Sphere Packings and List Decoding in Euclidean Space

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Joint work with Yihan Zhang (IST Austria)

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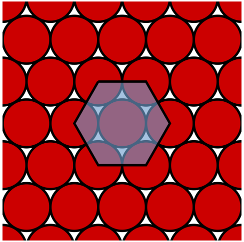
Sphere packings

What is the maximum number of n-dim spheres we can pack in a given volume?



Infinite sphere packings

- **Sphere packing:** Collection of infinitely many non-overlapping spheres of radius r in $\mathbb{R}^n$
- **Density:** Fraction of volume in $\mathbb{R}^n$ that is occupied by the spheres

Problem: Find a sphere packing with the largest density

- Open problem! Solutions only known for:

n	Optimality
1	trivial
2	Hexagonal [Lagrange 1773, Thue 1890]
3	BCC [Kepler 1611, Gauss 1831, Hales 1998]
8	E8 [Viazovska 2017]
24	Leech [Cohn-Kumar 2009, Cohn et al, 2017]

What about large n?

Open problem!

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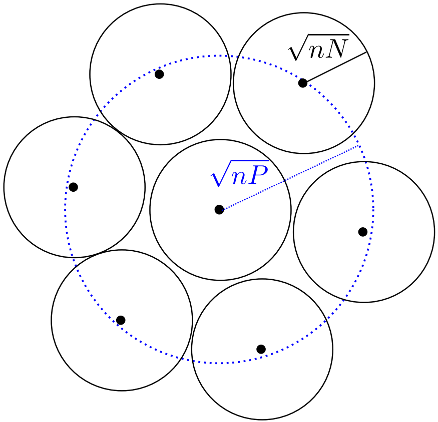
Best known bounds [Campos et al 2023, Kabatiyanskii-Levenshtein 1978]:

$$\frac{1}{2^n} \leqslant \frac{n \ln n}{2^{n+1}}(1 - o(1)) \leqslant \Delta_{opt}(n) \leqslant \frac{1}{2^{0.599n(1+o(1))}}$$

Nice survey: Cohn, “A conceptual breakthrough in sphere packings,” 2017

Sphere packings within bounded regions



How many nonoverlapping n-dimensional balls of radius $\sqrt{nN}$ can we pack in a larger ball of radius $\sqrt{nP}$?

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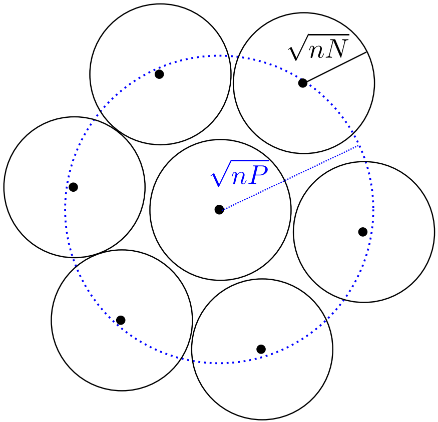

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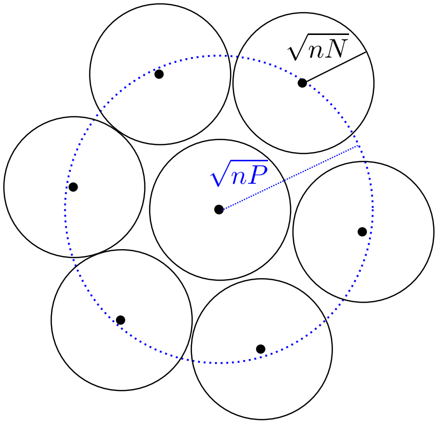
Open problem!

[Blachman 1962]

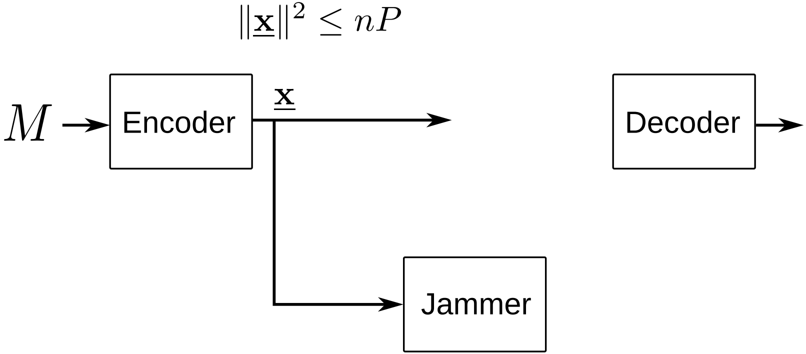
[Kabatiansky and Levenshtein 1978]

An equivalent formulation



How many points can we place within $\mathcal{B}_n(\sqrt{nP})$ such that the minimum pairwise distance between points is at least $2\sqrt{nN}$?

Connection to channel coding: Jamming adversary



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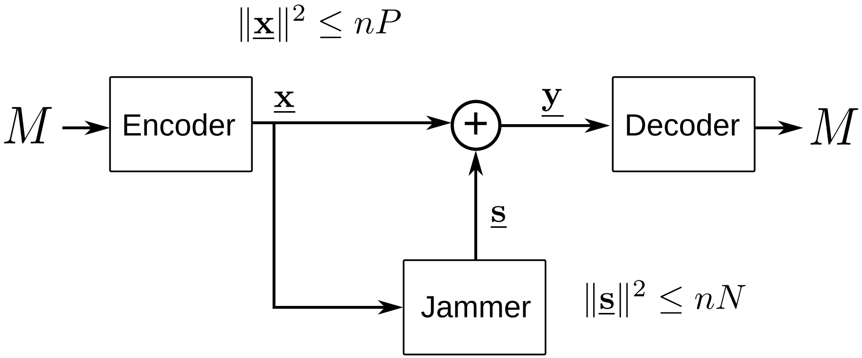
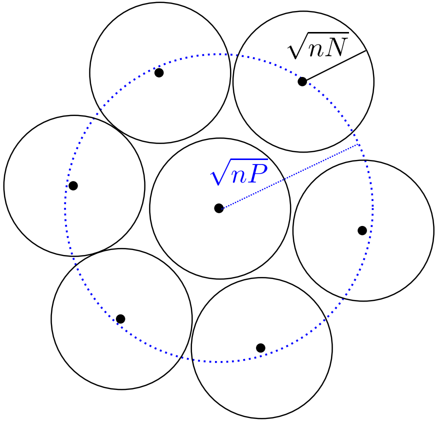


Want a code with minimum pairwise distance $2\sqrt{nN}$

Relaxation: Multiple packing



How many $\mathcal{B}(\cdot, \sqrt{nN})$ balls can we pack in $\mathcal{B}(0, \sqrt{nP})$ such that no more than L balls intersect at any given point?

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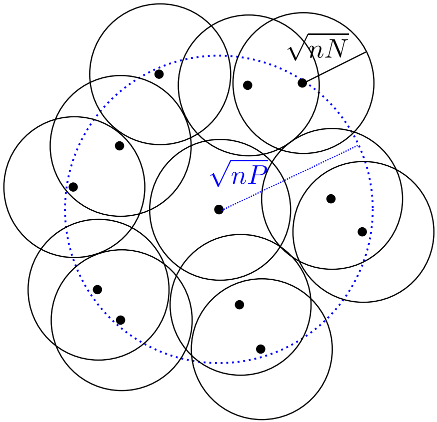

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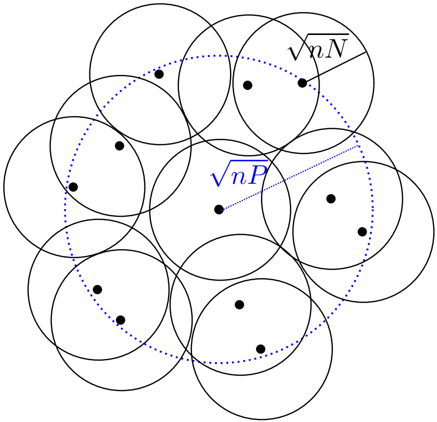

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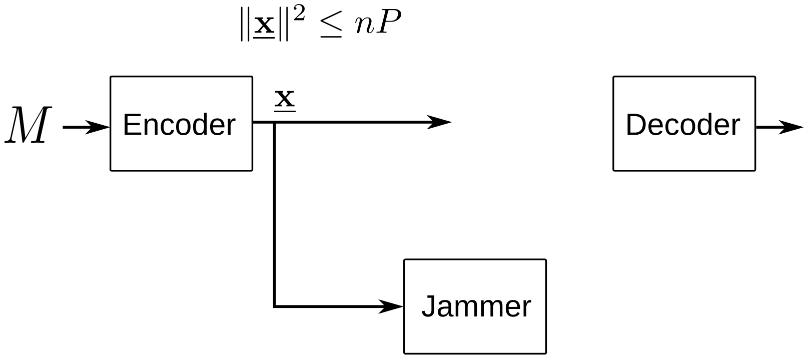
We can answer this! (partially)

(Adversarial) List decoding



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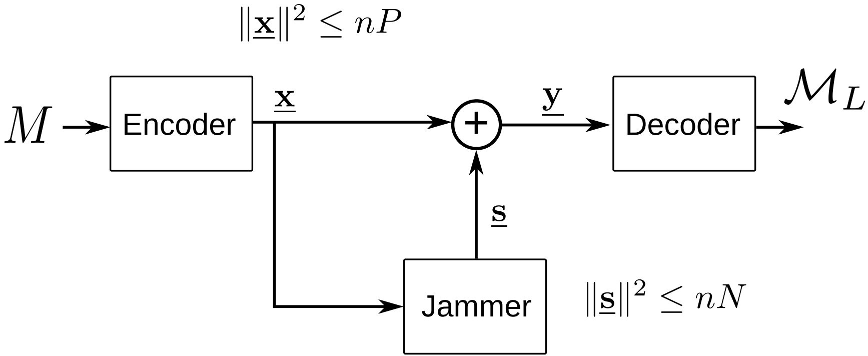

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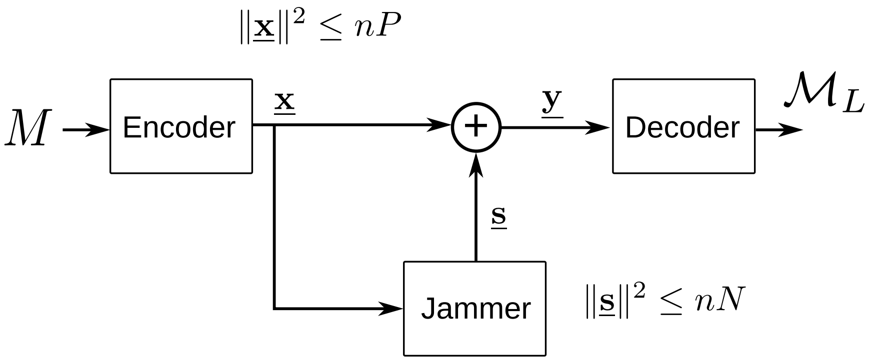

Output $\mathcal{M}_L = \{M_1, \ldots, M_L\}$ with the guarantee that $M \in \mathcal{M}_L$

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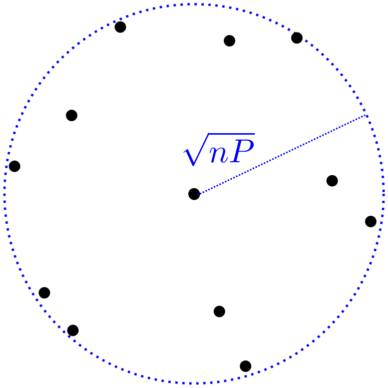
- Useful proof technique to prove achievability results for more complicated channels with jammers [Zhang *et al.* 2018, Zhang *et al.* 2020]
- Large body of work on list decoding over finite fields, and many connections to complexity theory, cryptography, combinatorics, quantum, etc.

Formal definition

Given $P, N > 0$ and $L \in \mathbb{Z}_{\geqslant 2}$, a (P, N, L) list decodable code $\mathcal{C}$ is a collection of points in $\mathbb{R}^n$ satisfying

- Power constraint: $\underline{x} \in \mathcal{B}_n(0, \sqrt{nP})$ for all $\underline{x} \in \mathcal{C}$



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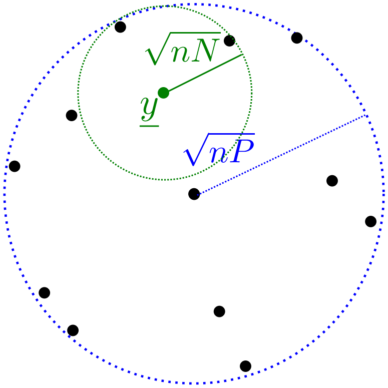

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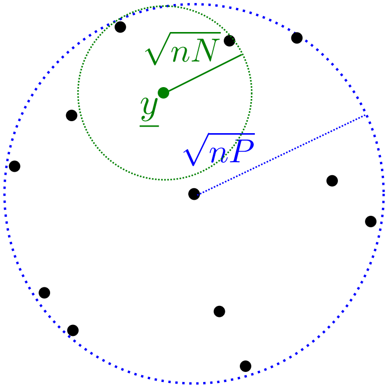

Rate of $\mathcal{C}$:

$$R(\mathcal{C}) \coloneqq \frac{1}{n} \ln |\mathcal{C}|$$

Fundamental question

Goal: Characterize the list decoding capacity $C_L(P, N)$: The limsup of achievable rates of (P, N, L)-list decodable codes.

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Theorem (List decoding capacity - folklore)

$$C(P, N) \stackrel{def}{=} \frac{1}{2} \ln \frac{P}{N}$$

- *There exist codes with rate $C(P, N) - \epsilon$ that are (P, N, L) list decodable with $L = \mathcal{O}(\frac{1}{\epsilon} \log \frac{1}{\epsilon})$.*

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- *There exist codes with rate $C(P,N)-\epsilon$ that are (P,N,L) list decodable with $L=\mathcal{O}(\frac{1}{\epsilon}\log\frac{1}{\epsilon})$.*
- *Any sequence of (P,N,L) list decodable codes with asymptotic rate greater than $C(P,N)$ must necessarily have $L=e^{\Theta(n)}$.*

Existence of codes

Take $\sqrt{N_\delta} = \sqrt{N} + \sqrt{\delta}$,

$$R = \frac{1}{2} \ln \frac{P}{N_\delta} - \epsilon$$

Random coding: Pick e^{nR} codewords $\underline{c}(1), \ldots, \underline{c}(2^{nR})$ i.i.d. uniform over $\mathcal{B}_n(\sqrt{nP})$.

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For any $\underline{y} \in \mathbb{R}^n$ and $i \in [2^{nR}]$,

$$\Pr[\underline{c}(i) \in \mathcal{B}_n(\underline{y}, \sqrt{nN_\delta})] \leqslant \frac{\mathrm{vol}(\mathcal{B}_n(\sqrt{nN_\delta}))}{\mathrm{vol}(\mathcal{B}_n(\sqrt{nP}))} = \left(\frac{N_\delta}{P}\right)^{n/2}$$

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For any $\underline{y} \in \mathbb{R}^n$ and $i_1, i_2, \ldots, i_{L+1}$,

$$\Pr[\underline{\mathbf{c}}(i_k) \in \mathcal{B}_n(\underline{y}, \sqrt{nN_\delta}) \text{ for } k = 1, 2, \ldots, L+1] \leqslant \left(\frac{N_\delta}{P}\right)^{n(L+1)/2}$$

Existence of codes (contd)

For any $\underline{y} \in \mathbb{R}^n$,

$$\begin{aligned}
&\Pr[\text{There are more than L codewords in } \mathcal{B}_n(\underline{y}, \sqrt{nN_\delta})] \\
&\leqslant \binom{e^{nR}}{L+1} \left(\frac{N_\delta}{P}\right)^{n(L+1)/2} \\
&\leqslant \exp\left(n(L+1)\left(R - \frac{1}{2}\ln\frac{P}{N_\delta}\right)\right) \\
&\leqslant \exp\left(-n(L+1)\epsilon\right)
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Final step: Take a δ-covering of $\mathcal{B}_n(\sqrt{nP} + \sqrt{n\delta})$ and use union bound

$$\Pr[\text{There are more than L codewords in } \mathcal{B}_n(\underline{y}, \sqrt{nN}) \text{ for any } \underline{y}]$$

$$\leqslant$$

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\end{aligned}$$

Comments

- $R > C(P, N)$: Show the existence of a "witness" $\underline{y}$ by picking one at random [Zhang *et al.* 2018]
- Structured codes: There exist nested lattice codes with rate $R = C(P, N) - \epsilon$ that are (P, N, L) list decodable with $L = 2^{O(\frac{1}{\epsilon} \log^2 \frac{1}{\epsilon})}$ [Zhang and Vatedka 2019]
- Unbounded L-packings: There exist unbounded L-packings with density $(1 + \delta)^{-n}$ for $L = O(\frac{1}{\delta} \log \frac{1}{\delta})$
- Unbounded lattice packings: There exist lattices with density $(1 + \delta)^{-n}$ that are L-packings for $L = 2^{O(\frac{1}{\delta} \log^2 \frac{1}{\delta})}$ [Zhang and Vatedka 2019]
- Open questions: Order-optimal dependence on ϵ, δ

What about $C_L(P, N)$ for fixed L?

- Using random (expurgated) Gaussian codebooks [Zhang and Vatedka 2022],

$$C_L(P, N) \geqslant \frac{1}{2}\left[\ln \frac{LP}{(L+1)N} + \frac{(L+1)N}{LP} + 1\right]$$

- Using random (expurgated) spherical codebooks [Zhang and Vatedka 2022],

$$C_L(P, N) \geqslant \frac{1}{2}\left[1 - \frac{(L+1)N}{LP} + \frac{1}{L}\ln \frac{P}{L(P-N)}\right]$$

- For $N \leqslant PL/(L+1)$ [Blinovsky 1999, Zhang and Vatedka 2022],

$$C_L(P, N) \leqslant \frac{1}{2}\ln \frac{LP}{(L+1)N}$$

Tighter lower bound

[Blinovsky 1999, Zhang and Vatedka 2021][1]

$$C_L(P,N) \geqslant \frac{1}{2}\left[\ln\left(\frac{LP}{(L+1)N}\right) + \frac{1}{L}\ln\left(\frac{P}{(L+1)(P-N)}\right)\right]$$

[1] Blinovsky, “Multiple packing of the Euclidean sphere,” *IEEE Trans Inf Theory*, 1999

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- Also holds under a stronger notion of average-radius list decoding
- As $L \to \infty$, the r.h.s. $\to \frac{1}{2}\ln\frac{P}{N}$
- $L = 1$: recovers best known lower bound for sphere packing [Blachman 1962]

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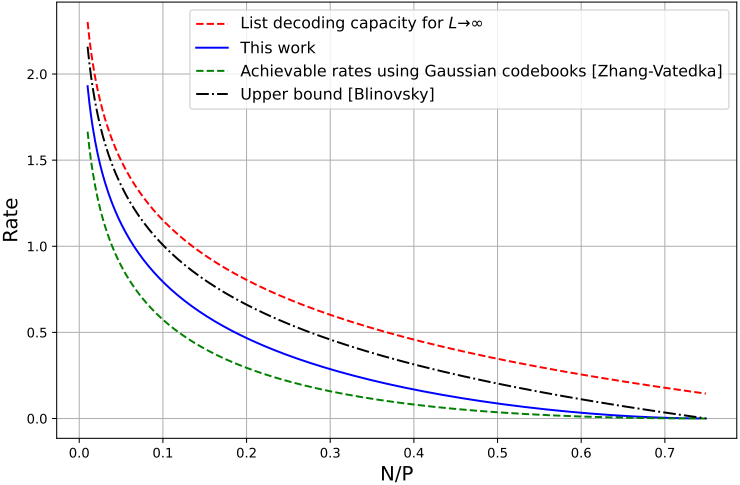
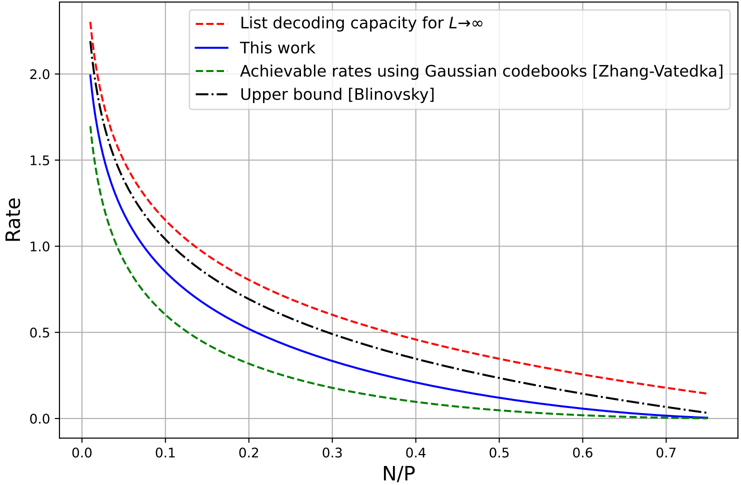
Bounds on $C_5(P, N)$



Bounds on $C_6(P, N)$



Ideas

Key tool: identifying a connection between adversarial list decoding and list decoding over Gaussian channels.

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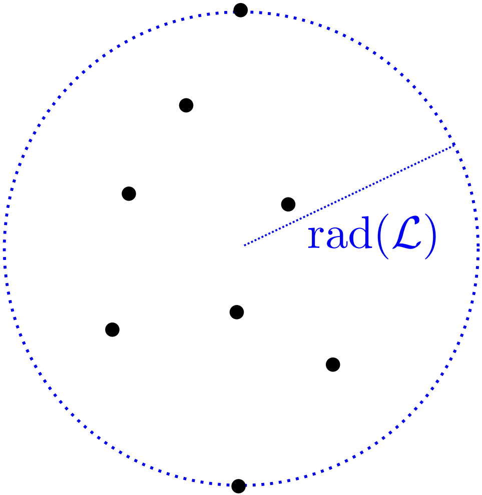
Good codes for (error exponents of) AWGN channels yield good (adversarial) list decodable codes! (with expurgation)

Careful analysis of higher-order Voronoi regions

Useful connection



Chebyshev radius: For any $\mathcal{L} \subset \mathbb{R}^n$, the Chebyshev radius $\mathrm{rad}(\mathcal{L})$ is the radius of the smallest closed ball containing $\mathcal{L}$.

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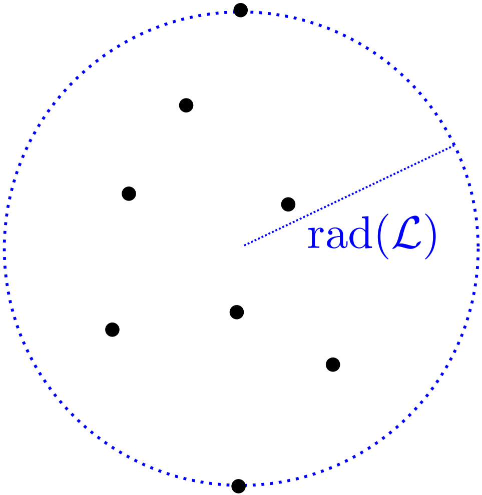

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A code $\mathcal{C}$ is $(P, N, L-1)$ list decodable iff it satisfies a power constraint of P, and every $\mathcal{L} \subset \mathcal{C}$ of size L has Chebyshev radius at least $\sqrt{nN}$.

Step 1: Key lemma

We show: for any code $\mathcal{C}$ of size M, there exists a subcode $\mathcal{C}'$ of size $M/2$ for which the following holds:

For every $\mathcal{L} \subset \mathcal{C}'$ with $|\mathcal{L}| = L$, we have

$$P^{ML}_{\mathrm{e,avg},L-1}(\mathcal{C}) \geqslant \exp\left(-\frac{\mathrm{rad}(\mathcal{L})}{2\sigma^2} - o(n)\right)$$

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or,

$$\mathrm{rad}(\mathcal{L}) \quad \geqslant \quad 2\sigma^2 \times \ln\left(\frac{1}{P^{ML}_{\mathrm{e,avg},L-1}(\mathcal{C})}\right) + o(n)$$

Connects Chebyshev radius of L-subsets of codes with error exponents for list decoding over Gaussian channels.

Step 2: Bounds on list decoding error exponent for Gaussian channels

Achievable expurgated error exponent: There exist sequences of codes for which

$$\frac{1}{n}\ln\frac{1}{P^{ML}_{\mathrm{e,avg},L-1}(\mathcal{C})} = E_{\mathrm{ex},L-1}(R) - o(1) = -\min_{s\geqslant 0,\rho\geqslant 1} F(s,\rho) - o(1)$$

where

$$F(s,\rho) := R(L-1)\rho - \rho\left[sLP + \frac{1}{2}\ln(1-2sP) + \frac{L-1}{2}\ln\left(1-2sP+\frac{P}{\sigma^2 L\rho}\right)\right].$$

Error exponents

- Proof: standard techniques [Gallager 1968]
- List decoding error exponents for **Gaussian channels**: [Gallager 1968, Merhav 2014][2]
- This work: more explicit expressions for the error exponent

[2] N. Merhav, "List decoding – random coding exponents and expurgated exponents," *IEEE Trans Inf Theory, 2014*

Achievable rate for adversarial list decoding

Set partial derivatives of $F(s,\rho)$ to 0. We get $\rho = \frac{L-1-2LPs}{2L^2 s(1-2Ps)\sigma^2}$, and

$$R = \frac{1}{2}\left[\ln \frac{(L-1)(1-2Ps)}{L(1-2Ps)-1} + \frac{1}{L-1}\ln(1-2Ps)\right].$$

Use this ρ, then

$$F(s,\rho) = -\frac{P(L(1-2Ps)-1)}{2L\sigma^2(1-2Ps)}$$

and

$$N \geqslant \frac{P(L(1-2Ps)-1)}{L(1-2Ps)}.$$

Choose the s that maximizes the r.h.s. above.

$$R = \frac{1}{2}\left[\ln \frac{(L-1)P}{LN} + \frac{1}{L-1}\ln \frac{P}{L(P-N)}\right].$$

Higher-order Voronoi regions

For any $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_L$, define

$$\mathcal{V}(\underline{x}_1, \ldots, \underline{x}_{L-1}) = \left\{ \underline{y} \in \mathbb{R}^n : \quad \|\underline{y} - \underline{x}_L\| > \|\underline{y} - \underline{x}_i\|,\ 1 \leqslant i \leqslant L-1 \right\}$$

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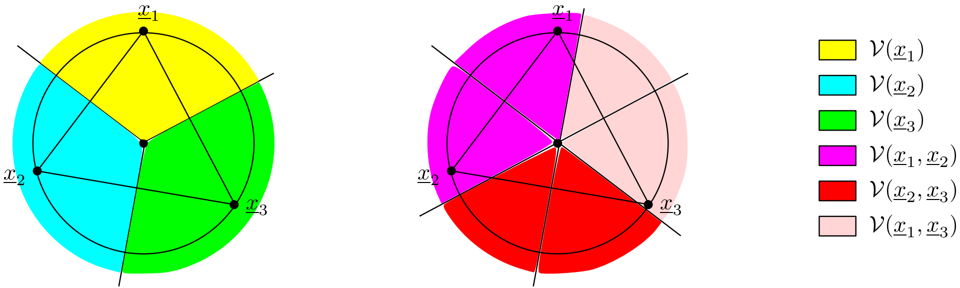
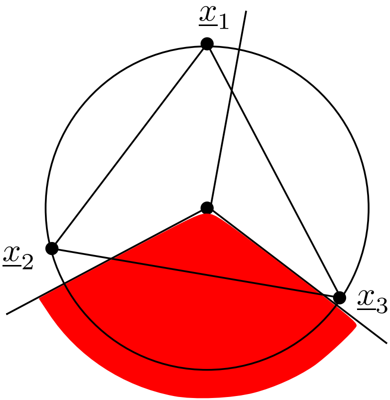


Probability of ML list-decoding error

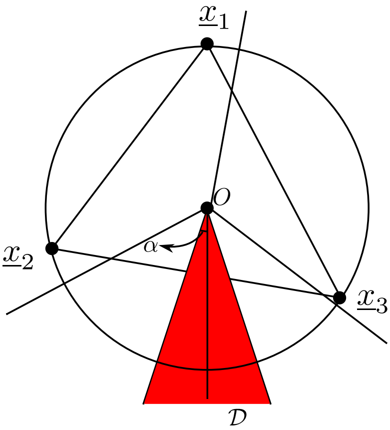
- When $\underline{x}_L$ is transmitted across AWGNC, ML decoder makes error if received vector lies in $\mathcal{V}(\underline{x}_1, \ldots, \underline{x}_{L-1})$
- Bound this probability from above
- But higher-order Voronoi regions are complicated!
- Idea: lower bound error probability using a simpler region

Lower bound error probability using simpler region



As long as minimum distance between codewords is $\Omega(\sqrt{n})$, higher order Voronoi region always contains a cone of certain radius.

Lower bound error probability using simpler region



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Use this to show

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Extensions: Unbounded packings

Infinite Constellations: Same setup, but no power constraint at the transmitter

"Rate": Normalized Logarithmic Density

$$R = \frac{1}{n} \limsup_{a \to \infty} \ln \left(\frac{|\mathcal{C} \cap [-a/2, a/2]^n|}{a^n} \right)$$

Extensions: Unbounded packings

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List decoding capacity: For $L = \omega(1)$,

$$C(N) = \frac{1}{2} \ln \frac{1}{2\pi e N}$$

Bounds for unbounded L-packings

Similar ideas yield

$$C_L(N) \geqslant \frac{1}{2} \ln\left(\frac{L}{2\pi e N(L+1)}\right) - \frac{\ln(L+1)}{2L}$$

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We can also show that

$$C_L(N) \leqslant \frac{1}{2} \ln \left(\frac{L}{2\pi e N(L+1)} \right)$$

Open questions

- Tighter upper/lower bounds on $C_L(P, N)$ and $C_L(N)$
- Improved bounds on list size for lattices and nested lattice codes
- Explicit codes/lattices that achieve list decoding capacity
- Connections of Euclidean list decoding to other problems

More details

- Yihan Zhang and Shashank Vatedka, “Multiple packing: Lower and upper bounds,” Arxiv, 2022
- Yihan Zhang and Shashank Vatedka, “Multiple packing: Lower bounds via infinite constellations,” IEEE Transactions on Information Theory, July 2023
- Yihan Zhang and Shashank Vatedka, “Multiple packing: Lower bounds using error exponents,” IEEE Transactions on Information Theory, Feb 2024
- Yihan Zhang and Shashank Vatedka, “List decoding random Euclidean codes and infinite constellations,” IEEE Transactions on Information Theory, Dec 2022

References

[1] N. Blachman, “On the capacity of bandlimited channel perturbed by statistically dependent interference,” *IRE Transactions on Information Theory*, vol. 8, pp. 48–55, 1962.

[2] G. A. Kabatiansky and V. I. Levenshtein, “On bounds for packings on a sphere and in space,” *Problemy Peredachi Informatsii*, vol. 14, no. 1, pp. 3–25, 1978.

[3] P. Elias, *List decoding for noisy channels*. Massachusetts Institute of Technology, Research Laboratory of Electronics, Cambridge, Mass., 1957, p. 12, Rep. No. 335.

[4] J. M. Wozencraft, “List decoding,” *Quarterly Progress Report*, vol. 48, pp. 90–95, 1958.

[5] M. Langberg, “Private codes or succinct random codes that are (almost) perfect,” in *Proc. IEEE Symp. Found. Comp. Sci.*, Rome, Italy, 2004.

[6] A. Sarwate, **"Robust and adaptive communication under uncertain interference,"** Ph.D. dissertation, University of California, Berkeley, 2008.

[7] S. Bhattacharya, A. J. Budkuley, and S. Jaggi, **"Shared randomness in arbitrarily varying channels,"** in *2019 IEEE International Symposium on Information Theory (ISIT)*, IEEE, 2019, pp. 627–631.

[8] Y. Zhang, S. Vatedka, S. Jaggi, and A. Sarwate, **"Quadratically constrained myopic adversarial channels,"** *arXiv preprint arXiv:1801.05951*, 2018.

[9] Y. Zhang, S. Vatedka, and S. Jaggi, **"Quadratically constrained two-way adversarial channels,"** *arXiv preprint arXiv:2001.02575*, 2020.

[10] Y. Zhang and S. Vatedka, **"List decoding random euclidean codes and infinite constellations,"** *arXiv preprint arXiv:1901.03790*, 2019.

[11] V Blinovsky, **"Multiple packing of the euclidean sphere,"** *IEEE Transactions on Information Theory*, vol. 45, no. 4, pp. 1334–1337, 1999.

[12] Y. Zhang and S. Vatedka, **“Bounds for multiple packing and list-decoding error exponents,”** *arXiv preprint arXiv:2107.05161*, 2021.

[13] R. G. Gallager, *Information Theory and Reliable Communication*. MIT Press, 1968.

[14] N. Merhav, **“List decoding—random coding exponents and expurgated exponents,”** *IEEE Transactions on Information Theory*, vol. 60, no. 11, pp. 6749–6759, 2014.