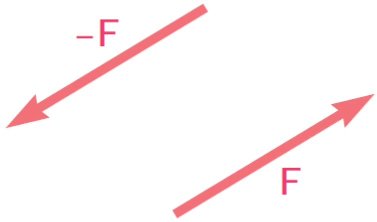
Moment of a Couple :-

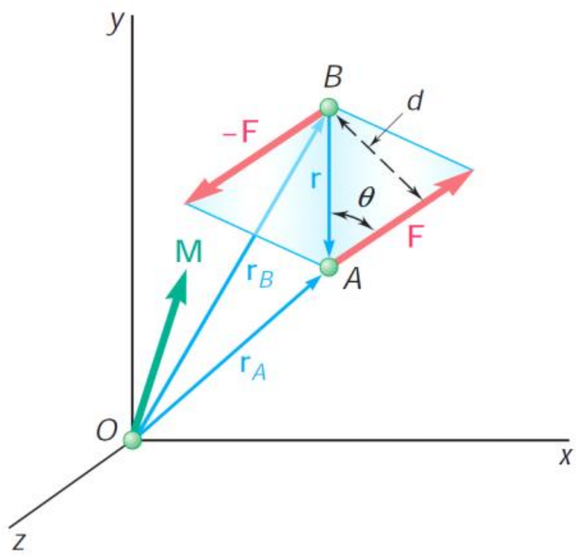


What is a Couple? Two forces $\vec{F}$ and $-\vec{F}$ having the same magnitude, parallel lines of action, and opposite sense are said to form a couple.

NOTE: Net resultant force $= 0$, but net moment $\neq 0$.

⇓ No translation

⇓ But the forces tend to rotate the object.



Denoting by $\vec{r}_A$ & $\vec{r}_B$, respectively, the position vectors of the points of application of $\vec{F}$ & $-\vec{F}$, the sum of moments of the two forces about O is :

$$\vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

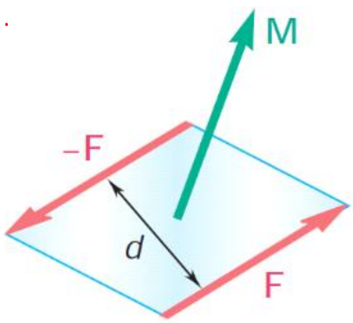
Defining $\vec{r}_A - \vec{r}_B = \vec{r}$, the sum of the moments of $\vec{F}$ & $-\vec{F}$ about O is :

$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

↳ Moment of the couple. } vector ⊥ar to the plane containing the two forces.

$$M = |\vec{M}| = rF\sin\theta$$
$$= Fd$$

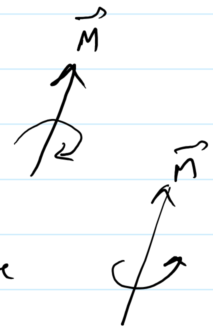
↳ ⊥ar distance between line of action of $\vec{F}$ & $-\vec{F}$.



Sense of $\vec{M}$ is defined by right-hand rule $\vec{M}$

Sense of $\vec{M}$ is defined by right-hand rule

- Clockwise : $-$ve
- Counter-clockwise : $+$ve

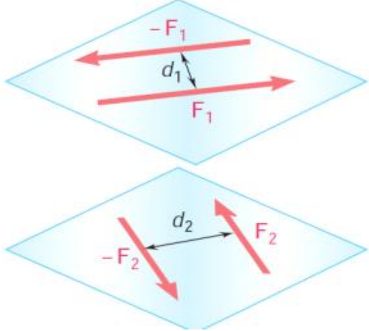


<u>Example</u>: Lug nut wrench

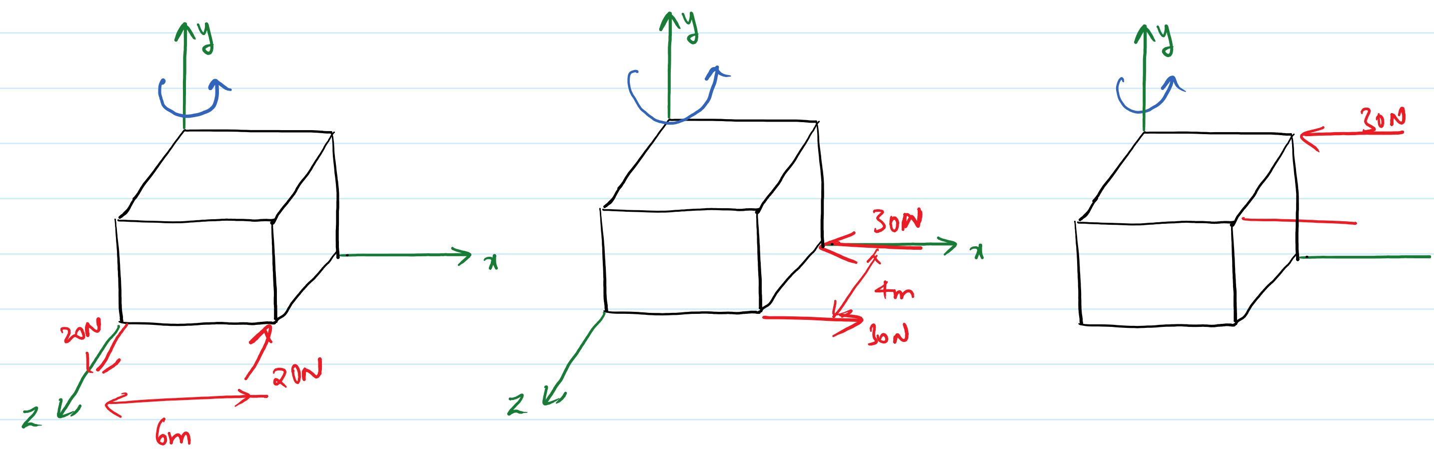
<u>**NOTE**</u>:

Two couples, one consisting of $\vec{F_1}$ & $-\vec{F_1}$ & the other of the forces $\vec{F_2}$ & $-\vec{F_2}$ will have equal moments if

$$F_1 d_1 = F_2 d_2$$

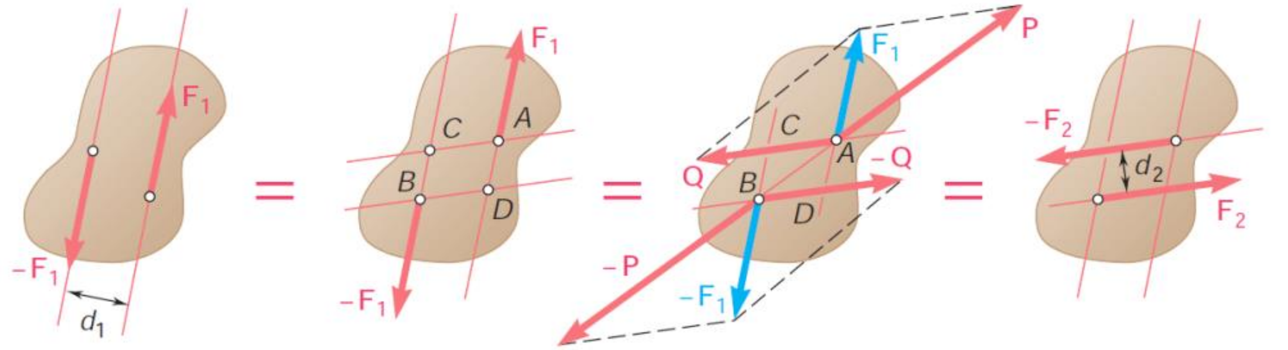


<u>EQUIVALENT COUPLES</u>



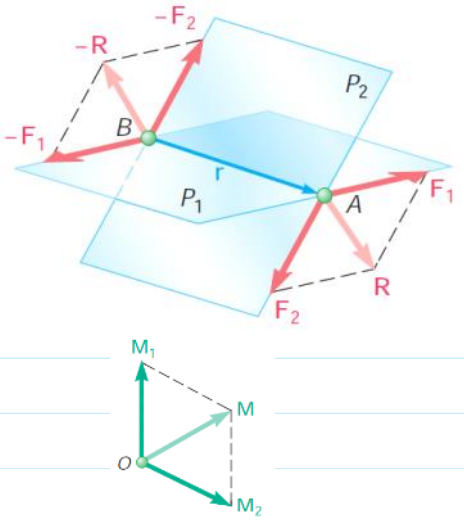
All the above couples are equivalent

Proof of $F_1 d_1 = F_2 d_2$

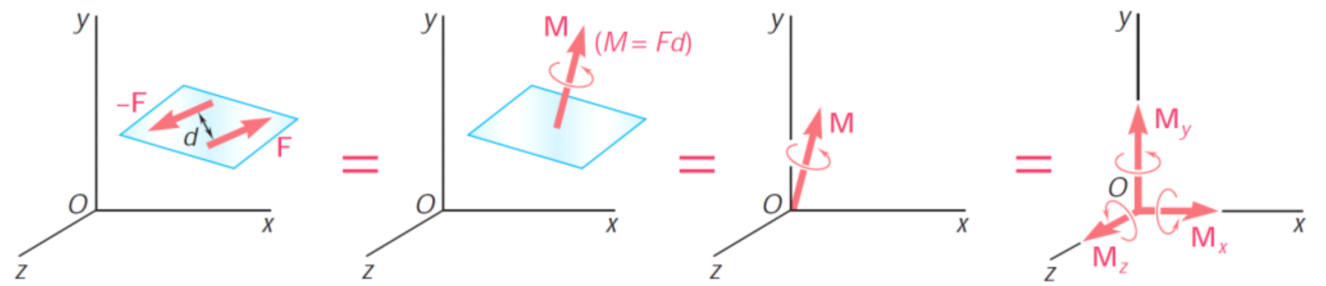


Addition of Couples

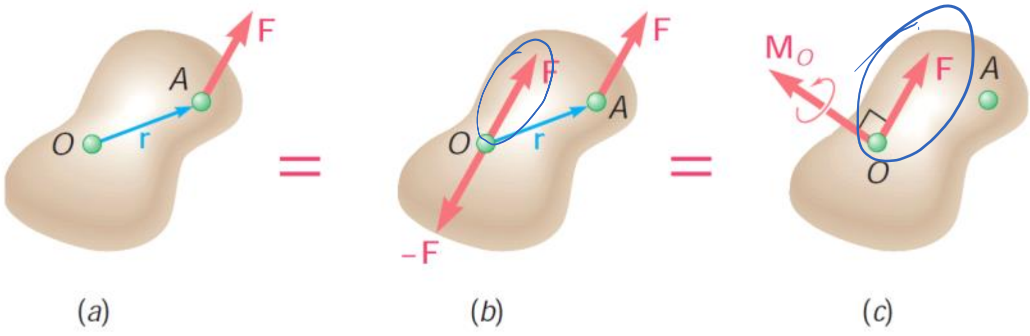
$$\vec{M} = \vec{r} \times \vec{R}$$
$$= \vec{r} \times (\vec{F_1} + \vec{F_2})$$
$$= \vec{r} \times \vec{F_1} + \vec{r} \times \vec{F_2}$$
$$= \vec{M_1} + \vec{M_2}$$



Represent Couples as vectors



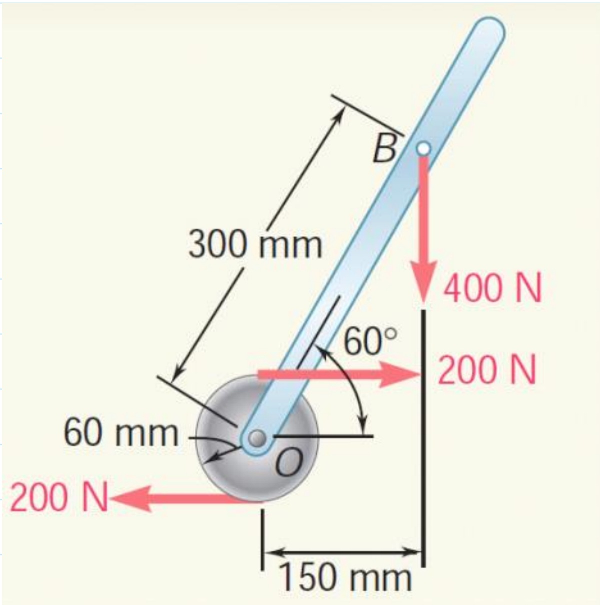
Resolution of a given force into force at 'O' and a Couple :-

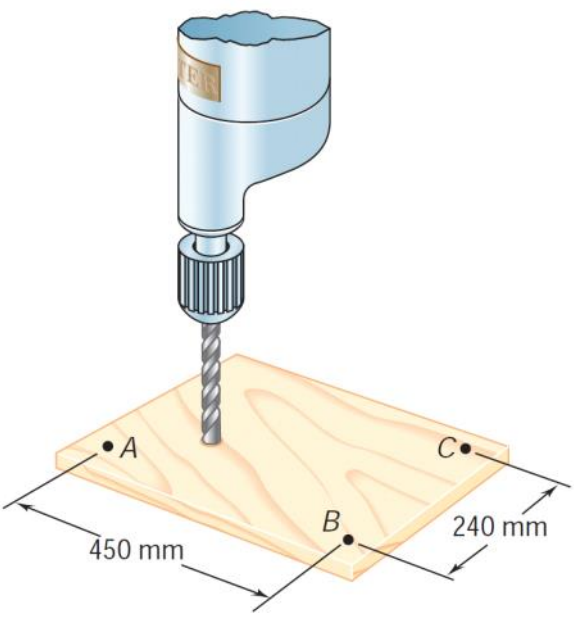


(a) (b) (c)

← force exerted by each hand on the wrench can be replaced with an equivalent force couple system acting on the nut.

Example:




A
C
B
450 mm
240 mm