

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 2

MA 1140 : Linear Algebra

Max Marks: 45

Each question carries 5 marks

1. Determine the value of a for which the following system of equation has a non-trivial solution.

$$a^3x + (a+1)^3y + (a+2)^3z = 0$$

$$ax + (a+1)y + (a+2)z = 0$$

$$x + y + z = 0$$

2. Determine for which values of a and b the following system has no solutions. (using the concept of determinants).

$$ax + y + 2z = 0$$

$$x + 2y + z = b$$

$$2x + y + az = 0$$

3. Determine the row-rank of

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

4. Determine the coefficients (i.e., component vector) of $P(x) = 5 + 7x - 3x^2$ relative to the following basis

1. The standard basis is $B = \{1, x, x^2\}$.

2. The ordered basis $C = \{1 + x, 2 + 3x, 5 + x + x^2\}$.

5. Find the matrix A of a linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that satisfies

$$T\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \& \quad T^2\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

6. If $T_1 : \mathbb{R}^n \rightarrow \mathbb{R}^m$, $T_2 : \mathbb{R}^m \rightarrow \mathbb{R}^p$ are two linear transformations, then prove that the composition $T = T_2 \circ T_1 : \mathbb{R}^n \rightarrow \mathbb{R}^p$ is also a linear transformation.

7. Find $\text{Ker}(S)$, $\text{Range}(S)$, and their dimensions for the linear transformation $S : M_{2 \times 2}(\mathbb{R}) \rightarrow M_{2 \times 2}(\mathbb{R})$ defined by $S(A) = A - A^T$.

8. Let V be the vector space of all real convergent sequences $\{x_n\}$. Define a transformation $T : V \rightarrow V$ as follows: If $x = \{x_n\}$ is a convergent sequence with limit a , let $T(x) = \{y_n\}$, where $y_n = a - x_n$ for $n \geq 1$. Prove that T is linear and describe the null space and range of T .

9. Let V be the space of all infinite sequences of real numbers. Consider the transformation

$$T(x_0, x_1, x_2, \dots) = (x_1, x_2, x_3, \dots)$$

from V to V (T drops the first term x_0 of the sequence).

1. Show that T is a linear transformation.
2. Find the kernel of T .
3. Is the sequence $(1, 2, 3, \dots)$ in the image of T ?
4. Find the image of T .