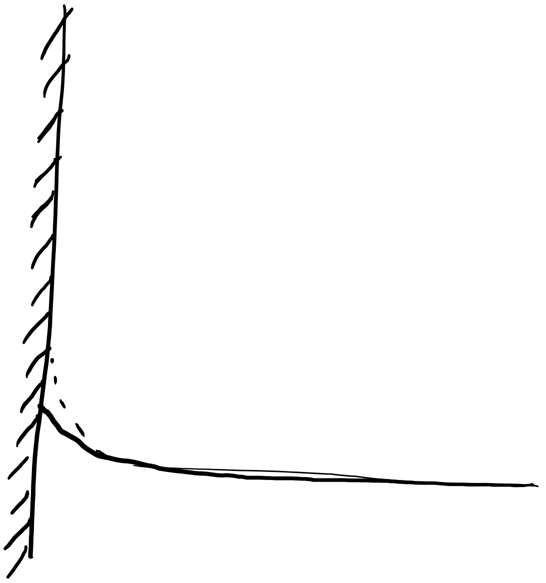
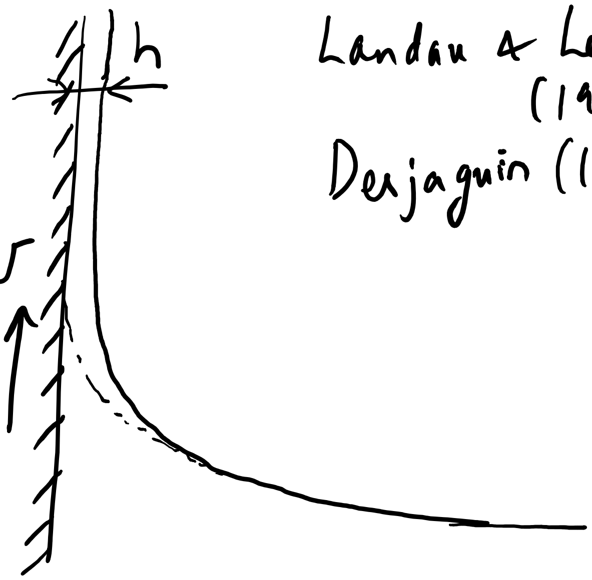
Forced Wetting :-





Landau & Levich (1942)
Derjaguin (1943)

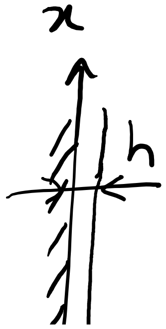
Parameters: $U, \mu, \sigma, g, \Delta\rho, h$

$$h = f(U, \mu, \sigma, g, \Delta\rho)$$

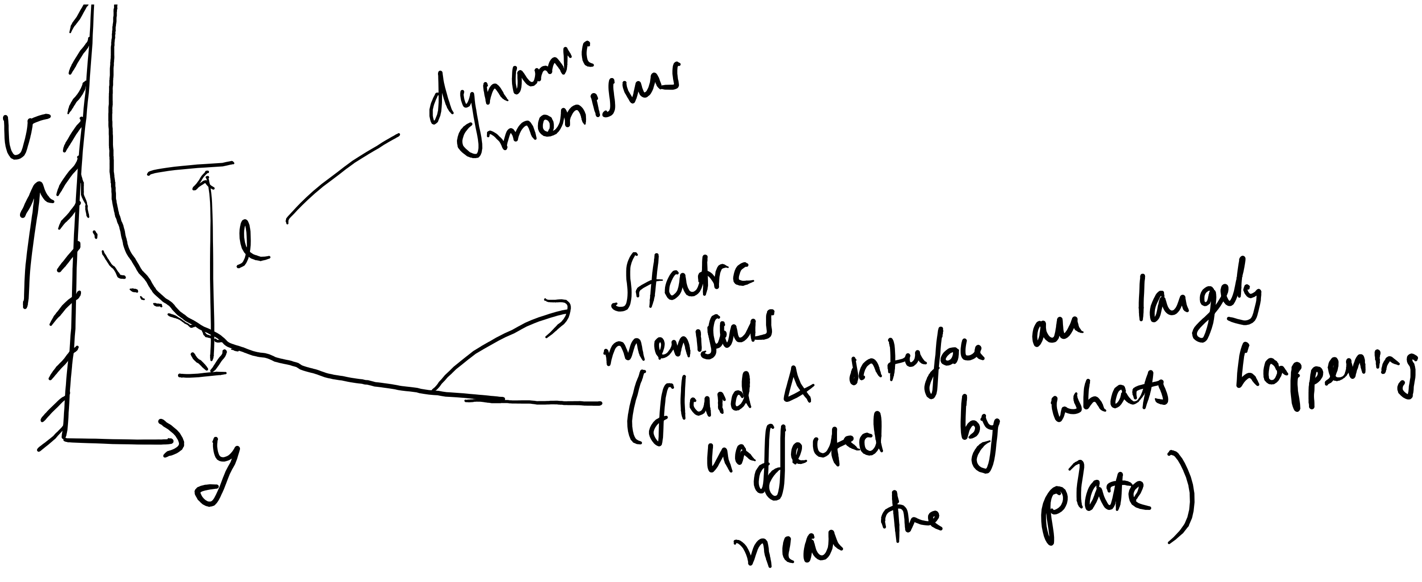
(or) $$\frac{h}{l_c} = g(Re, Ca)$$

We will look for films which are thin such that inertia is not important.

Then $$\boxed{\frac{h}{l_c} = g(Ca)}$$



dynamic



Viscous forces in the dynamic meniscus region is approximately : $\dfrac{\mu \cdot V}{h^2}$

Curvature forces scales as : $\dfrac{\sigma \cdot l_c^{-1}}{l}$

In this dynamic meniscus region, we expect viscous ~ curvature force.

$$\boxed{\frac{\mu V}{h^2} \sim \frac{\sigma \cdot l_c^{-1}}{l}}$$: h, l are unknowns

We need another relation to determine h & l:

match

To obtain the second equation, we match the static & the dynamic meniscus.

If $\frac{dh}{dx} << 1$, we can used the so called "lubrication approximation".

$$K_{dynamic} \approx \frac{d^2h}{dx^2} \sim O\left(\frac{h}{l^2}\right)$$

Curvature of static meniscus $\sim O\left(\frac{1}{l_c}\right)$

Matching Condition:-

$$\boxed{\frac{h}{l^2} \sim \frac{1}{l_c}}$$

We now have two equations with two unknowns:-

$$\frac{\mu U}{h^2} \sim \frac{\sigma}{l \cdot l_c} \quad \text{—— (i)}$$

$$\frac{h}{l^2} \sim \frac{1}{l_c} \quad \text{—— (ii)}$$

Eliminating l, we get :-

from (iii), $l^2 \sim h l_c$ or $l \sim h^{1/2} l_c^{1/2}$

$$\Rightarrow \frac{\mu V}{h^2} \sim \frac{\sigma}{h^{1/2} l_c^{1/2} \cdot l_c}$$

$$\Rightarrow \frac{\mu V}{\sigma} \cdot l_c^{1/2} \cdot l_c \sim h^{2-\frac{1}{2}}$$

$$\Rightarrow \left(\frac{\mu V}{\sigma}\right) l_c^{3/2} \sim h^{3/2}$$

$$\text{or} \quad \left(\frac{h}{l_c}\right)^{3/2} \sim \underbrace{\left(\frac{\mu V}{\sigma}\right)}_{Ca}$$

$$\text{or} \quad \boxed{\frac{h}{l_c} \sim Ca^{2/3}}$$

If we do a more rigorous theory, we find

$$\boxed{\frac{h}{l_c} = 0.9458\, Ca^{2/3}} \longrightarrow \text{Landau-Levich Law}$$

Length of the dynamic meniscus,

$$l \sim h^{1/2} l_c^{1/2}$$

$$\sim l_c^{1/2} \left\{ l_c^{1/2} Ca^{1/3} \right\}$$

$$\Rightarrow \boxed{l \sim l_c Ca^{1/3}}$$

For the above analysis to hold

$$Ca << 1.$$

More strictly, $Ca^{1/3} << 1$

For higher speeds, we get thicker films for which gravity cannot be ignored.. If gravity dominates surface tension, then we need to modify our force balance:-

Balancing viscous forces with gravity :-

$$\frac{\mu U}{h^2} \sim \rho g$$

$$\Rightarrow \quad h^2 \sim \frac{\mu V}{\rho g} \times \frac{\sigma}{\sigma}$$

$$\Rightarrow \quad h^2 \sim l_c^2 \cdot Ca$$

or $\boxed{\dfrac{h}{l_c} \sim Ca^{1/2}}$ → Derjaguin's Law