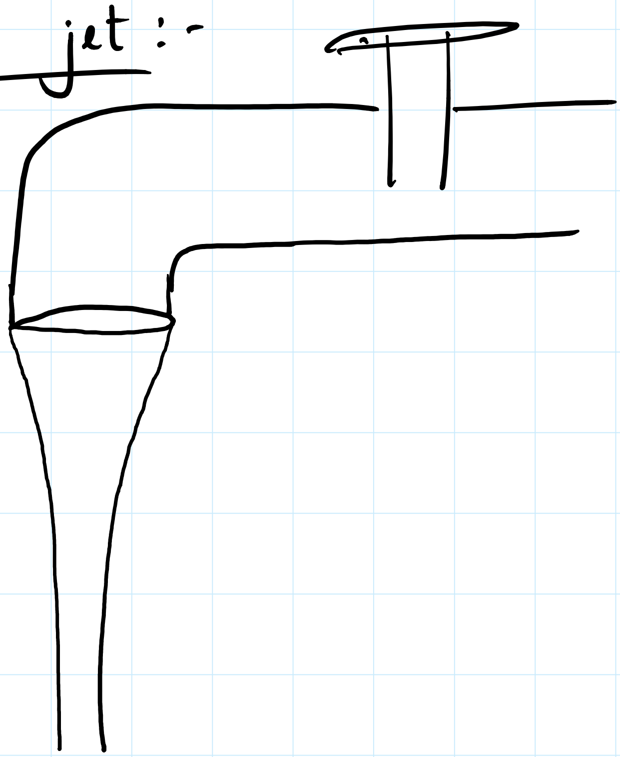
Shape of a falling jet:-



If $V \sim 1\ m/sec$

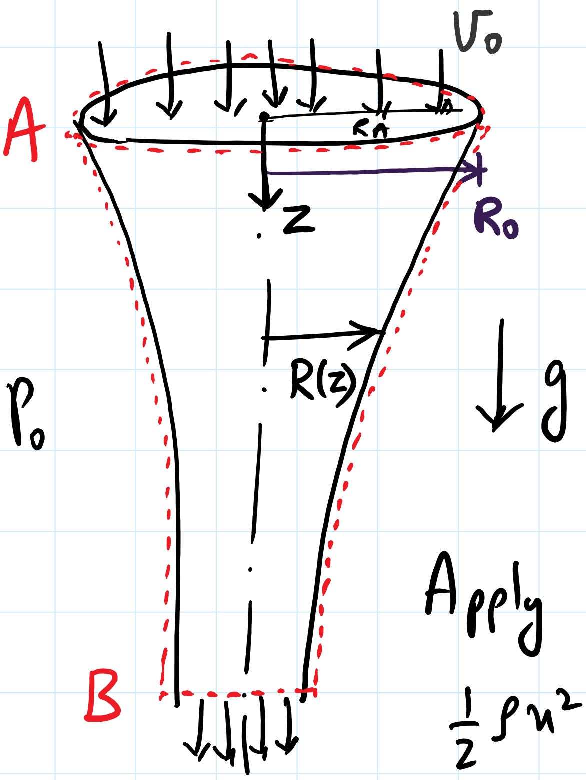
$R_0 \sim 2\ mm$

$\nu \sim 10^{-6}\ m^2/sec$

$$Re = \frac{V R_0}{\nu} \sim 2000$$

Since $Re \gg 1$, it is safe to ignore viscosity.

Now what we have is an inviscid jet.



Goal: To determine $R(z)$! (Shape of the jet)

We can use:
(i) Continuity equation

Apply Bernoulli's eqn. between A & B:-

$$\frac{1}{2}\rho u^2 + \rho g z + P = \text{constant}$$

$$\frac{1}{2}\rho V_A^2 + \rho g z + P_A = \frac{1}{2}\rho U_B^2 + P_B$$

$$P_A - P_0 = \frac{\sigma}{R_A} \Rightarrow P_A = P_0 + \frac{\sigma}{R_A}$$

$$P_B - P_0 = \frac{\sigma}{R_B} \Rightarrow P_B = P_0 + \frac{\sigma}{R_B}$$

$$\Rightarrow \frac{1}{2}\rho V_0^2 + \rho g z + \cancel{P_0} + \frac{\sigma}{R_0} = \frac{1}{2}\rho V(z)^2 + \cancel{P_0} + \frac{\sigma}{R(z)}$$

$$\Rightarrow \frac{1}{2}\rho\left(V^2 - V_0^2\right) = \rho g z + \sigma\left(\frac{1}{R_0} - \frac{1}{R}\right)$$

$$\frac{1}{2}\rho V^2 = \frac{1}{2}\rho V_0^2 + \rho g z + \sigma\left(\frac{1}{R_0} - \frac{1}{R}\right)$$

$$\Rightarrow \frac{V(z)^2}{V_0^2} = 1 + \frac{2gz}{V_0^2} + \frac{2\sigma}{\rho V_0^2}\left(\frac{1}{R_0} - \frac{1}{R}\right)$$

$$\frac{V(z)}{V_0} = \left[1 + \frac{2gz}{V_0^2} + \frac{2\sigma}{\rho V_0^2}\frac{1}{R_0}\left(1 - \frac{R_0}{R}\right)\right]^{1/2}$$

Defining $Fr = \dfrac{V_0^2}{g R_0}$ = Froude number

& $We = \dfrac{\rho V_0^2 R_0}{\sigma}$ = Weber number

$$\Rightarrow \boxed{\frac{V(z)}{V_0} = \left[1 + \frac{2z}{R_0}\cdot\frac{1}{Fr} + \frac{2}{We}\left(1 - \frac{R_0}{R}\right)\right]^{1/2}}$$

Mass-balance equation:-

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$$\dot{m}_{inlet} = \dot{m}_{outlet}$$

$$\Rightarrow \cancel{\rho \cdot} \pi R_0^2 \cdot V_0 = \cancel{\rho \cdot} \pi R(z)^2 \cdot V(z)$$

$$\Rightarrow \frac{R(z)}{R_0} = \left(\frac{V_0}{V(z)}\right)^{1/2} = \left(\frac{V(z)}{V_0}\right)^{-1/2}$$

$$\boxed{\frac{R(z)}{R_0} = \left[1 + \frac{2z}{R_0} \cdot \frac{1}{Fr} + \frac{2}{We}\left(1 - \frac{R_0}{R(z)}\right)\right]^{-1/4}}$$

Use
$V_0 = 1$ m/sec
$g = 10$ m/sec^2
$R_0 = 5$ mm
$\sigma = 0.07$ N/m
$\rho = 1000$ kg/m^3

$Fr \approx 20$

$We \approx 71$

$$\frac{R_0^4}{R^4} = 1 + \frac{2z}{R_0}\frac{1}{Fr} + \frac{2}{We}\left(1 - \frac{R_0}{R}\right)$$

$$R_0^4 = R^4 + \frac{2z}{R_0 \cdot Fr} \cdot R^4 + \frac{2}{We} \cdot R^4 - \frac{2}{We} \cdot R_0 \cdot R^3$$

$$\left(1 + \frac{2z}{R_0 \cdot Fr} + \frac{2}{We}\right) R^4 + \left(\frac{-2}{We} \cdot R_0\right) R^3 - R_0^4 = 0$$

$$aR^4 + bR^3 + c = 0$$

If $\underline{\underline{We \to \infty}}$:

$$a' R^4 + c = 0$$

$$R^4 = -\frac{c}{a'} = -\frac{(-R_0^4)}{\left(1 + \frac{2z}{R_0} \cdot Fr\right)}$$

$$\Rightarrow \quad R^4 = R_0^4 \left(1 + \frac{2z}{R_0} \cdot \frac{1}{Fr}\right)^{-1}$$

$$\Rightarrow \quad \boxed{\frac{R(z)}{R_0} = \left(1 + \frac{2z}{R_0} \cdot \frac{1}{Fr}\right)^{-1/4}}$$

$$\& \quad \frac{V(z)}{V_0} = \left(1 + \frac{2z}{R_0} \cdot \frac{1}{Fr}\right)^{1/2}$$

Radius decreases without surface tension, hence fluid accelerates more " " " . This is because $\frac{2}{We}\left(1 - \frac{R_0}{R}\right) < 0$ always.

If $\underline{\underline{Fr = \infty}}$:- Since $\frac{2}{We}\left(1 - \frac{R_0}{R}\right) < 0$,

$$\left(1 + \frac{2}{We}\right) R^4 + \left(-\frac{2}{We} R_0\right) R^3 - R_0^4 = 0$$

Let $\frac{1}{We} = \epsilon$

$$(1+2\epsilon)R^4 - 2R_0\epsilon R^3 - R_0^4 = 0$$

$$\boxed{(1+2\epsilon)y^4 - 2\epsilon y^3 - 1 = 0} \quad \text{when } y = \frac{R}{R_0}$$

Let $y = y_0 + \epsilon y_1 + \epsilon^2 y_2 + \cdots$

$$(1+2\epsilon)\left[y_0^4 + 4\epsilon y_0^3 y_1 + \cdots\right] - 2\epsilon\left[y_0^3 + 3\epsilon y_0^2 y_1 + \cdots\right] - 1 = 0$$

$\underline{O(1)}$: $y_0^4 - 1 = 0 \Rightarrow y_0 = 1$

$\underline{O(\epsilon)}$: $2y_0^4 - 4y_0^3 y_1 - 2y_0^3 = 0$

$$\Rightarrow 2 - 4y_1 - 2 = 0 \Rightarrow y_1 = 0$$