Summary : Equilibrium of a rigid body

We have seen that for equilibrium of a rigid body (in 3D), we require 6 scalar equations for 6 unknowns :

$$\sum F_x = 0 \quad , \quad \sum F_y = 0 \quad , \quad \sum F_z = 0$$

$$\sum M_x = 0 \quad , \quad \sum M_y = 0 \quad , \quad \sum M_z = 0$$

The above equations can be solved for no more than 6 unknowns, which generally represent reactions at supports.

In vector form : $\sum \vec{F} = 0$ & $\sum \vec{M}_O = \sum (\vec{r} \times \vec{F}) = 0$

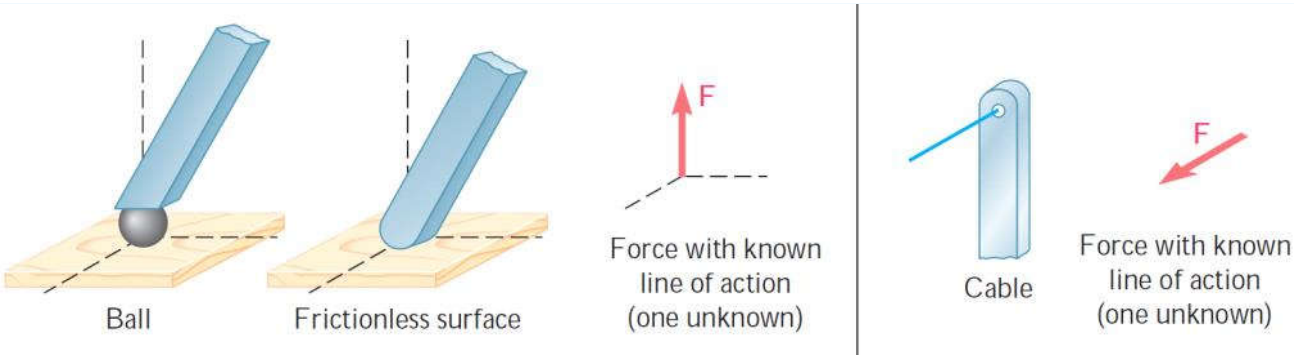
Reactions at supports & connections for a 3D - structure :-

The reactions on a 3D structure range from a single force ($\vec{F}$) of known direction (like at a frictionless surface) to a force - couple system ($\vec{F}$) ($\vec{M}$) (like at a fixed support).

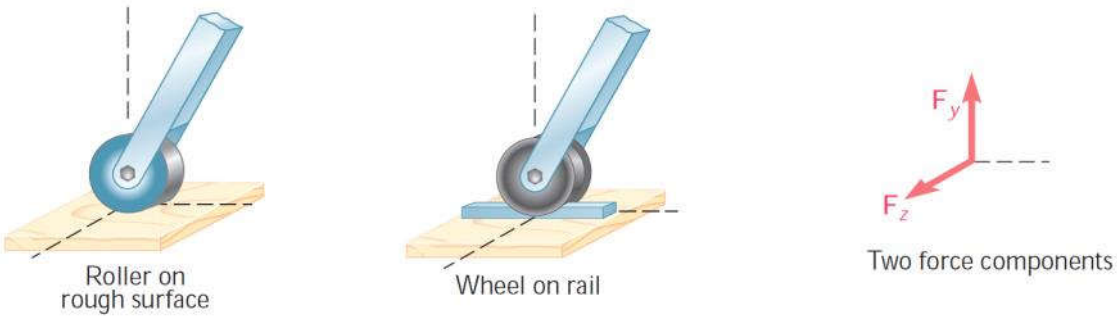
A simple way of determining the type of reactions at a given support is to find which of the 6 fundamental motions (3 translations in x, y, z & 3 rotations about x, y, z) are allowed & which are prevented / arrested.

Common types of supports & connections :-

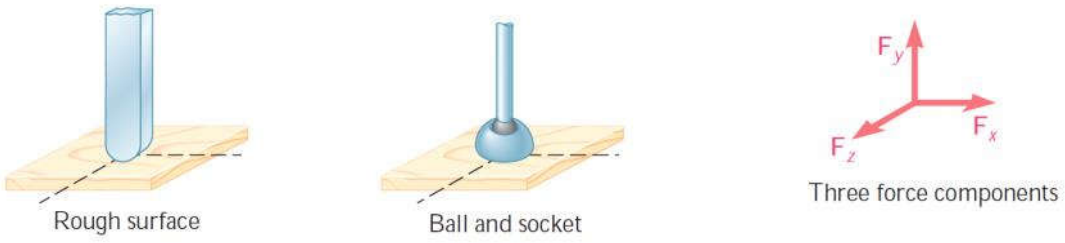
Single force :- Ball support, frictionless surface, Cables prevent translation in only one direction.



Two forces :- Roller on rough surface, Wheels on a rail
Both these prevent motion in two directions

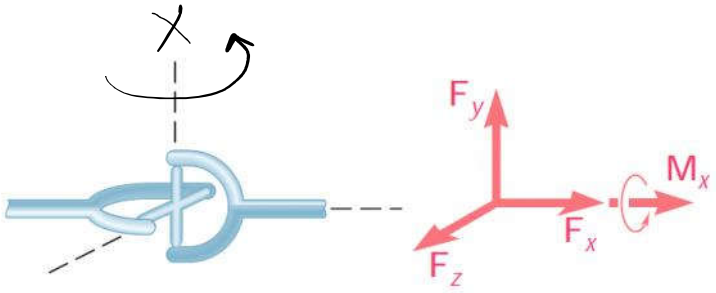


Three force components :- Direct contact with rough surface, ball & socket support prevents translation in 3 directions.



Universal joint :- To allow rotation about two axes.
Prevents translation completely & one rotation

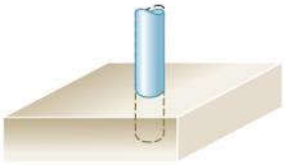
X ↑



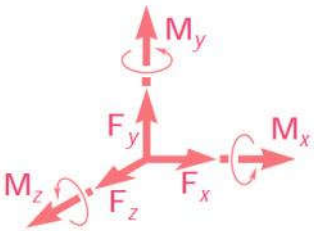
Universal joint

Three force components and one couple

Fixed joint :- All motion arrested.

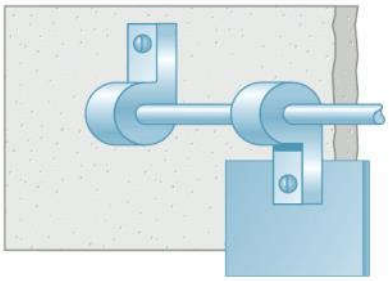


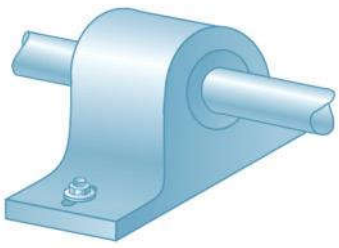
Fixed support

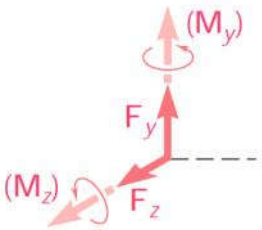


Three force components and three couples

Hinge & bearing supporting radial load only :-





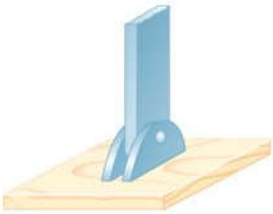


Two force components (and two couples)

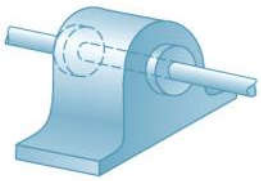
Hinge and bearing supporting radial load only

Translation & rotation along axis of shaft is allowed.

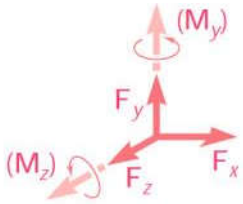
3-forces & 2-couples



Pin and bracket



Hinge and bearing supporting axial thrust and radial load

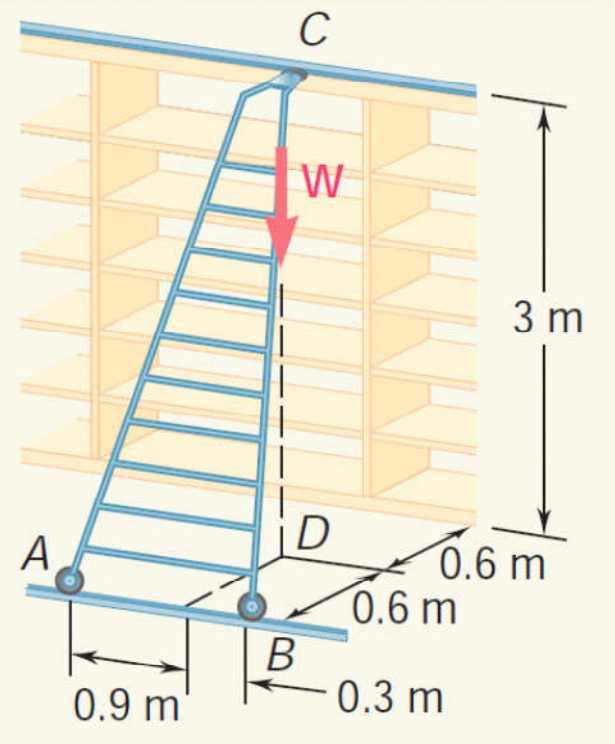


Three force components (and two couples)

Example:-

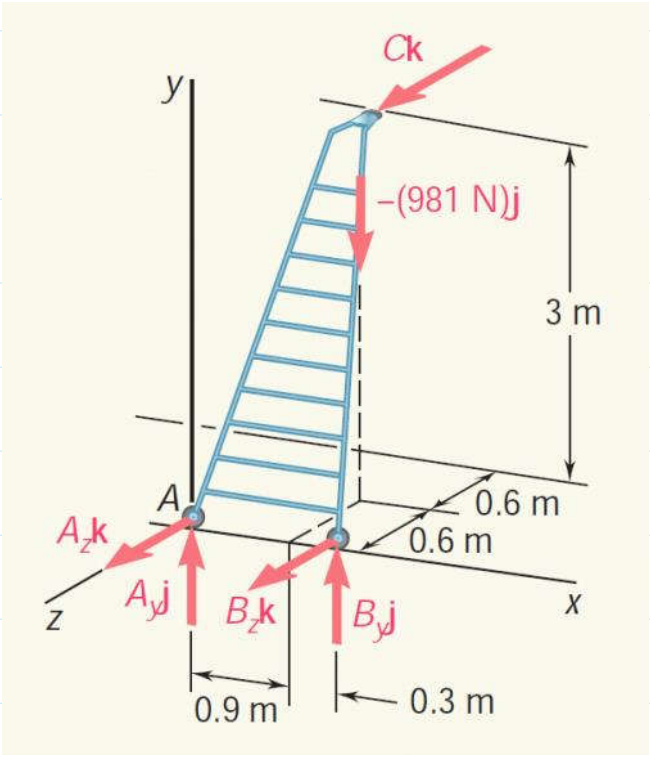
Ladder of weight $m = 20\,kg$ is used in a library.

Supported by two flanged wheels A & B mounted on a rail and by an unflanged wheel C resting against a rail fixed to the wall.



An 80 kg man stands on the ladder whose line of action (along with weight of ladder) intersects the floor at Point D.

Determine reactions at A, B, C.



$$\vec{R}_A = (245\,N)\,\hat{j} - (98.1\,N)\,\hat{k}$$

$$\vec{R}_B = (736\,N)\,\hat{j} - (98.1)\,\hat{k}$$

$$\vec{R}_C = (196.2\,N)\,\hat{k}$$