\mathbb{R}^4$. Describe an orthogonal basis for W by using Gram-Schmidt process.
2. For $p(x), q(x) \in P_2(\mathbb{R})$, the set of polynomials with degree ≤ 2 , show that $\langle p, q \rangle = \int_0^1 p(x)q(x)dx$ is an inner product.
3. Consider $P_2(\mathbb{R})$ equipped with $\langle p, q \rangle = \int_0^2 p(x)q(x)dx$ for $p(x), q(x) \in P_2(\mathbb{R})$. Find an orthonormal basis for $P_2(\mathbb{R})$ if one starts with $\{1, x, x^2\}$ as a basis of $P_2(\mathbb{R})$.
4. Find an orthonormal basis for the subspace $S \subseteq \mathbb{R}^4$, where S is the column span of

$$\begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

5. Find the distance from the point $y = (0, 0, 0, 1)$ to the subspace $V \subseteq \mathbb{R}^4$ spanned by vectors $x_1 = (1, -1, 1, -1), x_2 = (1, 1, 3, -1), x_3 = (-3, 7, 1, 3)$.
6. Let u and v be orthogonal vectors. If $u + v$ and $u - v$ are orthogonal, show that $\|u\| = \|v\|$, where $\|u\|$ denotes the norm of u .
7. Solve the following differential equations using diagonalization of matrices:

$$\begin{aligned} \frac{dy_1}{dt} &= y_1 - y_2 + 2y_3, \\ \frac{dy_2}{dt} &= 3y_1 + 4y_3, \\ \frac{dy_3}{dt} &= 2y_1 + y_2. \end{aligned}$$

8. Compute the eigenvalues and eigenvectors of A and A^{-1} , where $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

1. Calculate the trace for A and A^{-1} . What is the relation between the eigenvectors of A and A^{-1} ?
2. If A has eigenvalues λ_1 and λ_2 , what are the eigenvalues of its inverse?
3. In part (2), what about the case when $\lambda_1 = 0$?
9. For any real symmetric matrix $A \in M_{n \times n}(\mathbb{R})$, show that all eigen values of A are real.
10. If A is an $m \times n$ -matrix with linearly independent columns, then A can be factored as $A = QR$, where Q is an $m \times n$ matrix whose columns form an orthogonal basis for Columns of A and R is an $n \times n$ upper triangular invertible matrix with positive entries on its main diagonal. Find the QR

factorization of $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 0 & 3 \end{bmatrix}$.