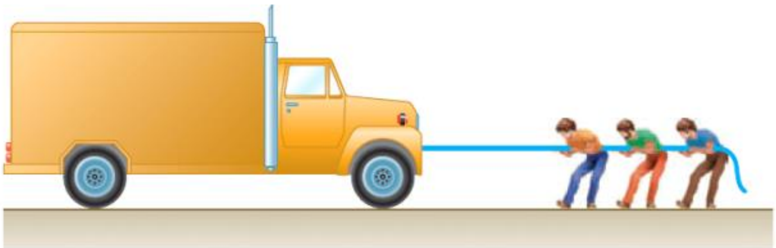
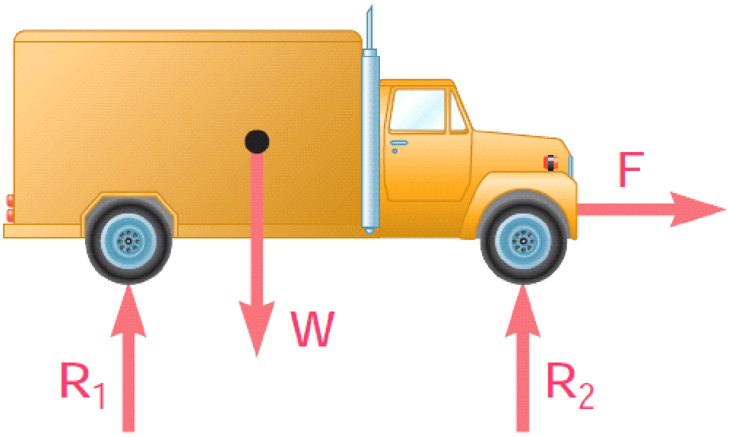
Rigid Bodies – Bodies which do not deform.

13 August 2018 13:59

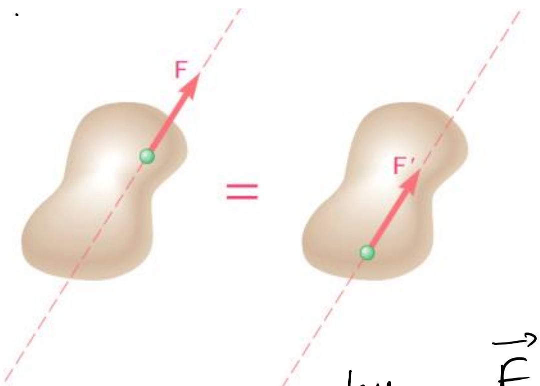
Forces on rigid bodies
- External forces – represent action of other bodies
- Internal forces – forces holding the body together





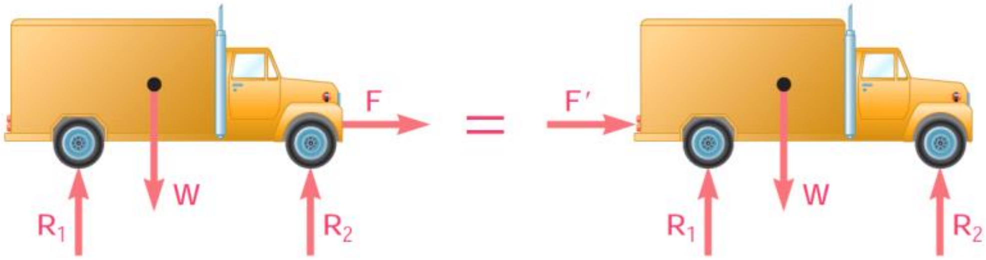
Principle of transmissibility : Equivalent forces

- States that conditions of equilibrium of motion remain unchanged if a force $\vec{F}$ acting at a point is replaced by a force $\vec{F}'$ of same magnitude & direction, but acting at a different point, provided that the two forces have the same line of action.



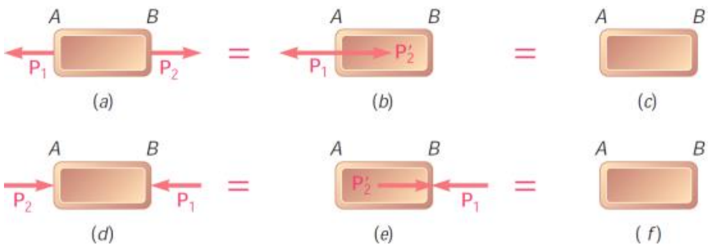
In figure below, we can replace $\vec{F}$ with $\vec{F}'$

In figure below, we can replace F with



force on front bumper   force on rear bumper

<u>Words of Caution</u> :- This principle may be freely used to determine conditions of motion or equilibrium of rigid bodies and to compute external forces on these bodies, it should be avoided to determine internal forces & deformations

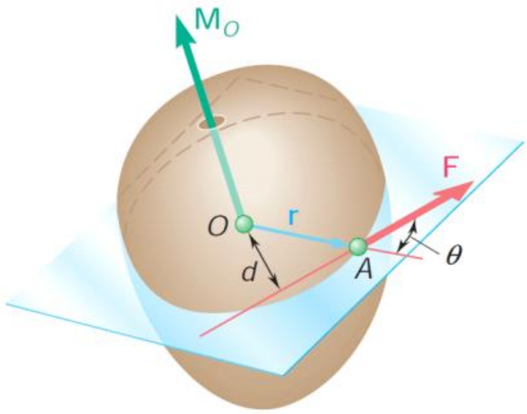


Case (a) & (d) are fundamentally different even though they both reduce to the same equivalents : (c) & (f).

(a) : Tension

(d) : Compression

<u>Moment of force about a point</u>



Moment of $\vec{F}$ about $\vec{O}$ is defined by

$$\boxed{\vec{M}_0 = \vec{r} \times \vec{F}}$$

$$M_0 = rF\sin\theta = Fd$$

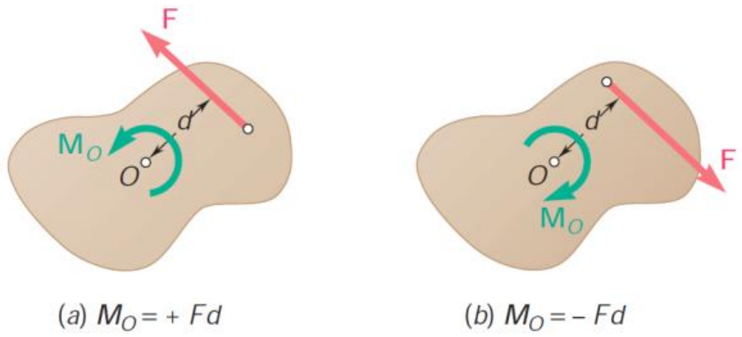
The magnitude of $\vec{M}_0$ measures the tendency of the force $\vec{F}$ to make the rigid body rotate about a fixed axis directed along $\vec{M}_0$

Principle of Transmissibility :- Two forces $\vec{F}$ & $\vec{F}''$ are equivalent if, and only if, they are equal (ie; same magnitude & direction) and have equal moments about a given point O.

Necessary & Sufficient condition for forces $\vec{F}$ & $\vec{F}'$ to be equivalent are :

$$\vec{F} = \vec{F}'$$
$$\vec{M}_0 = \vec{M}_0'$$

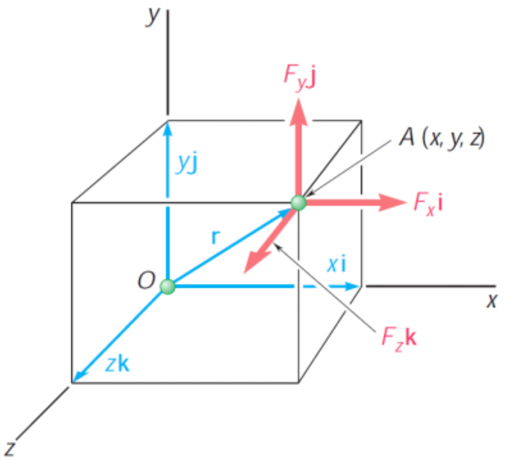
In 2D, we can simply use 'sign' to determine moment:



(a) $M_O = + Fd$ (b) $M_O = - Fd$

Rectangular components of moment:-

$$\vec{r} = \vec{r}_A - \vec{O}$$



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

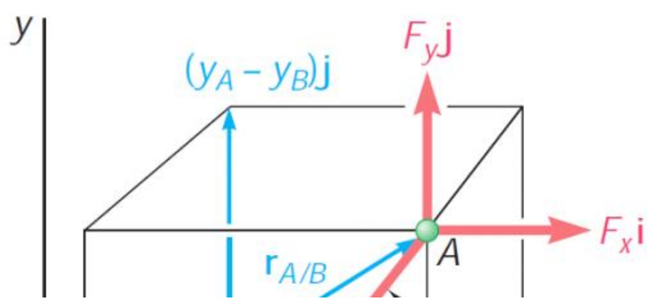
$$\vec{M}_O = \vec{r} \times \vec{F}$$

$$\begin{aligned} M_x &= yF_z - zF_y \\ M_y &= zF_x - xF_z \\ M_z &= xF_y - yF_x \end{aligned} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

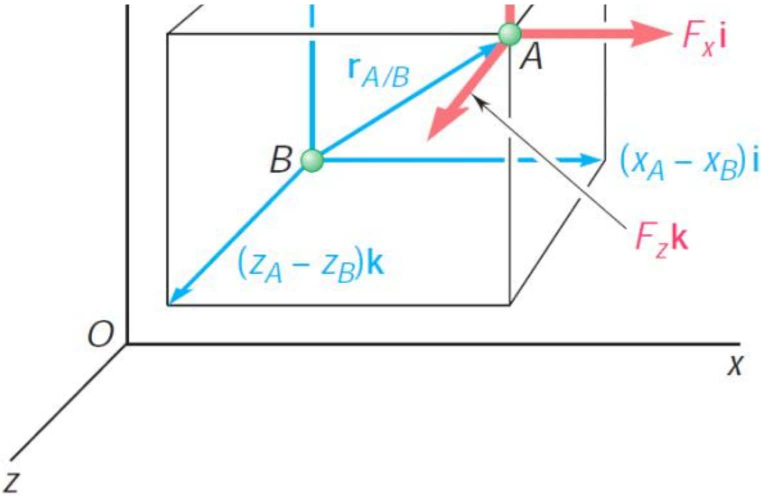
Moment about arbitrary point B:-

$$\vec{M}_B = r_{A/B} \times F = (\vec{r}_A - r_B) \times \vec{F}$$

$$M_B \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix}$$



$$\begin{vmatrix} F_x & F_y & F_z \end{vmatrix}$$



$$x_{A/B} = x_A - x_B$$

$$y_{A/B} = y_A - y_B$$

$$z_{A/B} = z_A - z_B$$

Moment about an axis:-

$$\vec{M}_{OL} = \hat{\lambda} \cdot \vec{M}_O = \hat{\lambda} \cdot (\vec{r} \times \vec{F})$$