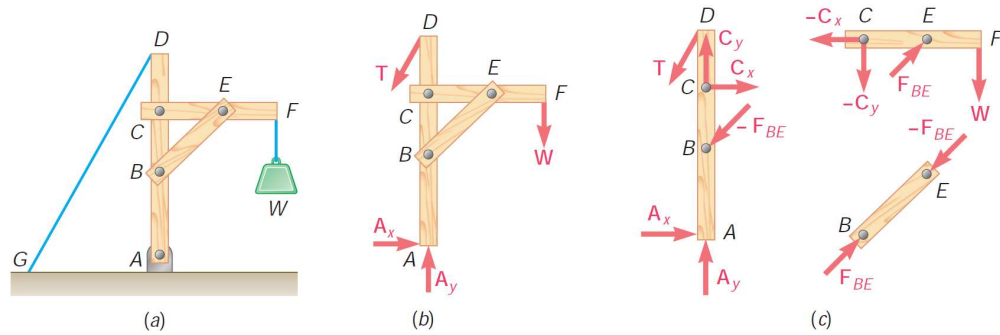


## Analysis of Frames & Machines:-

Unlike trusses, frames & machines contain at least one multi-force member, whereas trusses consist of pin joints & only two-force members.

## Analysis of frames:-



To find internal forces in all members, we will have to dismember the frame.

Member BE : 2-force member

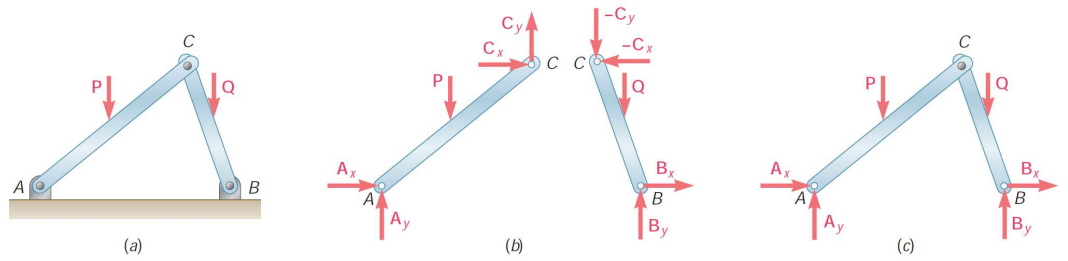
At C, two multi-force members are connected : AD & CF.

### Recipe:-

- (i) We first determine reactions by drawing FBD on the entire frame
- (ii) Determine internal forces in 2-force members.
- (iii) Determine internal forces in multi-force members.

The above whole example is for a "rigid frame". Not all frames are rigid, i.e., they will seek to retain their shape if support is removed.

Consider the frame shown below.

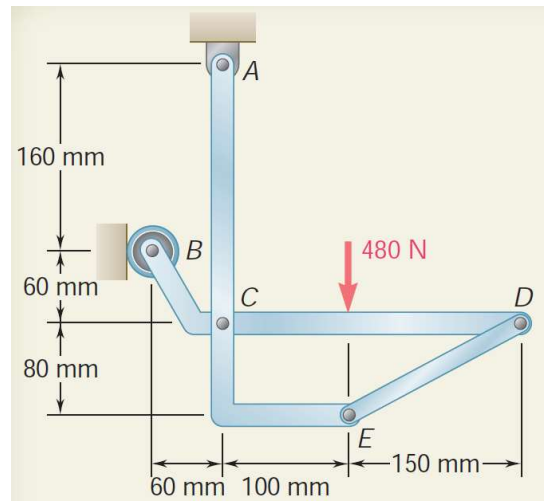


If supports are removed, the frame can deform. To treat such frames, we should treat it as two distinct rigid parts (AC & CB, mark all forces (including reactions) & determine the forces on them using FBD's. We should use action-reaction principles at points where the individual members connect, in the above case, at C.

Example:

Members ACE & BCD are connected at C & by link DE.

Determine force in link DE & components of force exerted at C on member BCD.

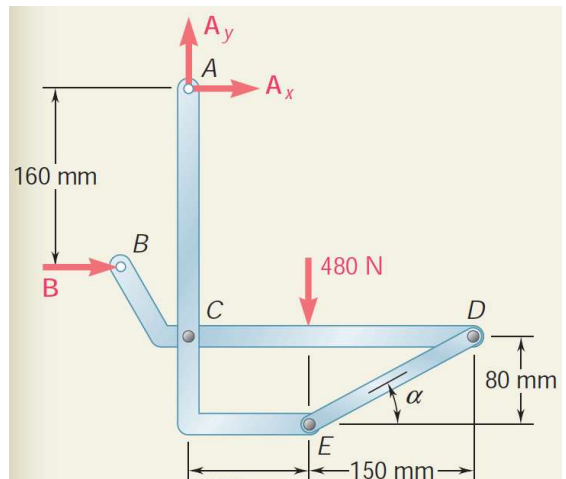


Soln: FBD of entire frame:-

$$A_x = -300 \text{ N}$$

$$A_y = 480 \text{ N}$$

$$\vec{B} = 300 \text{ N}$$

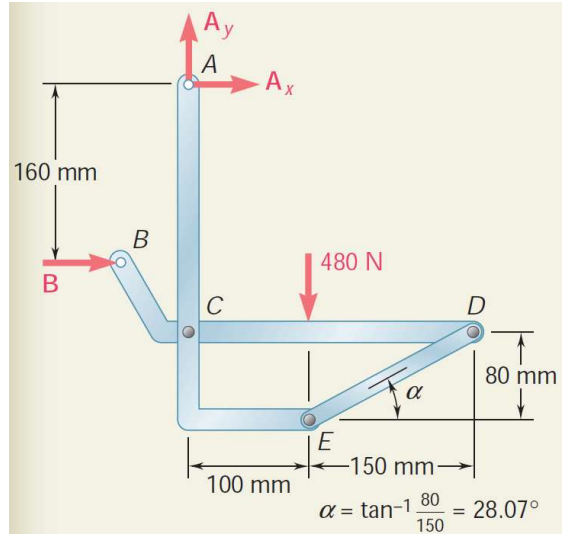


Soln: FBD of entire frame:-

$$A_x = -300 \text{ N}$$

$$A_y = 480 \text{ N}$$

$$\vec{B} = 300 \text{ N}$$



Forces in the members:-

Since only two members join at C, forces at C on each member are opposite to each other. We also assume link DE is in tension.

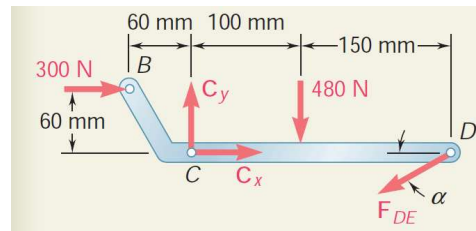
Member BCD:

$$\sum M_C = 0 \Rightarrow F_{DE} = -561 \text{ N}$$

(compression)

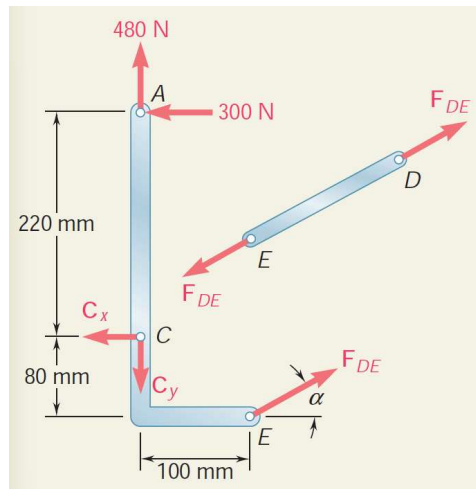
$$\sum F_x = 0 \Rightarrow C_x = -795 \text{ N}$$

$$C_y = +216 \text{ N}$$



FBD on ACE (check)

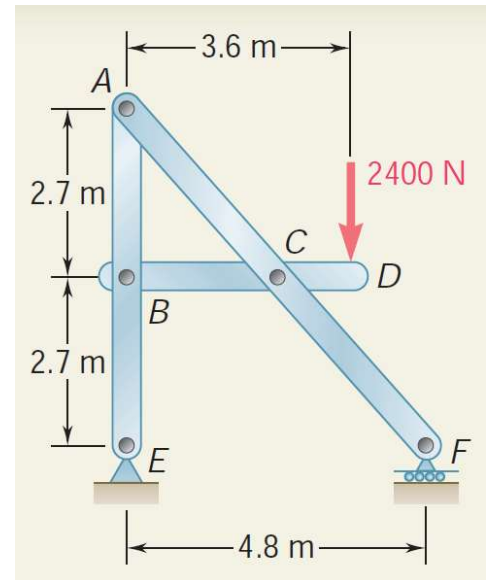
$$\sum M_A = 0$$



En: Determine components on each

Ex: Determine components on each member of the frame?

(A) We have pin-support at E  
 and roller support at F  
 $\Rightarrow$  We only have a total of 3 unknowns.



FBD of entire frame:-

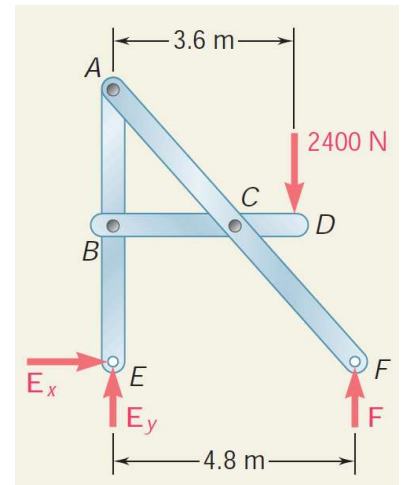
$$\sum M_E = 0 \Rightarrow (-2400\text{N})(3.6\text{m}) + F(4.8\text{m}) = 0$$

$$\Rightarrow F = 1800\text{N}$$

$$\sum f_y = 0 \Rightarrow E_y + F - 2400\text{N} = 0$$

$$\Rightarrow E_y = +600\text{N}$$

$$\sum F_x = 0 \Rightarrow E_x = 0$$



Forces in the members:-

Member BCD:

$$\sum M_B = 0 \Rightarrow C_y(2.4\text{m}) - (2400\text{N})(3.6\text{m}) = 0$$

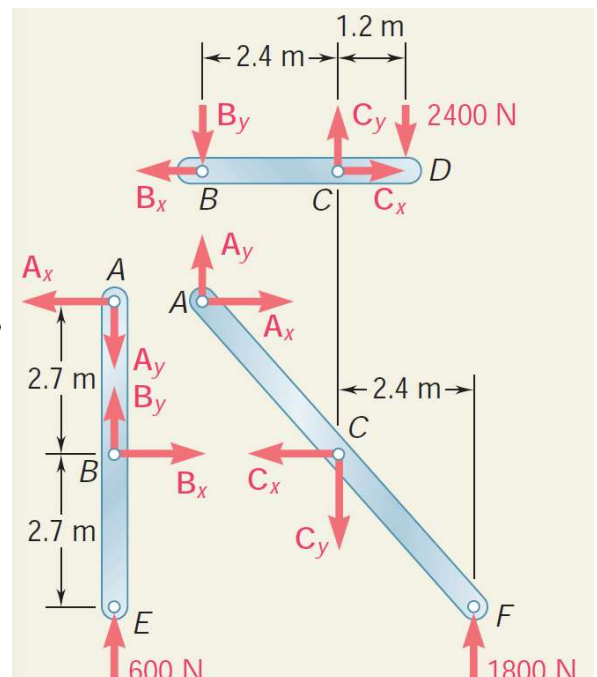
$$\Rightarrow C_y = 3600\text{N}$$

$$\sum M_C = 0 \Rightarrow B_y(2.4\text{m}) - (2400)(1.2\text{m}) = 0$$

$$\Rightarrow B_y = 1200\text{N}$$

$$\sum F_x = 0 \Rightarrow C_x - B_x = 0$$

Member ABE:



Member ADE

$$\sum M_A = 0 \Rightarrow B_x = 0$$

$$\sum M_B = 0 \Rightarrow A_x = 0$$

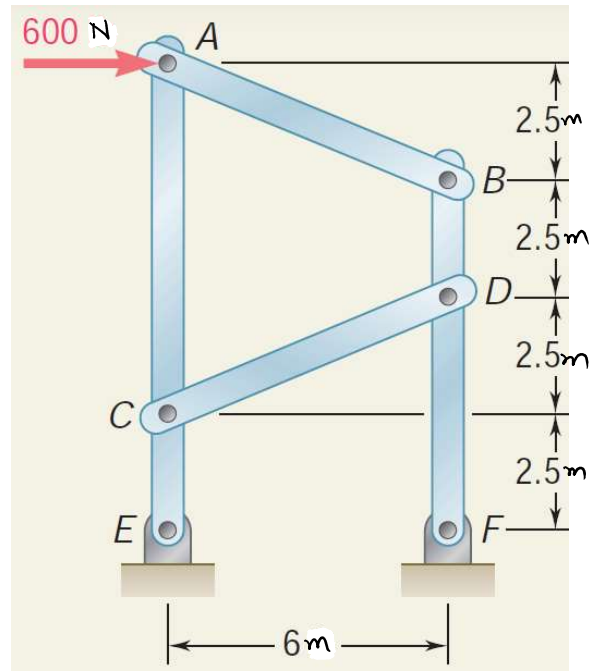
$$\sum f_y = 0 \Rightarrow -A_y + B_y + 600 = 0 \Rightarrow A_y = 1800 \text{ N}$$

Also since  $B_x = 0$ ,  $C_x = 0$



Member ACF: Can easily verify that  $\sum M = 0$  & all  $\sum f_x = 0$  &  $\sum f_y = 0$

Ex: A 600 N horizontal force is applied to pin A. Determine the forces on the two vertical members of the frame.

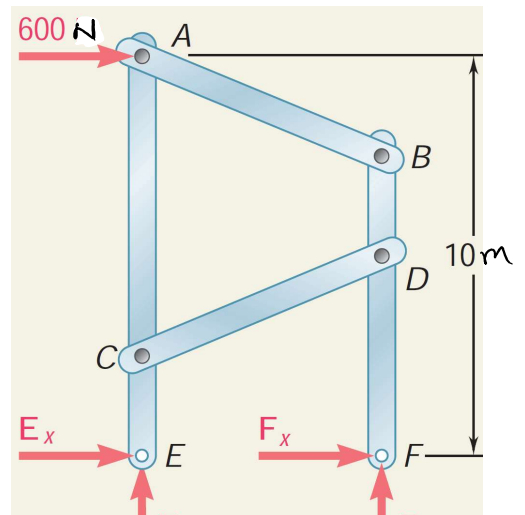


(A) FBD of entire frame:-

We have four unknowns, but we can still determine two of them.

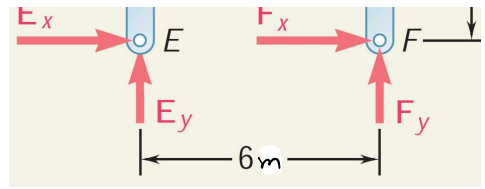
$$\sum M_E = 0 \Rightarrow (-600 \text{ N})(10 \text{ m}) + f_y(6 \text{ m}) = 0$$

$$\Rightarrow f_y = 1000 \text{ N}$$



$$\Rightarrow f_y = 1000 \text{ N}$$

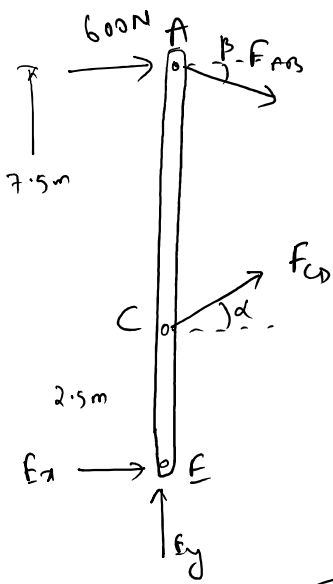
$$\sum f_y = 0 \Rightarrow E_y + f_y = 0 \Rightarrow E_y = -1000 \text{ N}$$



To determine  $E_x$  &  $F_x$ , we must analyse forces on individual members.

Member 1: When dismembering the frame, we have to determine where pin-A is attached to ACE & AB. We will assume that pin-A is attached to ACE.

Member ACE:



$$\cos \alpha = \frac{6}{\sqrt{6^2 + \frac{5^2}{2^2}}} = \frac{12}{\sqrt{144 + 25}} = \frac{12}{13}$$

$$\sin \alpha = \frac{5}{13}$$

$$\cos \beta = \frac{12}{13}, \quad \sin \beta = \frac{5}{13}$$

$$\sum f_y = 0 \Rightarrow E_y + F_{CD} \sin \alpha - F_{AB} \sin \beta = 0$$

$$\Rightarrow -1000 \text{ N} + F_{CD} \left(\frac{5}{13}\right) - F_{AB} \left(\frac{5}{13}\right) = 0$$

$$\sum M_E = 0 \Rightarrow (-F_{CD} \cos \alpha)(2.5 \text{ m}) - (F_{AB} \cos \beta)(10 \text{ m}) - (600 \text{ N})(10 \text{ m}) = 0$$

Solving:  $F_{CD} = -1040 \text{ N}$  &  $F_{AB} = 1560 \text{ N}$

$$\sum f_x = 0 \Rightarrow E_x + F_{CD} (\cos \alpha) + 600 \text{ N} + F_{AB} \cos \beta = 0$$

$$\Rightarrow \boxed{E_x = -1080 \text{ N}}$$

We now have  $F_2 = 480 \text{ N}$