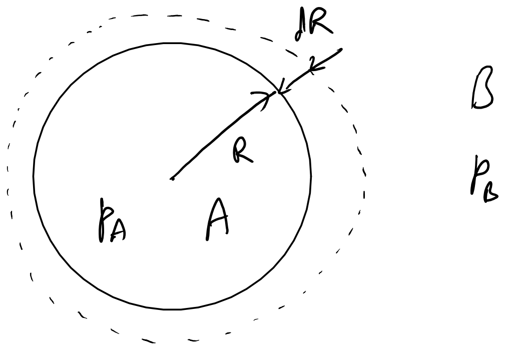
The notion of Laplace Pressure :-

Consider a drop of fluid A in B.



Work is required to raise the size of the drop from R to $R+dR$

$$dW = -p_A\, dV_A - p_B\, dV_B + \gamma_{AB}\, dA$$

$$dV_A \approx 4\pi R^2\, dR = -dV_B$$

$$dA = 4\pi (R+dR)^2 - 4\pi R^2$$
$$= 8\pi R\, dR$$

$$\Rightarrow dW = -p_A \cdot 4\pi R^2\, dR + p_B \cdot 4\pi R^2\, dR + \gamma_{AB} \cdot 8\pi R\, dR$$
$$= -(p_A - p_B)\, 4\pi R^2\, dR + \gamma_{AB} \cdot 8\pi R\, dR$$

For mechanical equilibrium, $dW = 0$

$$\Rightarrow \quad p_A - p_B = \frac{2\gamma_{AB}}{R}$$

or $$\boxed{\Delta P = \frac{2\gamma}{R}}$$

For a soap bubble in air, there are two

For a soap bubble in air, there are two soap-air interfaces $\Rightarrow$ $\Delta P = \dfrac{4\gamma}{R}$ or $\dfrac{4\sigma}{R}$

Ex: $\sigma = 35\ \dfrac{mN}{m}$, $R = 0.5\ cm$,

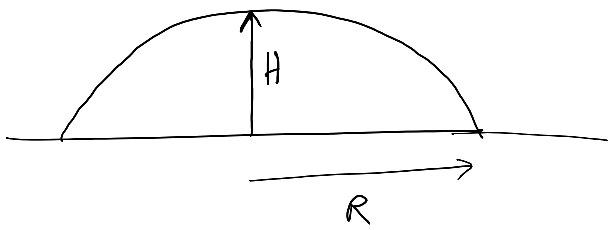
$$\Delta P = \frac{4 \times 35 \times 10^{-3}}{0.5 \times 10^{-2}} = 28\ \frac{N}{m^2} = 28\ Pa.$$

The above expression can be generalised to arbitrarily curved interfaces:

$$\Delta P = \sigma \underbrace{(\nabla \cdot \hat{n})}_{\text{curvature.}}$$

Wetting Phenomena:-

Balance the Laplace pressure with the hydrostatic pressure:-



$$\sigma \cdot \frac{1}{H} \sim \rho g H \Rightarrow H^2 \sim \frac{\sigma}{\rho g}$$

$$\sim l_c^2$$

$l_c = \sqrt{\dfrac{\sigma}{\rho g}}$: capillary length.

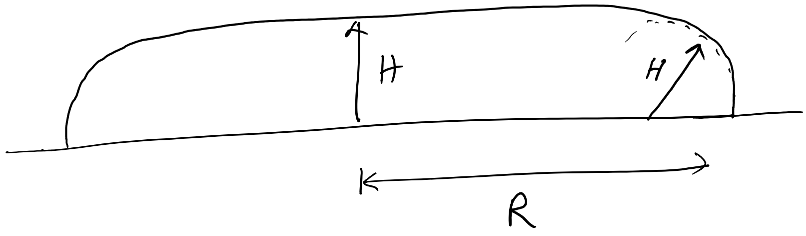
(i) If there is only one length scale,

$H \sim l_c \Rightarrow R \sim l_c$

If $H \leq l_c$, surface tension dominates and [spher]ical shape.

If ... the drops assume a spherical ...

(ii) If there is more than one length scale?



For large drops, drops spread till the height H becomes comparable to l_c.

Laplace + Hydrostatic pressure are in balance at the drop edge.

$$\Rightarrow \quad H \sim l_c$$

$$\text{Volume} \approx \pi R^2 \cdot H$$

$$\Rightarrow \quad R^2 \sim \frac{V}{\pi H} \quad \text{or} \quad \boxed{R \sim \sqrt{\frac{V}{\pi l_c}}}$$

For $V = 10\ ml$, $l_c = 2.7\ mm$,

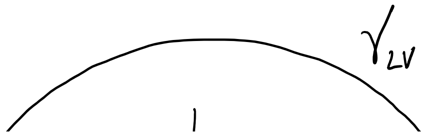
$$R \sim 34\ mm$$

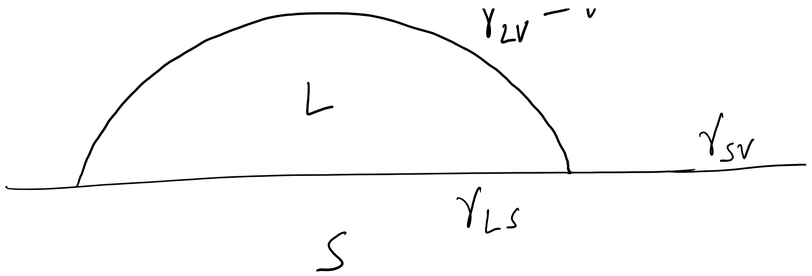
But in general, wetting properties on the fluid-solid surface cannot be ignored.

Two possibilities exist :- Total Wetting
or
Partial Wetting.

V

$\gamma_{LV} = \sigma$





Degree of wetting is determined by "Spreading parameter".

$$S = \left(E_{substrate}\right)_{dry} - \left(E_{substrate}\right)_{wet}$$

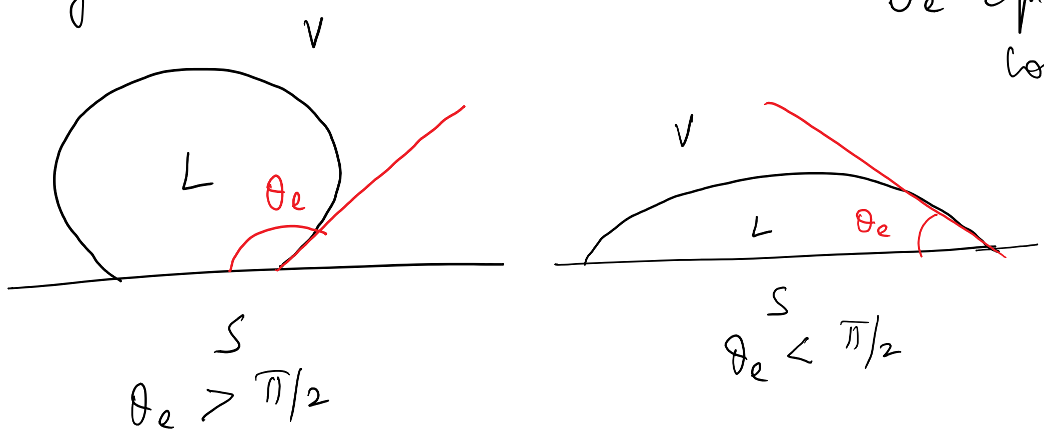
$$S = \gamma_{SV} - \left(\gamma_{SL} + \gamma_{LV}\right)$$

(i) If $S > 0$, complete wetting, i.e.; $\theta_e = 0$

Liquids wet the substrate resulting in a lower energy configuration : Total Wetting

(ii) If $S < 0$, $\theta_e > 0$

θ_e : Equilibrium contact angle.



If the liquid is water then :

(i) for $\theta_e > \pi/2$, surface is hydrophobic

(ii) for $\theta_e < \pi/2$; surface is hydrophilic

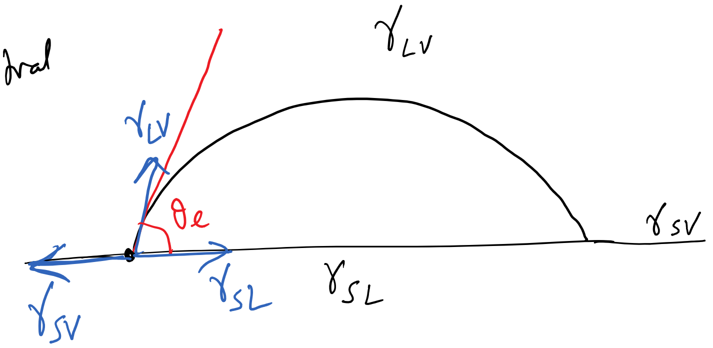
For water on a clean glass surface,

$\theta_e < \pi/2$

for mercury on glass ; $\theta_e > \pi/2$

Young's Law:- (Tomas Young, 1805).

Doing a tangential force balance :-



$$\gamma_{SV} = \gamma_{SL} + \gamma_{LV} \cos(\theta_e)$$

or

$$\boxed{\cos\theta_e = \frac{\gamma_{SV} - \gamma_{SL}}{\gamma_{LV}}}$$: Young's Law.

Since $S = \gamma_{SV} - (\gamma_{SL} + \gamma_{LV})$, we get

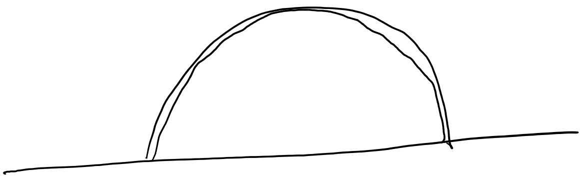
$$\cos\theta_e = \frac{S + \gamma_{LV}}{\gamma_{LV}} = 1 + \frac{S}{\gamma_{LV}}$$

Note:

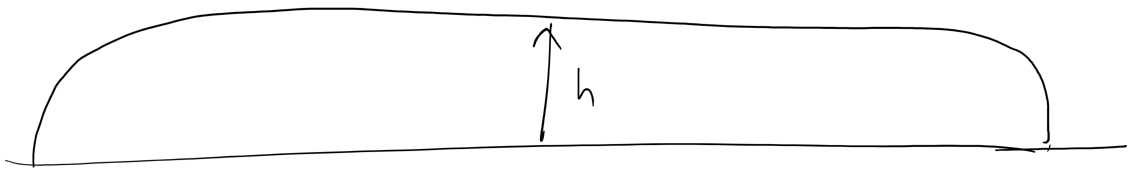
(i) If $S > 0$, $\cos\theta_e > 1$: Young's equation fails.
This results in complete spreading.

(ii) Vertical balance not satisfied at the contact line.

— Dimples on paint after bubble bursting is because of this vertical force.



Puddle Example:-



Contact area $= A$

Volume $= V \approx A \cdot h$

Total energy

$$E \approx \underbrace{(\gamma_{SL} - \gamma_{SV})A + \gamma_{LV} A}_{\text{Surface Energy}} + \underbrace{\frac{1}{2}\rho g h^2 A}_{\text{Gravitational Potential energy}}$$

$$E = -S\frac{V}{h} + \frac{1}{2}\rho g h \cdot V$$

Minimizing E w.r.to h:

$$\frac{dE}{dh} = +S \cdot \frac{V}{h^2} + \frac{1}{2}\rho g \cdot V = 0$$

$$\frac{S}{h^2} = -\frac{\rho g}{2} \Rightarrow h = \sqrt{\frac{-2S}{\rho g}}$$

(for partial wetting $S < 0$)

$$\cos\theta_e = 1 + \frac{S}{\sigma}$$

$$\Rightarrow \sigma\cos\theta_e - \sigma = S$$

$$\Rightarrow \sigma(\cos\theta_e - 1) = S$$

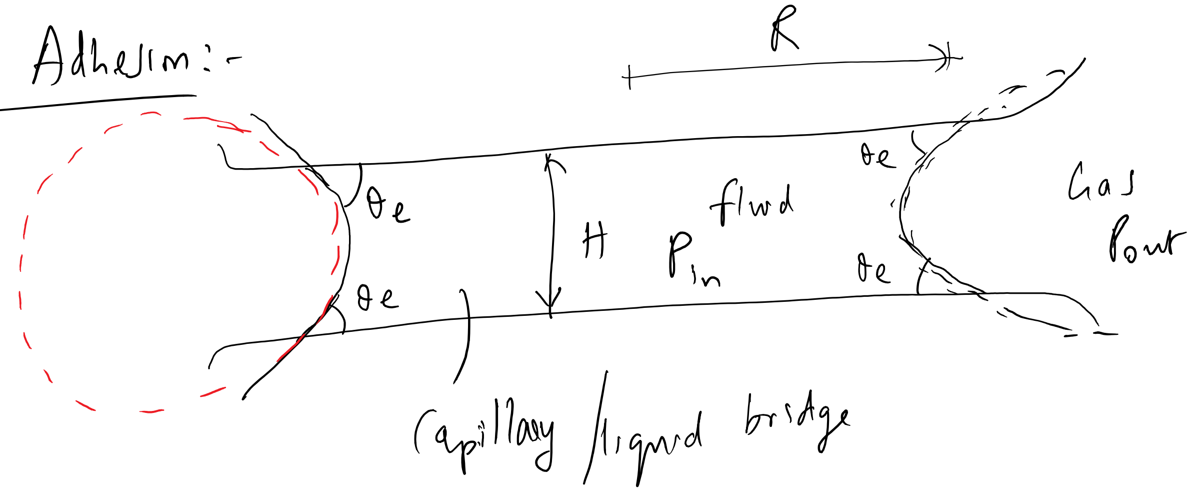
$$\Rightarrow \sigma\left(\cancel{1} - 2\sin^2\left(\frac{\theta_e}{2}\right) - \cancel{1}\right) = S$$

$$\Rightarrow 2\sigma\sin^2\left(\frac{\theta_e}{2}\right) = -S$$

$$h = \sqrt{\frac{+2}{\rho g} \times 2\sigma\sin^2\left(\frac{\theta_e}{2}\right)} = 2\sin\left(\frac{\theta_e}{2}\right)\sqrt{\frac{\sigma}{\rho g}}$$

$$\boxed{h = 2\sin\left(\frac{\theta_e}{2}\right)\cdot l_c}$$

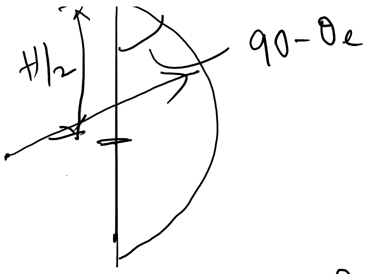
Capillary Adhesion:-



$R >> H$

$$\text{Capillary pressure} = \Delta P = \sigma\left(\frac{1}{R} - \frac{\cos\theta_e}{(H/2)}\right)$$

$$= \sigma\left(\frac{1}{R} - \frac{2\cos\theta_e}{H}\right)$$



for $R >> H$, $$\Delta P \approx -\sigma \frac{2\cos\theta_e}{H}$$

If $\theta_e < \pi/2$, $\cos\theta_e > 0 \Rightarrow \Delta P < 0$

$$\Delta P = P_{in} - P_{out} < 0$$
$$\Rightarrow P_{in} < P_{out}$$

$$\text{Force} = \Delta P \cdot A = \pi R^2 \cdot \Delta P$$
$$= \pi R^2 \times -\frac{2\sigma\cos(\theta_e)}{H}$$

for water-air interface, $H = 10\,\mu m$, $\theta_e = 60^\circ$, $\sigma = 70\,\frac{mN}{m}$,
$R = 10$ cm,

$$\Delta P = -\frac{\cancel{2}\sigma}{H} \times \frac{1}{\cancel{2}} = -\frac{70\times10^{-3}}{10\times10^{-6}} = -7000\ N/m^2$$

$F = -200$ N $\approx$ weight of 20 litres of water