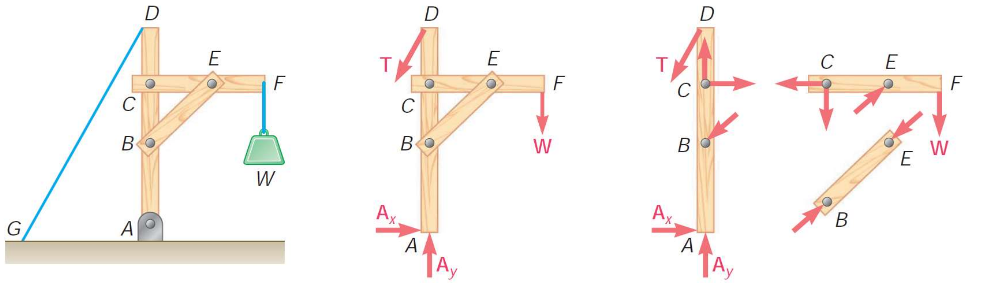
Analysis of Trusses, Frames & Structures:-

In earlier lectures, we considered analysis of single bodies. In many applications, it is more realistic to connected "simple" bodies to create a more complicated & functional load bearing structure.

In the figure below, you see a crane which can hold a load W. The weight W is now shared by various "members" CD, BE, AB & string GD along with reactions at the ground.



To analyse the load on each member, we can "dismember" the structure and draw free-body diagrams for each member separately. Then all the "internal" loads on the structure become "external" loads on each member.

Note that Newton's III law has been used in drawing the FBD on individual members.

Common Categories of Structures:-

(i) Trusses: These are designed to support loads & usually consist of straight members connected with joints at the ends of each member. Each member of a truss is exclusively a two-force system.

Bridge with pin-joints.

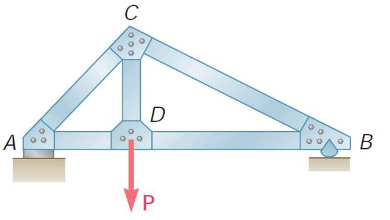
(ii) Frames: These are similar to trusses, [illegible] have a few members [illegible] directed along the member.

with three or four forces not necessarily directed along

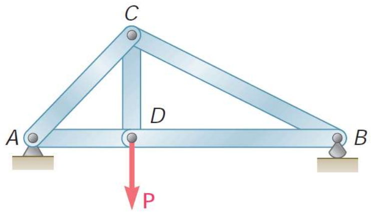
(iii) __Machines:__ These are designed to modify & transmit forces & usually contain moving parts. Like frames, they too contain multiforce members.

Definition of a Truss..

A typical truss is shown here →



(a)

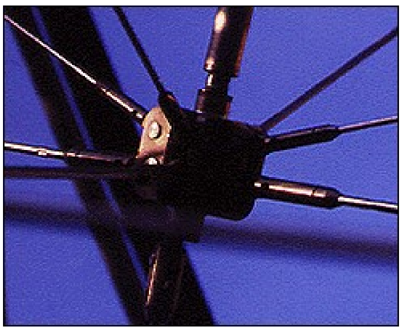


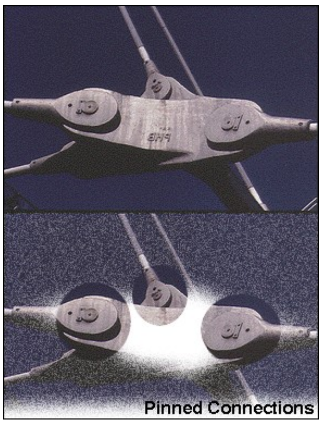
It contains of straight members always joined at their ends. So in this truss, there is No member AB, instead we have 5 smaller members AC, AD, CD, DB & CB.

Typically, trusses are designed to carry loads in its plane, hence trusses can largely be treated as two-dimensional structures.

The members themselves can support very little lateral loads, hence loads on trusses are largely applied at their joints. The weight of members are also assumed to act at such joints.

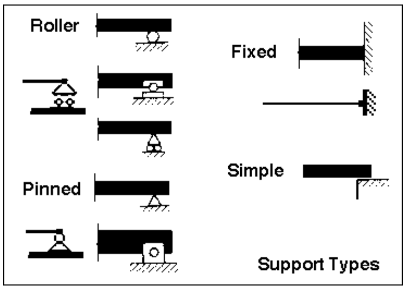
Though members are welded / bolted / riveted together, it is customary to treat all joints as "pin joints", i.e. joints where there are only forces and no moments / couples.

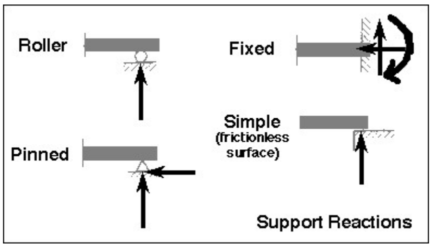


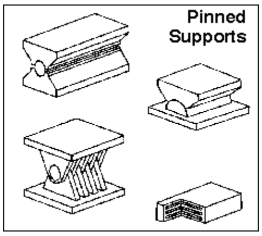


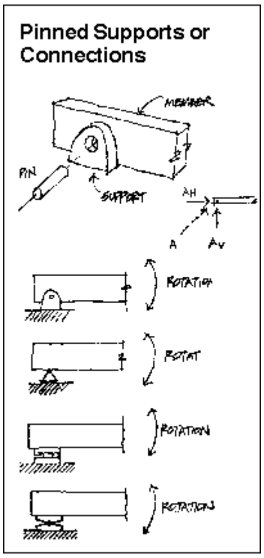
Common types of joints:-

- with **pin joints**. A pin joint allows relative rotation of the parts; it does not transmit moments. Forces are transmitted in all directions. The pin connection is, by far, the most common model for connections in structures.
- a **round pin in a slot**. A pin in slot allows relative rotation of the two parts and relative motion in one direction. The only force transmitted is orthogonal to the slot.
- **square pin in a slot** (or shaft around a rod). This connection allows sliding in the slot but does not allow rotation. A force orthogonal to the slot is transmitted; so is a moment.

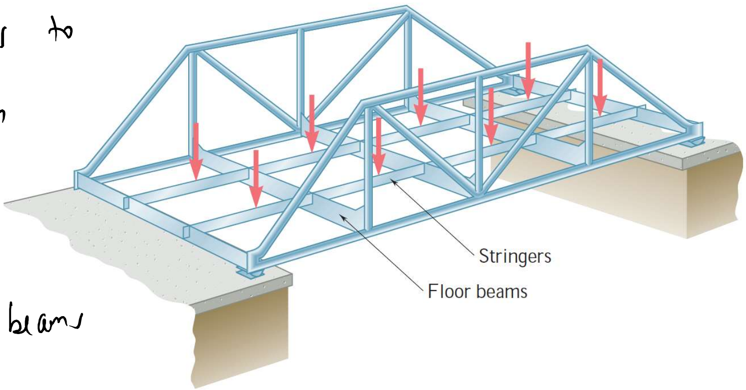




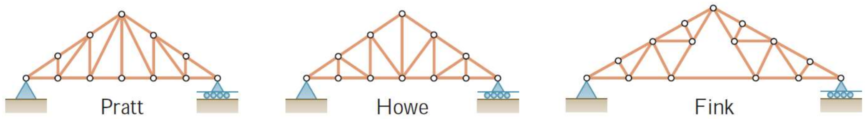




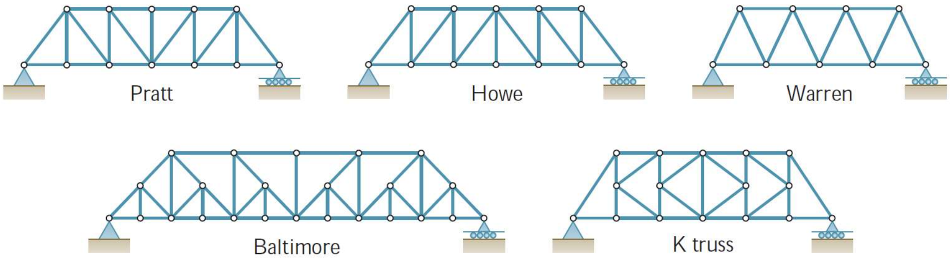
If a structure needs to carry a load away from the plane of the truss, load transmitting member such as floor beams & stringers are used.



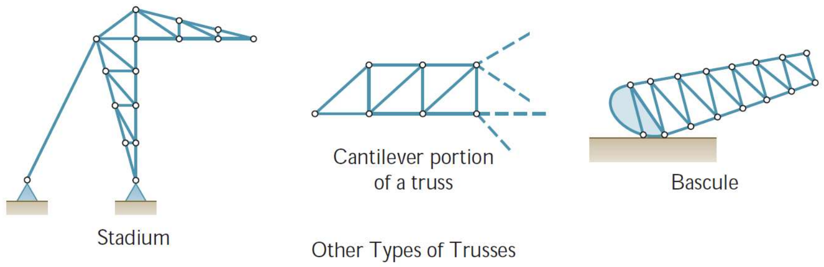
Common types of trusses



Typical Roof Trusses



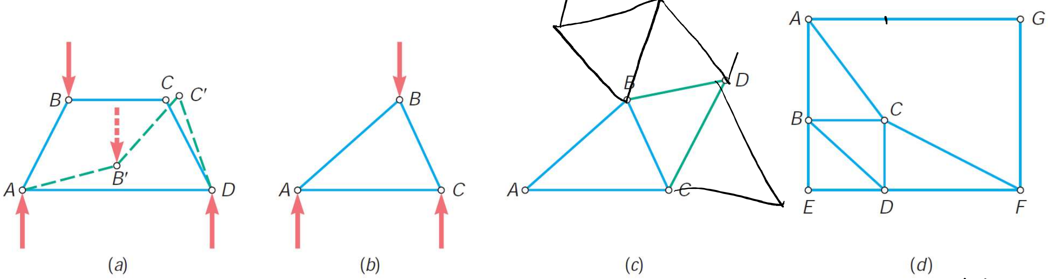
Typical Bridge Trusses



Other Types of Trusses

An ideal truss only consists of two-force members.

Simple Truss :-

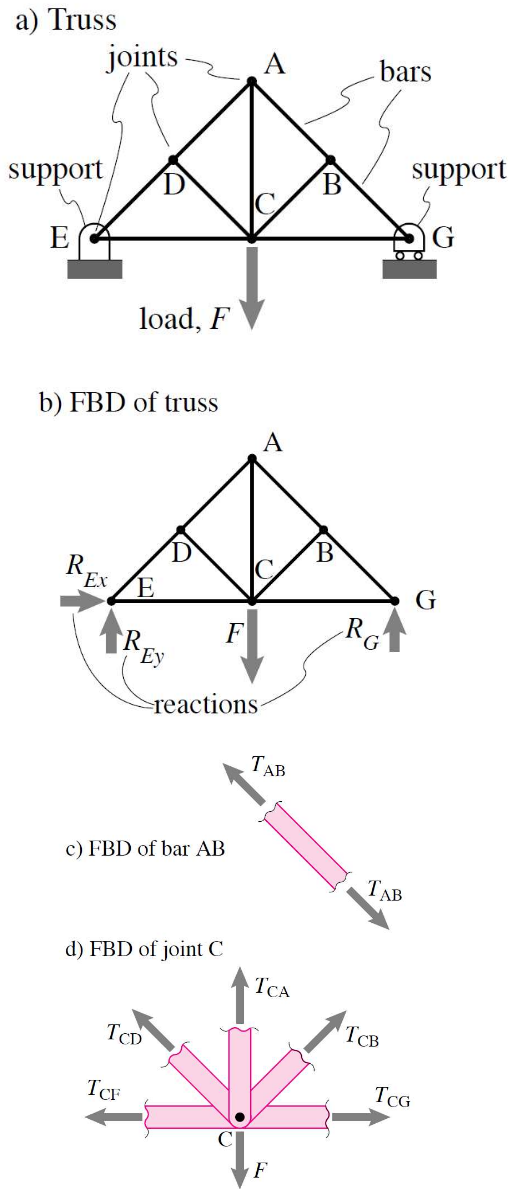


A larger truss can be constructed by simply adding two new members at each step - such a truss is called a simple truss.

Typically if there are n joints, then number of bars or members, $\boxed{m = 2n-3}$

Recipe for determining forces in a truss:-

(i) Identify & label all bars/members, supports, joints & forces.



(ii) Treat truss as one single rigid structure & draw its FBD to determine reactions.

(iii) One can proceed to draw FBD's of bars & joints to determine all necessary forces.

Two broad ways of studying trusses:

Method of joints

Method of sections

Method of Joints:

For n pin-joints, we will have 2n forces (3n in 3D), i.e.; 2 forces at each pin × n joints. So we can write equilibrium equation for all n-joints to obtain 2n equations.

No. of unknowns that can atmost be determined is:

→ m forces in m members/bars

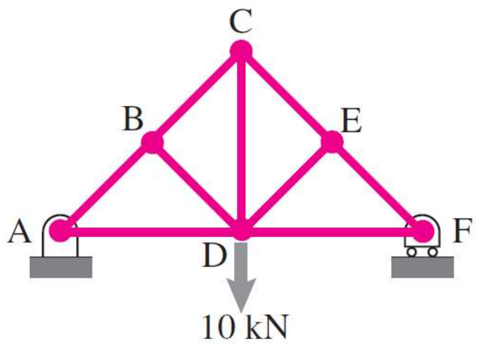
→ 3 reaction forces

Since $m + 3 = 2n$

Example:

$F = 10$ KN

All inclined rods have same length, $d = 2$ m



(i) find reactions at A & F

(ii) Find forces in BD & BC.

Solution

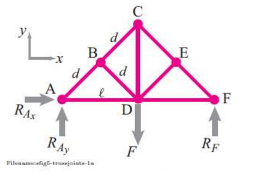


Figure 5.15:

1. **Support reactions:** To find the support reactions at A and F, we draw the free-body diagram of the entire truss (see fig. 5.15). We are given that $d = 2\,\mathrm{m}$ and that $\angle ABD = \angle DEF = \pi/2$. Therefore, $\ell = \sqrt{2}d = 2\sqrt{2}\,\mathrm{m}$.

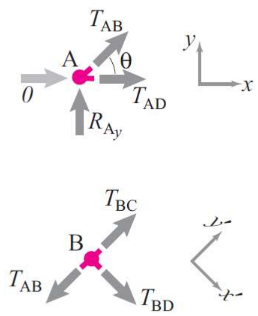
   The scalar force-balance equation in x-direction readily gives $R_{A_x} = 0$. The scalar moment-balance equation about point A gives

$$2\ell R_F - \ell F = 0 \quad \Rightarrow \quad R_F = \frac{F}{2} = 5\,\mathrm{kN}.$$

   Now, from the scalar force balance in the y-direction, we have

$$R_{A_y} + R_F - F = 0 \quad \Rightarrow \quad R_{A_y} = F - R_F = 5\,\mathrm{kN}.$$

$$R_{A_x} = 0, \quad R_{A_y} = 5\,\mathrm{kN}, \quad R_F = 5\,\mathrm{kN}$$



2. **Tensions in BD and BC:** We can find the tensions in rods BC and BD by analyzing the equilibrium of joint B. As you can see, joint B has three unknown forces acting on it, namely the tensions of rods AB, BC and BD. Since the joint equilibrium equations (only two scalar equations) can only solve for two unknowns, we need to start at joint A, determine T_{AB} first and then move on to joint B.

   The free-body diagrams of the joints A and B are shown in fig. 5.16. Let us first consider the equilibrium of joint A. From the scalar force-balance equations, we have

$$\begin{aligned}
\sum F_y = 0 \quad &\Rightarrow \quad R_{A_y} + T_{AB}\sin\theta = 0 \\
&\Rightarrow \quad T_{AB} = -R_{A_y}/\sin\theta = -5\,\mathrm{kN}/(1/\sqrt{2}) = -7\,\mathrm{kN}. \\
\sum F_x = 0 \quad &\Rightarrow \quad T_{AB}\cos\theta + T_{AD} = 0 \\
&\Rightarrow \quad T_{AD} = -T_{AB}\cos\theta = -7\,\mathrm{kN}(1/\sqrt{2}) = 5\,\mathrm{kN}.
\end{aligned}$$

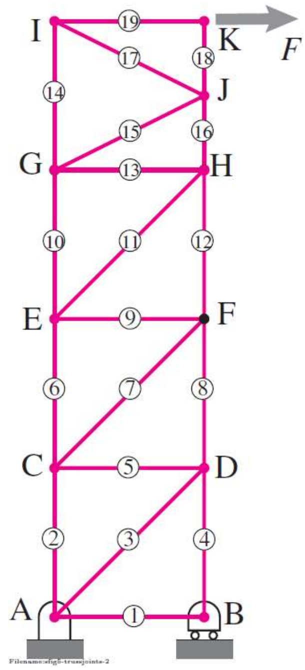
   Now, we analyze joint B. From the geometry of forces, it is clear that writing scalar force-balance equations in the x' and y' directions will be advantageous. For example, the force balance in the x' direction immediately gives $T_{BD} = 0$. The force balance in the y' direction gives

$$-T_{AB} + T_{BC} = 0 \quad \Rightarrow \quad T_{BC} = T_{AB} = -7\,\mathrm{kN}.$$

$$T_{BC} = -7\,\mathrm{kN}, \quad T_{BD} = 0$$

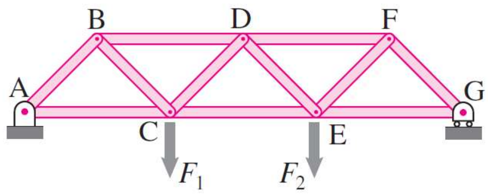
   Note that it is easy to spot bar BD as a zero-force member since it is perpendicular to rods AB and BC.

Example: Assume all horizontal & vertical rods to be 1 m long. Rods 16 & 18 are 0.5 m long. If F = 500 N, find force in rod 15



Example:

All inclined rods are 1m long & are at right angles to each other.

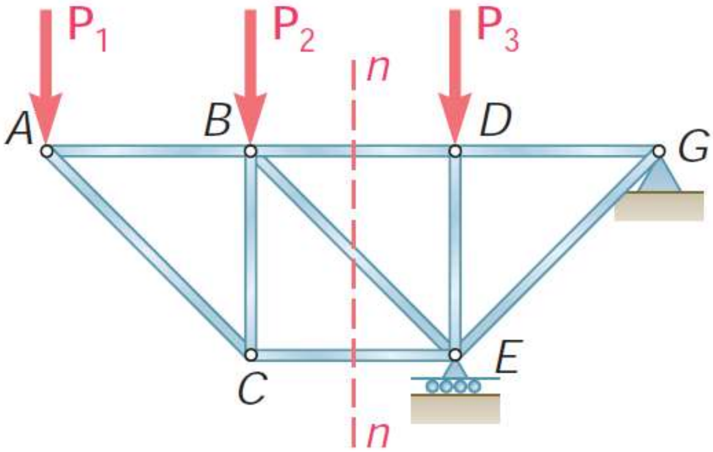


$F_1 = 4$ kN

$F_2 = 1$ kN.

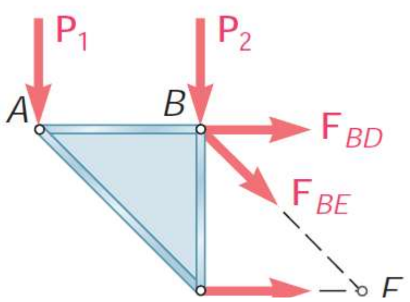
Determine tensions in CE, DE & DF.

METHOD OF SECTIONS

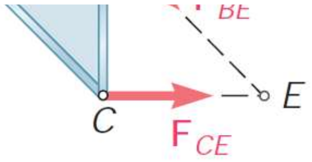


Method of Joints is useful if you wish to determine all the forces in all members.

But if you wish to determine force in just one or two members, method of joints can be cumbersome.



Consider the above truss shown where you may be interested in force in only BD.



force in only BD.

Since every part of the truss is in equilibrium, we can cut the truss at section n-n.

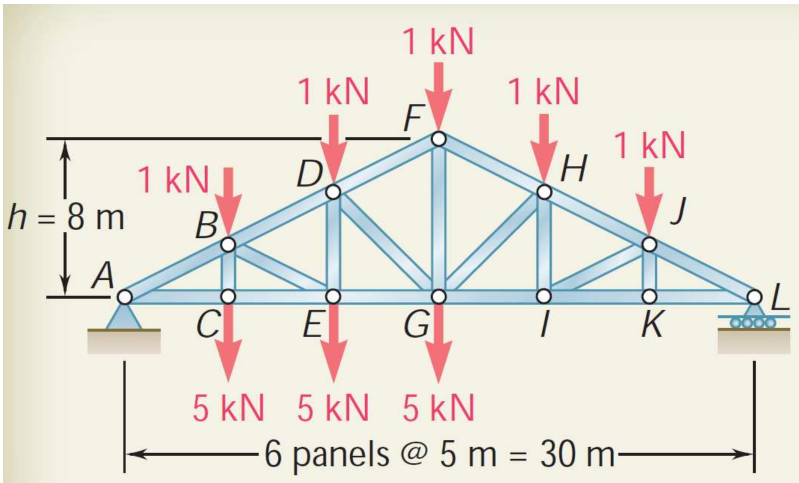
To determine force F_{BD}, simply take $\vec{M}_E = 0$

To determine force F_{CE}, simply take $\vec{M}_B = 0$

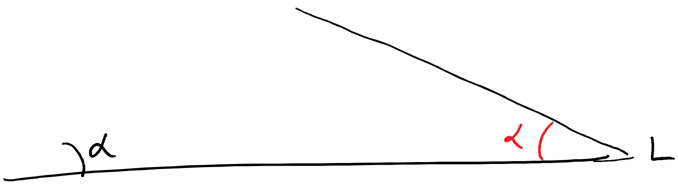
To determine force F_{BE}, simply set $\sum f_y = 0$ on the entire section.

Example :-

Determine force in members FH, GH & GI of the roof truss.



Soln: (i) FBD of the entire truss:



$\sum f_x = 0$

$\Rightarrow A_x = 0$

$\sum f_y = 0$ } $A_y = 12.5$ KN & $L_u = 7.5$ KN

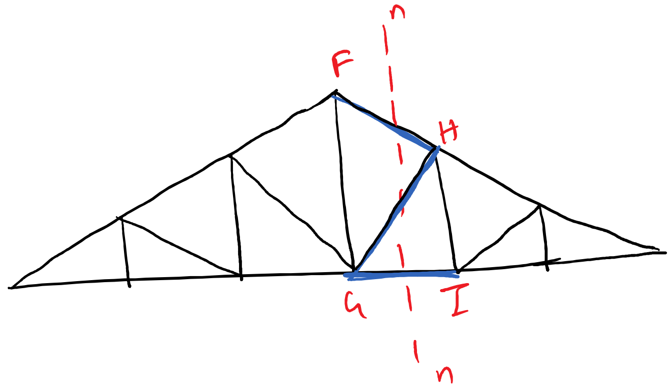
6

$\Sigma F_y = 0$

$\quad 4 \quad L_y = 7.5 \text{ kN}$

Also $\tan\alpha = \dfrac{FG}{GL} = \dfrac{8\,m}{15\,m} = 0.5333 \Rightarrow \alpha \approx 28.07°$

(iii) <u>Force in member GI</u>: Cut the truss as shown:

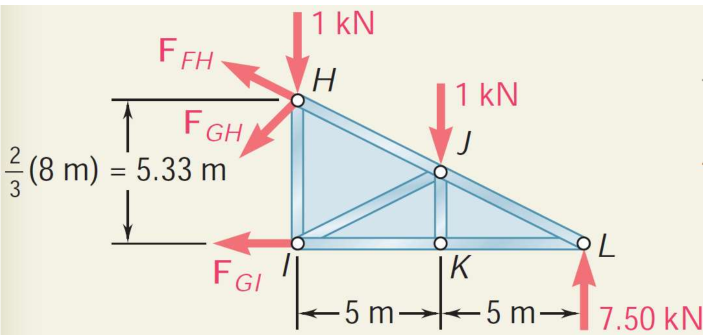


Using section HLI & taking moments about H:

$\Sigma M_H = 0$

$\Rightarrow (7.5\text{ kN})\,10\,m - (1\text{ kN})\,5\,m - F_{GI}\,(5.33\,m) = 0$

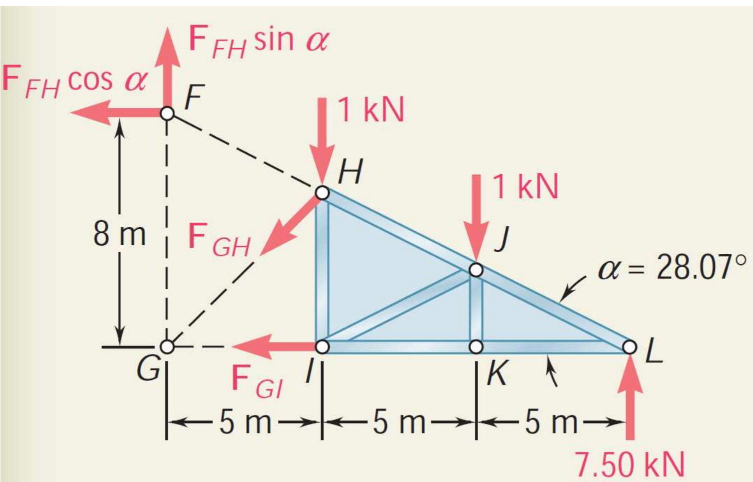
$\rightarrow F_{GI} = +13.13\text{ kN}$ (tension)



<u>Force in member FH</u>:

Taking moments about G:

$\Sigma M_G = 0$

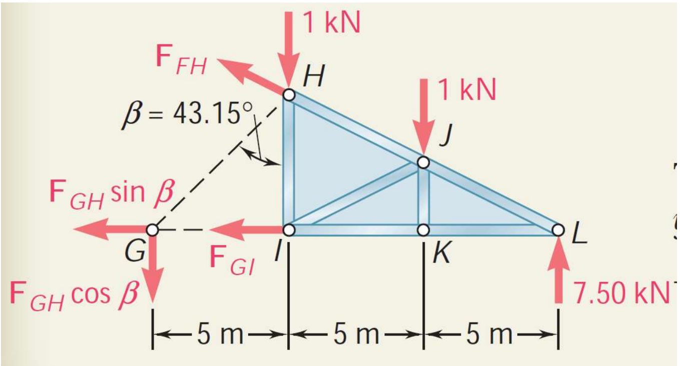


$(7.5\text{ kN})(15\,m) - (1\text{ kN})(10\,m) - (1\text{ kN})(5\,m) + (F_{FH}\cos\alpha)(8\,m) = 0$

in compression.

$\Rightarrow \quad f_{FH} = -13.81\,kN \quad$ M $\quad F_{FH} = 13.81\,kN$

Force in member GH:-



Note that

$$\tan\beta = \frac{GI}{HI} = \frac{5\,m}{\frac{2}{3}(8\,m)}$$

$$\Rightarrow \beta = 43.15°$$

Resolving F_{GH} into x & y-components at G & setting

$\sum M_L = 0 \quad \Rightarrow \quad F_{GH} = -1.371\,kN \quad$ M $\quad F_{H} = 1.371\,kN$ in compression.