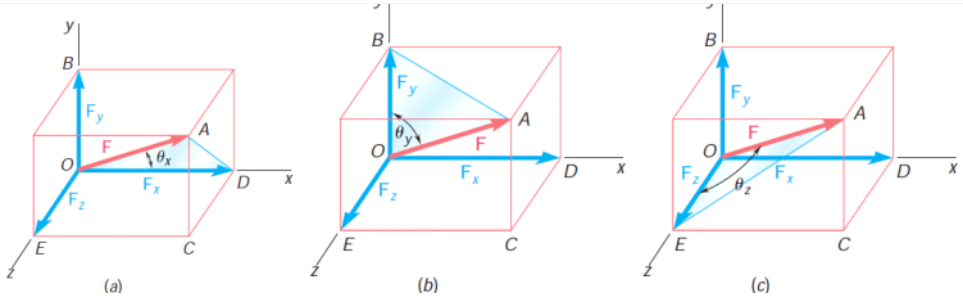
Forces in Space

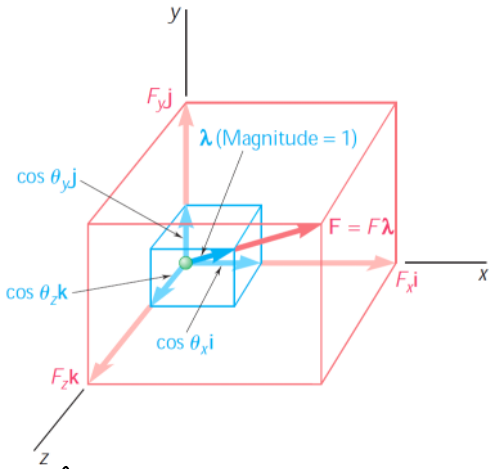


$$\vec{F} = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$$

where $f_x = F\cos\theta_x$

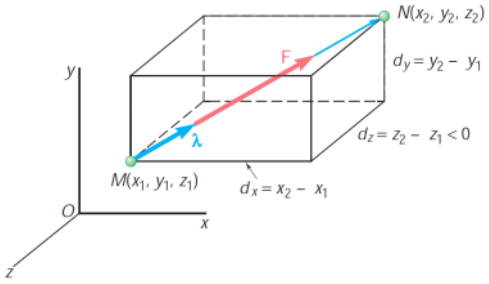
$f_y = F\cos\theta_y$

$f_z = F\cos\theta_z$



Direction of the force is given by the unit vector $\hat{\lambda} = \cos\theta_x \hat{i} + \cos\theta_y \hat{j} + \cos\theta_z \hat{k}$

If a force is defined by its magnitude & its line of action as shown below:



then we have: $\vec{MN} = d_x \hat{i} + d_y \hat{j} + d_z \hat{k}$

$$\therefore \hat{\lambda} = \frac{\vec{MN}}{|MN|} = \frac{1}{d}\left(d_x \hat{i} + d_y \hat{j} + d_z \hat{k}\right)$$

where $d = \sqrt{d_x^2 + d_y^2 + d_z^2}$

Since $\vec{F}$ $F\hat{\lambda}$, we have

$$\vec{F} = \frac{F}{d}\left(d_x \hat{i} + d_y \hat{j} + d_z \hat{k}\right)$$

Comparing with the earlier expression, we also have

$$\cos\theta_x = \frac{dx}{d} \quad ; \quad \cos\theta_y = \frac{dy}{d} \quad ; \quad \cos\theta_z = \frac{dz}{d}$$

Addition of forces :-

The resultant force $\vec{R}$ is simply given by adding up 'vectorially' all the forces in space :

$$\vec{R} = \Sigma \vec{F}$$

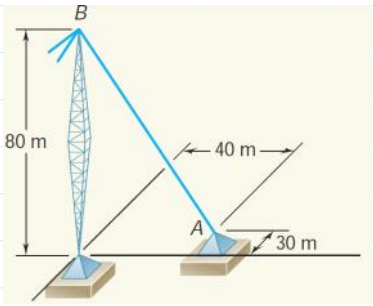
In component form, we have

$$R_x = \Sigma F_x$$

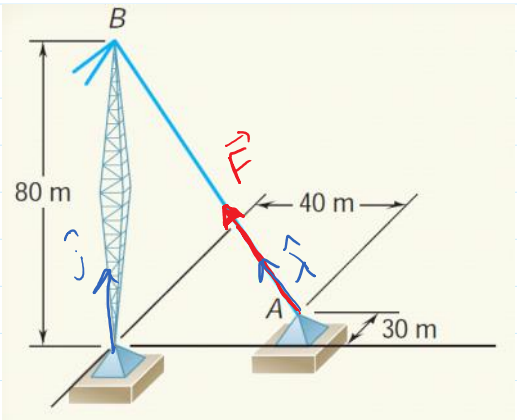
$$R_y = \Sigma F_y$$

$$R_z = \Sigma F_z$$

EXAMPLE :



A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 2500 N. Determine (*a*) the components F_x, F_y, F_z of the force acting on the bolt, (*b*) the angles θ_x, θ_y, θ_z defining the direction of the force.



The force is along AB & is directed from A to B.

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (0\hat{i} + 80\hat{j} + 0\hat{k}) - (40\hat{i} + 0\hat{j} - 30\hat{k})$$

$$= -40\hat{i} + 80\hat{j} + 30\hat{k}$$

$$= d_x\hat{i} + d_y\hat{j} + d_z\hat{k}$$

$$AB = |\vec{AB}| = d = \sqrt{d_x^2 + d_y^2 + d_z^2} \approx 94.3\ m$$

$$\vec{F} = F\hat{\lambda} = F \cdot \frac{\vec{AB}}{|AB|} = F \quad \frac{d_x\hat{i} + d_y\hat{j} + d_z\hat{k}}{}$$

$$\therefore \vec{F} = \frac{2500N}{94.3m}\left[(-40m)\hat{i} + (80m) + (30m)\hat{k}\right]$$

$$= (-1060N)\hat{i} + (2120N)\hat{j} + (795N)\hat{k}$$

F_x F_y F_z

Direction cosines:-

$$\cos\theta_x = \frac{f_x}{F} = \frac{d_x}{d} = \frac{-1060}{2500} \Rightarrow \theta_x \approx 115^\circ$$

$$\cos\theta_y = \frac{f_y}{F} = \frac{2120}{2500} \Rightarrow \theta_y \approx 32^\circ$$

$$\cos\theta_z = \frac{f_z}{F} = \frac{795}{2500} \Rightarrow \theta_z \approx 71.5^\circ$$