ME2240: Fluid Mechanics
Assignment - 1

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Due date: 6th September 2022, before the class begins.

Note: Please write your solutions on a neat blank sheet of paper (preferably on both sides) with the roll number and name clearly written at the top. Loose sheets will not be accepted, so make sure you staple the sheets. And don't forget to number the sheets. Any evidence of copying will result in getting a zero mark for the entire assignment. Make sure that you work out the solutions on your own. Draw the plots on a graph paper or neatly on a plane sheet of paper with the axis clearly marked.

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1. A velocity field is given by $\vec{\mathbf{V}} = ax^4\hat{\boldsymbol{\imath}} + bx^2y^2\hat{\boldsymbol{\jmath}}$. What are the units of a and b in SI convention. The coordinates x and y are measured in meters.

2. A velocity field is given by $\vec{\mathbf{V}} = a^2x^2\hat{\boldsymbol{\imath}} + bx^3y^2\hat{\boldsymbol{\jmath}}$. What are the units of a and b in SI convention. The coordinates x and y are measured in meters.

3. A velocity field is given by $\vec{\mathbf{V}} = Axy\hat{\boldsymbol{\imath}} + ABxy\hat{\boldsymbol{\jmath}}$. What are the units of A and B in SI convention. The coordinates x and y are measured in meters.

4. For the velocity field given by $\vec{\mathbf{V}} = Ax^2y\hat{\boldsymbol{\imath}} + Bxy^2\hat{\boldsymbol{\jmath}}$ with $A = 2m^{-2}s^{-1}$ and $B = 1m^{-2}s^{-1}$, obtain an equation for the streamlines. Plot several sample streamlines in the first quadrant. Note that the coordinates are measured in meters.

5. A velocity field is given by $\vec{\mathbf{V}} = ax^3\hat{\boldsymbol{\imath}} + bxy^3\hat{\boldsymbol{\jmath}}$ with $a = 1m^{-2}s^{-1}$ and $b = 1m^{-3}s^{-1}$. Find the equation of the streamlines. Plot several streamlines in the first quadrant.

6. A velocity field is given by $\vec{\mathbf{V}} = axt\hat{\boldsymbol{\imath}} - by\hat{\boldsymbol{\jmath}}$, where $a = 0.1s^{-2}$ and $b = 1s^{-1}$. For the particle that passes through the point $(x, y) = (1, 1)$ at instant $t = 0$ s, obtain the expression for the streamline and pathline (i.e. obtain expressions for $x(t)$ and $y(t)$). Make a rough sketch of the pathline during the interval from $t = 0$ to $t = 3$ s. Compare with the streamlines plotted through the same point at the instants $t = 0, 1,$ and 2 s. Coordinates are measured in meters.

7. A velocity field is given by $\vec{\mathbf{V}} = axt\hat{\boldsymbol{\imath}} + b\hat{\boldsymbol{\jmath}}$, where $a = 0.1s^{-2}$ and $b = 4m/s$. For the particle that passes through the point $(x, y) = (3, 1)$ at instant $t = 0$ s, obtain expressions for the streamline and pathline. Make a rough sketch of the pathline during the interval from $t = 0$ to $t = 3$ s. Compare with the streamlines plotted through the same point at the instants $t = 0, 1,$ and 2 s. Coordinates are measured in meters.

8. In class, we have seen that shear stress is related to strain-rate by the Newton's law of viscosity, written as

$$\tau = \mu \frac{du}{dy}.$$

   Using SI units all quantities, express the unit of viscosity μ in terms of fundamental quantities, i.e. kg, m, s.

9. Let us consider a more complex fluid which has a different stress-strain rate relationship given by the equation

$$\tau = \tau_o + \mu \left(\frac{du}{dy} \right)^n,$$

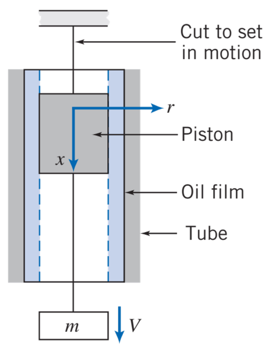
   where n is a positive real number. Using SI units for all the quantities, express the unit of viscosity μ in terms of fundamental units, i.e. kg, m, s, and n.

10. A Newtonian motor oil, with specific gravity of 0.85 and a dynamic viscosity $\mu = 0.008 kgm^{-1}s^{-1}$ flows steadily down an inclined surface with inclination $\theta = 30°$ in the form of a thin film with thickness $h = 4mm$. The velocity profile in the thin film is given by

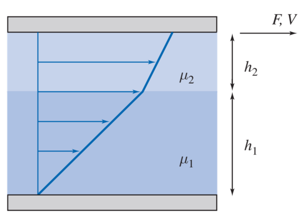
$$u = \frac{\rho g}{\mu}\left(hy - \frac{y^2}{2}\right)\sin\theta,$$

where y is measured normal to the inclined surface. Make a rough sketch of the velocity profile. Determine the magnitude of the shear stress that acts on the surface.

11. A 73-mm-diameter aluminium piston of specific gravity 2.64 with length 100 mm resides in a stationary 75-mm-inner-diameter steel tube lined with SAE 10W-30 oil at $25°C$ with a viscosity, $\mu = 0.06 kgm^{-1}s^{-1}$. A mass $m = 2$ kg is suspended from the free end of the piston. The piston is set into motion by cutting a support cord. What is the terminal velocity of mass m? Assume a linear velocity profile within the oil. Also assume the tube to be very long and the oil film to be of uniform thickness. You can take $g = 10m/s^2$.



12. Fluids of viscosities $\mu_1 = 0.1 N\cdot s/m^2$ and $\mu_2 = 0.15 N\cdot s/m^2$ are contained between two plates (each plate is 1 m^2 in area). The thicknesses are $h_1 = 0.5mm$ and $h_2 = 0.3mm$, respectively. Find the force F to make the upper plate move at a speed of $1m/s$. What is the fluid velocity at the interface between the two fluids?



13. When a fluid at uniform speed U flows against a stationary surface, the fluid is slowed rapidly near the surface due to the no-slip condition. The region where the fluid slows down is called the 'boundary layer'. Away from the surface, typically at a distance $y = \delta$, the fluid attains the free-stream uniform velocity. The velocity profile in the viscous boundary layer can be approximated by the formula

$$u(y) = a + b\left(\frac{y}{\delta}\right) + c\left(\frac{y}{\delta}\right)^2.$$

where y is the wall-normal direction. Using the no-slip condition at the solid surface and assuming that the fluid is both stress-free and attains uniform flow speed at $y = \delta$, find the values of the constants a, b and c.