

INDIAN INSTITUTE OF TECHNOLOGY HYDERABAD

DEPARTMENT OF MATHEMATICS

ASSIGNMENT 1

MA 1140 : Linear Algebra

Max Marks: 50

Each question carries 5 marks

1. The upward velocity of a rocket is given at three different times as below:

Time, t (s)	Velocity, v (m/s)
5	106.8
8	177.2
12	279.2

The velocity data is approximated by a polynomial as

$$v(t) = a_2 t^2 + a_1 t + a_3 \quad \text{for } 5 \leq t \leq 12.$$

Find the values of a_1 , a_2 , and a_3 using the Gauss elimination method and also find the velocity at $t = 6, 7.5, 9, 11$ seconds.

2. A liquid-liquid extraction process conducted in the Electrochemical Materials Laboratory involved the extraction of nickel from the aqueous phase into an organic phase. A typical set of experimental data from the laboratory is given below:

Ni aqueous phase 'a' (g/l)	2	2.5	3
Ni organic phase 'g' (g/l)	8.57	10	12

Assuming 'g' is the amount of Ni in the organic phase and 'a' is the amount of Ni in the aqueous phase, the quadratic interpolant that estimates 'g' is given by

$$g = x_1 a^2 + x_2 a + x_3, \quad (2 \leq a \leq 3).$$

Find the values of x_1 , x_2 , x_3 using LU-decomposition. Estimate the amount of nickel in the organic phase, when 2.3 g/l is in the aqueous phase using quadratic interpolation.

3. Find A^{-1} and use it solve $Ax = b$, where $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$.

4. Let $V = \mathbb{R}^2$. For any $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) := (a_1 + b_1, a_2 b_2),$$
$$c.(a_1, a_2) := (ca_1, a_2).$$

Is V a vector space over $\mathbb{R}$ under the above operations? Justify your answers.

5. Let $A_n = (a_{ij})_{n \times n}$ be a $n \times n$ -matrix defined as follows

$$a_{ij} := \begin{cases} 1 & \text{if } i = j \text{ or } i = j + 1, \\ -1 & \text{if } j = i + 1, \\ 0 & \text{otherwise.} \end{cases}$$

Show that $\det(A_n) = \det(A_{n-1}) + \det(A_{n-2})$ and tell us what is the famous quantity that $\det(A_n)$ computes?

6. Is the set $H = \left\{ \begin{bmatrix} 2a & b \\ 3a+b & 3b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ a subspace of $M_{2 \times 2}(\mathbb{R})$? Justify your answers.

7. Is the set $\{(1, -1, 2), (1, -2, 1), (1, 1, 4)\}$ linearly independent?. If so, does this set also span $\mathbb{R}^3$?

8. Let $S = \{(4, 5, 6), (r, 5, 1), (4, 3, 2)\}$ be a subset of $\mathbb{R}^3$. For what values of $r \in \mathbb{R}$, the span of S is not $\mathbb{R}^3$.

9. Let $v_1 = (1, 3, 5), v_2 = (2, 5, 9), v_3 = (-3, 9, 3)$ be vectors in $\mathbb{R}^3$. Are they dependent vectors in $\mathbb{R}^3$? If so, then produce a dependence relation?

10. Let $A := \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) Show that A is an invertible matrix and calculate A^n for $n \in \mathbb{N}$.

(b) Calculate the $a, b \in \mathbb{R}$ such that $A^2 + aA + bI = 0$, where I is a 3×3 -identity matrix.

(c) Show that there exists real numbers $c_0, c_1, \dots, c_n \in \mathbb{R}$ such that

$$A^{-n} = \sum_{i=0}^n c_i A^i,$$

for any $n \in \mathbb{N}$.