

Density functional study of elastic and vibrational properties of the Heusler-type alloys Fe_2VAl and Fe_2VGa

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The structural and elastic properties as well as phonon-dispersion relations of the Heusler-type alloys Fe_2VAl and Fe_2VGa are computed using density functional and density-functional perturbation theory within the generalized-gradient approximation. The calculated equilibrium lattice constants agree well with the experimental values. The elastic constants of Fe_2VAl and Fe_2VGa are predicted. From the elastic constants the shear modulus, Young's modulus, Poisson's ratio, sound velocities, and Debye temperatures are obtained. By analyzing the ratio between the bulk and shear moduli, we conclude that both Fe_2VAl and Fe_2VGa are brittle in nature. The computed phonon-dispersion relation shows that both compounds are dynamically stable in the $L2_1$ structure without any imaginary phonon frequencies. The isomer shifts in Fe in the two compounds are discussed in terms of the Fe s partial density of states, which reveal larger ionicity/less hybridization in Fe_2VGa than in Fe_2VAl . For the same reason the Cauchy pressure is negative in Fe_2VAl but positive in Fe_2VGa .

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I. INTRODUCTION

The Heusler-type intermetallic compounds Fe_2VAl and Fe_2VGa have recently attracted great attention due to their fascinating thermal, electrical, magnetic, and transport properties,¹⁻⁴ not only from the basic science perspectives but also from the potential application as thermoelectric materials.⁴⁻⁶ Though Heusler-type intermetallics commonly appear as metals,⁷ semiconducting behavior has been observed in Fe_2VAl and Fe_2VGa , as evidenced by their negative temperature coefficient of resistivity.¹ This feature has been attributed to the appearance of a pseudo gap in the density of states in the vicinity of the Fermi level, and these materials have been characterized as semimetals. Nuclear magnetic resonance experiments and optical conductivity measurements further confirmed the existence of deep pseudo gaps near the Fermi energy (E_F) in both compounds.^{8,9} Recently, it has been reported by Lue *et al.*¹⁰ that the partial substitution of Ga by Ge in Fe_2VGa effectively dopes electrons to the system thereby leading to a dramatic decrease in the electrical resistivity. A similar study for Fe_2VAl was carried out by Nishino *et al.*⁵ with the same conclusion. The variation of the Seebeck coefficient with a sign change from positive to negative can be understood by means of a rigid band shifting in the Fermi level across the pseudogap.

Several band structure calculations have been reported providing various insights into Fe_2VAl and Fe_2VGa , all confirming that Fe_2VAl is a paramagnetic semimetal with a pronounced pseudogap at the Fermi level.¹¹⁻¹⁷ The calculated pseudogap seems however too wide to explain the experimental data. Guo *et al.*¹¹ suggested the negative temperature coefficient of resistance to arise from carrier localization,

while Singh and Mazin¹² pointed toward the interaction of carriers with fluctuating magnetic moments, and Weht and Pickett¹³ proposed dynamic correlations between holes and electrons to be responsible for the unusual resistivity behavior. The importance of magnetic moment formation in off-stoichiometric compounds have been confirmed experimentally,¹⁸ and recent theoretical studies of the $\text{Fe}_{2+x}\text{V}_{1-x}\text{Z}$ ($Z=\text{Al, Ga}$) compounds^{19,20} with a supercell approach predicted that antisite defects or excess atoms in the Heusler lattice may induce a radical change in the electronic and magnetic properties. Similarly, Antonov *et al.*²¹ studied the electronic structure and x-ray magnetic circular dichroism in $\text{Fe}_{2-x}\text{V}_{1+x}\text{Al}$ for various x using first-principles calculations, which confirm the formation of magnetic moments on vanadium impurities.

Alloys based on Fe_2VAl are good candidates for thermoelectric materials.⁴⁻⁶ Although they have a high thermoelectric power, because of their high electrical conductivities and high Seebeck coefficients, they have poor figures of merit compared with other state-of-the-art thermoelectric materials due to their high thermal conductivity, κ . Hence reduction of κ is required for practical applications,⁶ which may be achieved by substitution of Al by a heavier element.⁵ Large experimental efforts have been devoted to characterization of Fe_2VAl based materials,²² including x-ray absorption²³ and photoemission^{24,25} to reveal their surface and bulk electronic structure and electrical and thermal properties.²⁶⁻²⁸ Yet, the lattice dynamics and mechanical properties have not been explored, which is taken as the objective of the present work. Here we present results of computations of the elastic constants and phonon-dispersion relations for Fe_2VAl and Fe_2VGa using density functional and density-functional perturbation theory within the generalized-gradient approxima-

tion. From the modulus of elasticity, we predict the mechanical behavior of these compounds.

II. COMPUTATIONAL DETAILS

The $L2_1$ crystal structure of Heusler alloys has space group $Fm\bar{3}m$ (No. 225). The Bravais lattice is face-centered cubic with four atoms in the unit cell: two Fe atoms in positions $\pm(1/4, 1/4, 1/4)a$, a V atom at $(0,0,0)$, and an Al or Ga atom in position $(1/2, 1/2, 1/2)a$, where a designates the cubic lattice constant.

The all-electron linear muffin-tin orbital method²⁹ in a full-potential (FP-LMTO) implementation³⁰ is used to calculate the total energies, and basic ground-state and elastic properties of Fe_2VAl and Fe_2VGa . In this method, the crystal volume is split into two regions: nonoverlapping muffin-tin spheres surrounding each atom and the interstitial region between the spheres. We used a double κ spdf LMTO basis to describe the valence bands, i.e., Hankel functions characterized by decay constants κ are smoothly augmented with numerical radial functions within the atomic spheres. The calculations included the $4s$, $4p$, $3p$, and $3d$ basis functions for iron and vanadium, the $3s$, $3p$, and $3d$ basis for aluminum, and the $4s$, $4p$, and $3d$ basis for gallium. The exchange correlation potential was calculated within the generalized-gradient approximation (GGA) scheme.³¹ Inside the muffin-tin spheres, the charge density and potential were expanded in terms of spherical harmonics up to $l_{\text{max}}=6$, while in the interstitial region, they were expanded in plane waves, with 12 050 waves (energy up to 171 Ry) included in the calculation. The energy curves were fitted to a second-order Birch equation of state³² to obtain the ground-state properties. The elastic constants were obtained from the variation of the total energy under volume-conserving strains, as outlined in Ref. 33. To obtain well converged total energies and elastic constants we used a $(28 \times 28 \times 28)$ Monkhorst-Pack³⁴ k mesh, which for $Fm\bar{3}m$ leads to 624 k points in the irreducible wedge of the Brillouin zone.

In order to calculate the vibrational properties of the Heusler alloys we used the density functional perturbation theory as implemented in the plane-wave pseudopotential method, through the QUANTUM-ESPRESSO package.³⁵ We have used the Troullier-Martins norm-conserving pseudopotentials.³⁶ In

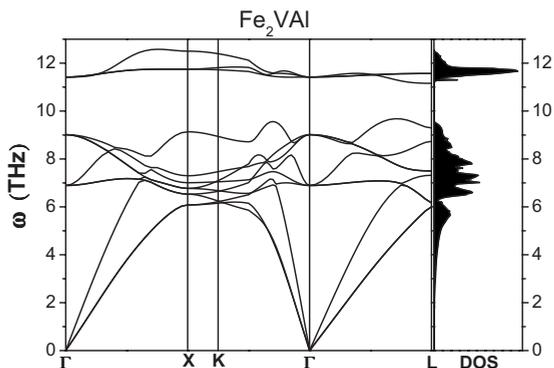


FIG. 1. Calculated phonon dispersion curves and phonon density of states of Fe_2VAl

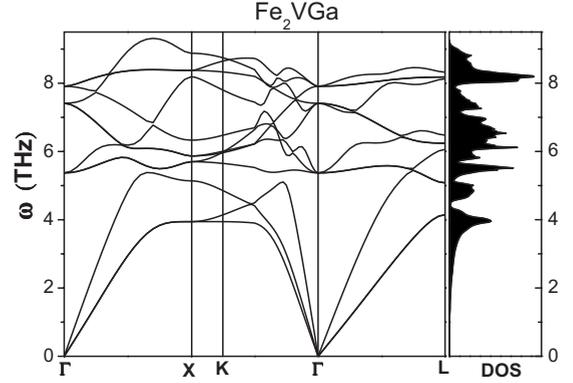


FIG. 2. Calculated phonon dispersion curves and phonon density of states of Fe_2VGa

this case, convergence tests (phonon frequencies stable within 0.05 THz) lead to the choice of kinetic energy cutoffs of 80 Ry, and a $(8 \times 8 \times 8)$ Monkhorst-Pack grid of k points for the Brillouin zone integration.

III. PHONONS

The calculated phonon dispersion curves of Fe_2VAl and Fe_2VGa are presented in Figs. 1 and 2, respectively. We do not find any imaginary phonon frequency in the whole Brillouin zone for the two compounds. This supports the dynamical stability of both compounds in the Heusler structure, which is not *a priori* evident in view of instabilities found in similar alloys, e.g., Ni_2MnAl and Ni_2MnGa .^{37,38} It is interesting to note that the optical phonons for Fe_2VGa are coupled, falling in a broad frequency range between 5.2 and 9.1 THz. This is in contrast to Fe_2VAl , where three optical branches are separated from the lower frequency phonons. Inspection of the atomic masses of Fe, V, Ga, and Al allows us to understand this difference between the two compounds. The similar atomic masses of Fe, V, and Ga lead to the coupled phonon dispersions, while the significantly lower atomic mass of Al results in the phonon separation. The three highest optical phonon modes of Fe_2VAl have their major amplitude contribution on Al atoms. At the Γ point there are three threefold degenerate optical modes, since there are four

TABLE I. Calculated lattice constants in \AA , bulk modulus in GPa, and its pressure derivative B' of Fe_2VAl and Fe_2VGa at the theoretical equilibrium volume.

Compound		Lattice constant	Bulk modulus	B'
Fe_2VAl	Expt.	5.76 ^a		
	Theory, this work	5.712	220.8	5.3
	Other theory	5.712 ^a	212 ^a	
Fe_2VGa	Expt.	5.77 ^a		
	Theory, this work	5.727	228.5	4.9
	Other theory	5.726 ^a	214 ^a	

^aReference 23.

TABLE II. Calculated elastic constants, shear modulus (G), and Young's modulus (E) all expressed in GPa, and Poisson's ratio ν for Fe₂VAl and Fe₂VGa at the theoretical equilibrium volume

	C_{11}	C_{12}	C_{44}	G	E	ν
Fe ₂ VAl	415.7	125.3	170.7	160.5	387.6	0.208
Fe ₂ VGa	363.1	161.0	152.3	131.8	331.7	0.258

atoms in the unit cell, and there is no splitting between the transversal and longitudinal modes in these semimetallic compounds.

The lattice thermal conductivity is inversely proportional to the sound velocity,³⁹ so assuming similar phonon scattering properties (relaxation times) for Fe₂VAl and Fe₂VGa, the former compound thus should have the smaller thermal conductivity, which might be an observation of relevance in efforts of minimization of thermal conductivity for good thermoelectric figures of merits. Note that in the study of Ge substitution for Al in Fe₂VAl in Ref. 5 the opposite trend was found, i.e., κ is reduced when the heavier Ge substitutes for Al, but this was attributed to the change in phonon relaxation time due to the incorporation of impurities with a large mass difference.

IV. GROUND-STATE AND ELASTIC PROPERTIES

The calculated equilibrium lattice constants and the bulk moduli along with results of experimental and other theory work are given in Table I. The lattice constant obtained from the present calculation is underestimated by 0.8% for Fe₂VAl and 0.7% for Fe₂VGa when compared with the experimental value. The present results on lattice constant and bulk modulus agree quite well with the recent full potential linear augmented plane wave calculations.²³ The calculated elastic constants of Fe₂VAl and Fe₂VGa are presented in Table II. This is the first theoretical prediction of the elastic constants, which still awaits experimental confirmation. It is noticeable that the Cauchy pressure, $C_p \equiv C_{12} - C_{44}$ is negative for Fe₂VAl ($C_p = -45$ GPa) but positive for Fe₂VGa ($C_p = +9$ GPa). According to Pettifor⁴⁰ this reflects a

more covalent character of bonding in Fe₂VAl. A simple relationship, empirically linking the plastic properties of materials with their elastic moduli has been proposed by Pugh.⁴¹ The shear modulus G represents the resistance to plastic deformation, while the bulk modulus B represents the resistance to fracture. A high B/G ratio may then be associated with ductility, whereas a low value would correspond to a more brittle nature. The critical value separating ductile and brittle materials is around 1.75 according to Ref. 41. In the case of Fe₂VAl the value of B/G is 1.37 and for Fe₂VGa the value is 1.73 from our calculated values in Table II, and therefore Fe₂VAl can be classified as a brittle material, while Fe₂VGa resides on the borderline between brittle and ductile. Frantsevich⁴² has a similar criterion based on the Poisson ratio, namely, $\nu < 1/3$ ($\nu > 1/3$) for brittle (ductile) character, hence categorizing both compounds as brittle (see Table II). The difference merely reflects the imprecise nature of the concept.⁴³

The sound velocities of Fe₂VAl and Fe₂VGa may be derived from the calculated elastic constants³³ (see Table III). The properties of the two compounds are quite similar apart from the heavier mass of Ga compared to Al, which is the dominating factor leading to smaller sound velocities in Fe₂VGa compared to Fe₂VAl. The Debye temperature may be estimated from the simple isotropic approximations, $k_B \Theta_D = \hbar v_m k_D$, where k_B and k_D are the Boltzmann constant and the Debye vector, respectively, and v_m the average sound velocity. Hence, the smaller sound velocity in Fe₂VGa directly leads to a smaller Debye temperature. The sound velocities may also be read from the acoustic phonon branches in Figs. 1 and 2. They are listed in the Table as well. One notices quite some anisotropy, the sound velocities along

TABLE III. Calculated longitudinal, shear, and average wave velocity (v_l , v_s , and v_m , respectively) in km/s, and the Debye temperature, θ_D , in Kelvin from the average elastic wave velocity for Fe₂VAl and Fe₂VGa at the theoretical equilibrium volume. Also listed are the directional sound velocities read from the acoustic phonon branches along directions [100], [111], and [110].

Compound		v_l	v_s	v_m	θ_D
Fe ₂ VAl	Isotropic	8.13	4.93	5.45	447
	[100]	7.49	4.44		
	[111]	7.72	4.46		
	[110]	10.23	6.41		
Fe ₂ VGa	Isotropic	7.09	4.05	4.50	369
	[100]	6.29	3.52		
	[111]	6.26	3.13		
	[110]	8.15	4.52, 4.19		

TABLE IV. Calculated and measured isomer shifts of Fe₂VAl and Fe₂VGa, in mm/s relative to α -Fe. Also listed are the partial charges inside the Fe muffin-tin sphere (radius $R=2.305$ a.u.). The Fe s charge within the lowest s -like band is listed quoted in parentheses (see text).

	Fe ₂ VAl	Fe ₂ VGa
I.S. theor.	+0.021	+0.121
I.S. expt. ^a	+0.058(5)	+0.161(5)
Fe n_s	0.394(0.140)	0.370(0.111)
n_p	0.437	0.419
n_d	6.112	6.121
n_f	0.015	0.023

^aReference 44.

[110] being $\sim 30\%$ larger than along the [100] and [111] directions.

The isomer shift in Fe in Fe₂VAl and Fe₂VGa have been measured by Ref. 44. The isomer shift is proportional to the difference in electron contact density, $\rho(0)$, between the sample and reference materials,

$$I.S. = \alpha[\rho(0) - \rho_{ref}(0)].$$

We calculated the electron contact density of Fe in the Fe₂VAl and Fe₂VGa compounds and compared to that of α -Fe. Using the calibration constant $\alpha = -0.22a_0^3$ mm/s (Ref. 45), this leads to the isomer shifts listed in Table IV. The difference in contact density is quite small, being larger in Fe₂VAl than in Fe₂VGa by $0.45a_0^{-3}$. Thus the isomer shift in the latter compound is larger by 0.10 mm/s, in excellent agreement with the experimental finding. However, the calculated isomer shifts relative to α -Fe are about 0.04 mm/s smaller than measured, which could be a temperature related effect, since the experiments are conducted at room temperature, while our theory pertains to 0 K. The origin of the different value for the isomer shift in Fe₂VAl and Fe₂VGa is the difference in electronegativity of Al and Ga. Although the electronic structures of these compounds are very similar,¹⁵ the greater electronegativity of Ga compared to Al transcripts into a deeper position of the Ga-dominated deepest s band in Fe₂VGa compared to the position of the equivalent Al s band in Fe₂VAl. This leads to reduced hybridization of the Fe $4s$ states with this band in Fe₂VGa compared to Fe₂VAl. To illustrate this, we plot in Fig. 3 the partial density of states of Fe $4s$ character for the two compounds. This is the relevant quantity to consider, since the dominant contribution to the electron contact density *variations* in solids stems from the valence s electrons (only s states and relativistic $p_{1/2}$ states extend their amplitude into the nuclear region). Thus, in Fig. 3 the relevant s band is the lower part of the density of states

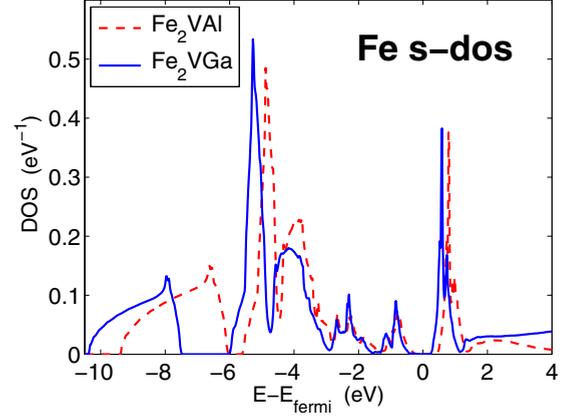


FIG. 3. (Color online) Calculated Fe $4s$ partial density of states (in units of electrons per eV and per Fe atom) in Fe₂VAl (dashed and red) and Fe₂VGa (full line and blue). The zero of energy is placed at the Fermi level, which falls in the pseudo gap within the Fe and V d bands. The full density of states for these compounds have been discussed in Refs. 12–14.

(between -10.3 and -7.5 eV in Fe₂VGa, between -9.3 and -6 eV in Fe₂VAl, relative to the Fermi level). The reduced hybridization of the Fe $4s$ states with this band leads to the smaller bandwidth in Fe₂VGa compared to Fe₂VAl, as well as the fewer integrated number of Fe $4s$ electrons for this band (Table IV), which almost exactly reflects the difference in isomer shifts. In contrast, the upper part of the Fe $4s$ partial density of states in Fig. 3, which describes the hybridization of Fe $4s$ states with the Fe and V d bands, is much more similar in the two cases, and also integrates to almost the same number of Fe $4s$ charge. The larger hybridization in Fe₂VAl compared to Fe₂VGa also explains the difference in sign of the Poisson ratio, discussed above.

V. CONCLUSIONS

Ab initio electronic structure and phonon frequency calculations based on the density functional theory have been presented for the Heusler alloys Fe₂VAl and Fe₂VGa. The elastic constants have been obtained, while no experiments exist for comparison. The differences in phonon dispersions for the two compounds can be explained by the heavier mass of Ga compared to Al, while the difference in Fe isomer shift is attributed to the higher electronegativity of Ga compared to Al, the ensuing reduced covalency in Fe₂VGa compared to Fe₂VAl also explaining the difference in Cauchy pressure.

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