Flow past a rectangular cylinder in a stratified fluid

Harish Dixit, A Sameen [1], Rama Govindarajan

Engineering Mechanics Unit,
Jawaharlal Nehru Centre for Advanced Scientific Research, Bangalore, India
hdixit@jncasr.ac.in, asameen@ictp.it, rama@jncasr.ac.in

Abstract

A 2D numerical simulation of flow past rectangular cylinders has been carried out in a stratified fluid using lattice Boltzmann methods (LBM). Stratification is achieved using a conventional two-component LBM model. Simulations correspond to a range of Reynolds numbers from 20 to 500 and Froude numbers of ∞, 10 and 5. It is shown that stratification could allow for merging of like-signed vortices downstream of the bluff body and creates a top-down asymmetry in the wake.

1. Introduction

The breakdown of a wake behind a bluff body is a classic example of shear flow instability. A comprehensive overview of the subject is provided in review papers by Williamson (1996); Zdravkovich (1996); Matsumoto (1999). When the Reynolds number, Re exceeds a critical value, a Karman vortex street associated with a periodic shedding appears.

Traditionally, studies on bluff body wakes have been restricted to those behind circular cylinders and spheres. Much less work seems to exist for flow past rectangular cylinders. A detailed experimental study was undertaken by Okajima (1982) to calculate the Strouhal numbers for vortex shedding behind rectangular cylinders. Davis et al. (1984) carried out a numerical simulation for flow past a square cylinder for a large range of Reynolds numbers, and the simulations were restricted to 2D. Mukhopadhyay et al. (1992) carried out two-dimensional simulations for a square cylinder to address the effect of confining walls on the wakes. Saha et al. (1999) studied the effect of inlet shear on wakes for a square cylinder, while Saha et al. (2000) carried out a DNS of flow past a square cylinder at Re=100 and examined the kinetic energy budget in the cylinder wake. All studies on rectangular cylinders, to our knowledge, have been carried out in unstratified flow.

For the case of vortex shedding in a stratified medium, Boyer et al. (1989) carried out an experimental study on flow past a cirular cylinder and proposed a scaling model for the separation angle and the length of the upstream blocked region. A large number of studies have been devoted to turbulent shedding behind towed bodies (see Spedding (1997, 2002), Xu et al. (1995)). Voropayev et al. (1999) studied the case of wake behing a maneuvering body and observed the formation of large scale coherent structures in a stratified fluid. An even larger volume of literature is concerned with the dynamics of self-propelled bodies, but is beyond the scope of the present work. Due to multiple separations both at the leading as well as the trailing edge, the shedding characteristics for a square cylinder could be very sensitive to density variations across it.

The present work is part of a ongoing effort to understand the influence of stratification on vortical structures, centered around studies on bluff body wakes. Here, we present

[1] Presently at the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy

results from two-dimensional numerical simulations of flow past rectangular cylinders in the presence of stratification. Note that in this study, the stratification is perpendicular to the direction of the vorticity.

2. Numerical Simulations

To simulate flow past a rectangular cylinder in a stratified medium, a two-component lattice Boltzmann method has been employed. The details of the method can be found in Dixit and Babu (2006) and Chen and Doolen (1998). The lattice Boltzmann method (LBM hereafter) is a discrete form of the continuous Boltzmann equation with suitable modifications to the collision operator and the equilibrium distribution function. Two distribution functions are used for simulating hydrodynamic and thermal fields respectively and the Boussinesq approximation is employed. The collision operator employs the Bhatnagar-Gross-Krook (BGK) approximation with polynomial equilibrium distributions for both the distribution functions.
The flow can be characterised by three non-dimensional numbers: Froude number, $Fr = \frac{U_\infty}{Nh}$; Prandtl number, $Pr = \frac{\nu}{\chi}$ and Reynolds number, $Re = \frac{U_\infty h}{\nu}$. Here N is the buoyancy frequency, U_∞ is the free stream velocity, h is the height of the bluff body, ν is the kinematic viscosity and χ is the thermal diffusivity. The aspect ratio also changes the shedding characteristics, and is defined as $A = h/d$ where d is the width of the body. In the present simulations, a very modest parameter range is covered: $Pr = 0.71$, $20 \leq Re \leq 400$, $Fr = 5,10,\infty$. The vortex shedding can be characterised by another non-dimensional number called the Strouhal number, $St = fh/U$ where f is the dominant shedding frequency.

3. Results

We compare the $St - Re$ relationship shown in fig.1 to the experimental result of Okajima (1982). The agreement is good except at high Re where the deviations could be because the signal was monitored for insufficient time. It can be observed in the fig.1 that the Strouhal number is not altered much due to stratification. However, stratification effects the critical Reynolds number at which shedding first appears. The shedding is delayed to Re=80 when Fr=5, w

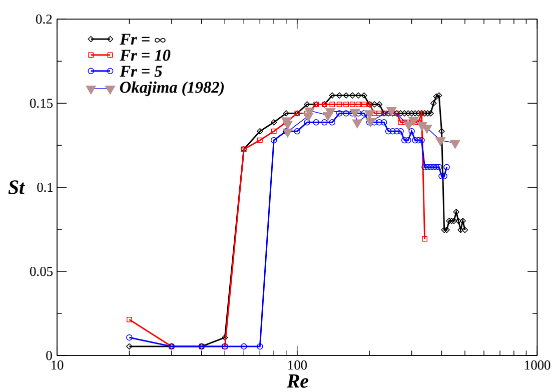


Figure 1: Variation of Strouhal number with inlet Reynolds number for Fr=∞, Fr=5 and Fr=10

3.1. Merging Events

The merging of similar signed vortices in the late wake has been observed. For example, Spedding (2002) towed a sphere in a stratified tank and showed the presence of merger events downstream of the sphere. He attributes the longevity of wakes in a stratified fluid to these merger events. We observe similar merger events (fig.2), but the main difference from the work of Spedding (2002) is that the direction of stratification is perpendicular to the vorticity. In the unstratified case (fig.2(a)) no merging events occur, atleast within the computational domain. In the stratified case with Fr=10 (fig.2(b)), the wake appears to be distorted but is otherwise similar to the unstratified case. With a further increase of stratification to Fr=5, merging events appear as can be seen in fig.2(c). Also, stratification is found to create a top-down asymmetry in the shedding downstream of the body. We don't completely understand the physical reasoning for this asymmetry, but would be discussed during the conference.

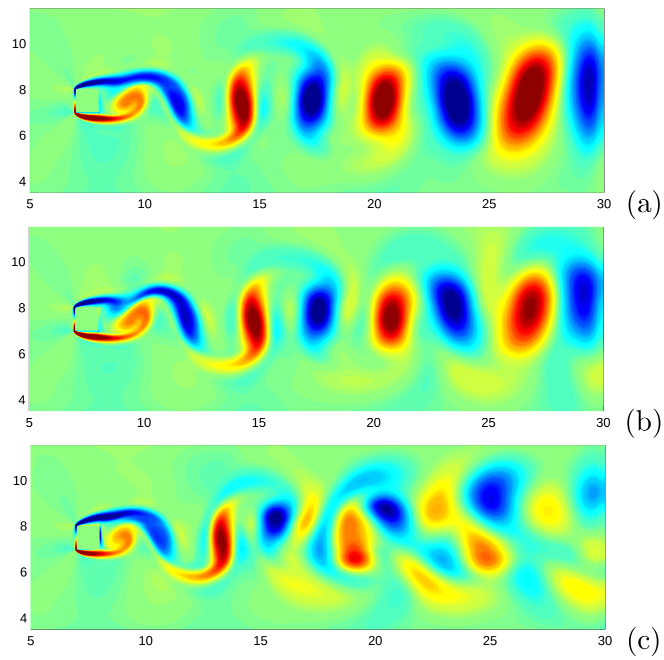


Figure 2: Vorticity contours for Re = 200, A = 1, at various Froude numbers: (a) $Fr = \infty$, (b) $Fr = 10$, (c) $Fr = 5$

There is a qualitative change between $Fr = 10$ and $Fr = 5$; the significance of this Froude number range is being studied. The shedding at $Fr = 10$ is very similar to the unstratified case as can be seen in the vorticity contours of fig.2.

The greater tendency for like-signed vortices to merge as stratification increases is similar to the secondary vortex streets observed by Inoue and Yamazaki (1999) for unstratified cylinder wakes. Stratification tends to push the counter-rotating vortices apart and in this aspect is similar to flow behind a body of higher aspect ratio. To verify this, we carry out simulations with a bluff body of aspect ratio 3. A schematic view of the domain is shown in fig.3. Figure 4 shows the formation of secondary vortex streets during the initial stages of the shedding process. It can be seen that the location where the secondary street first appears moves upstream as stratification is increased.
Figure 5 shows the vorticity contours for a bluff body of A = 3, Re = 200 and $Fr = \infty$ after the shedding has reached an equlibrium at t = 3,00,000. As can be clearly seen,

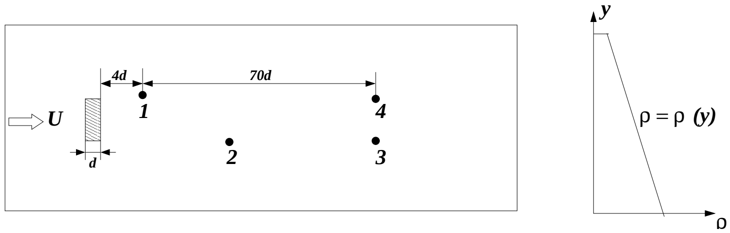


Figure 3: A schematic view of the computational domain showing four monitor points. The figure is not to scale.

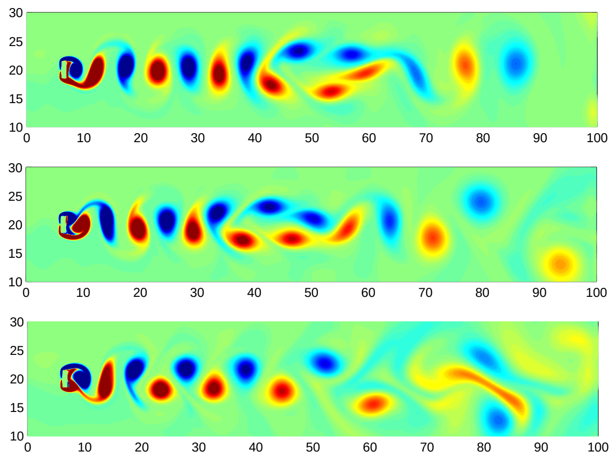


Figure 4: Vorticity contours for A = 3, Re = 200, time = 1,00,000 in LBM units, at various Froude numbers: (a) Fr = ∞, (b) Fr = 10, (c) Fr = 5

the separation between the vortex blobs has increased and therefore, there is a greater tendency for the merger of like-signed vortices. The unsteady time traces of total velocities from monitoring locations 1 and 4 is given in fig.6 along with the corresponding Strouhal numbers. The Strouhal number at location 4 being nearly half the value measured at location 1 is evidence of merger. We have verified that there is no top-down asymmetry.

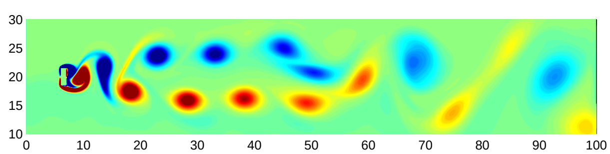


Figure 5: Vorticity contours for $A = 3$, $Fr = \infty$, $Re = 200$ and time = 3,00,000

However, a distinct asymmetry was found in a stratified case especially at $Fr = 5$. In fig.7, vorticity contours for a body with $A = 3$ at $Fr = 5$ and $Re = 200$ is shown. In fig.8, time traces of total velocity from monitor points 3 and 4 is presented. There is a discernible drop in the amplitude of oscillation between the two time traces, but the Strouhal number is found to be identical upto the fourth decimal place. Hence the asymmetry is in the amplitude of oscillation and the dominant frequency of vortex shedding is not altered.

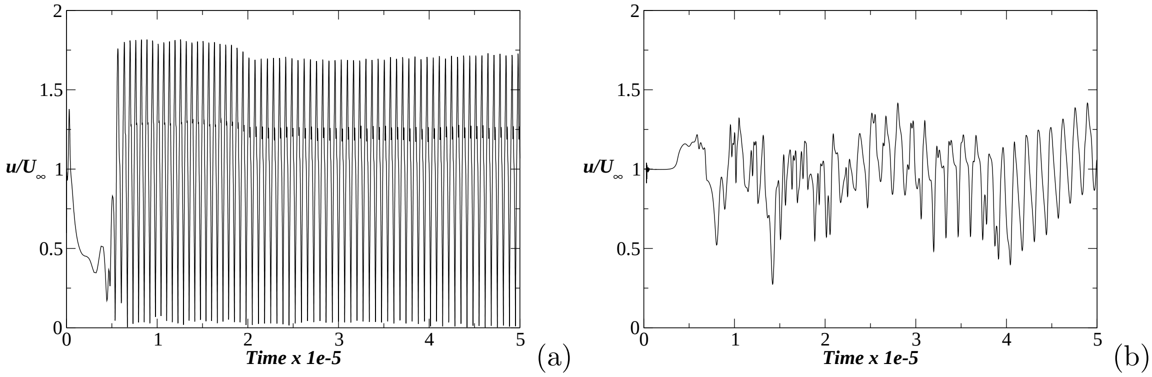
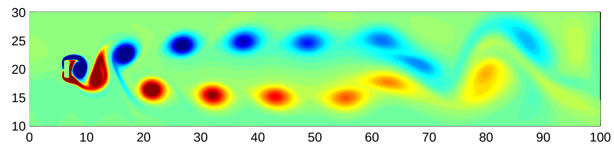


Figure 6: Time trace of total velocity downstream of the body for $A = 3$, $Fr = \infty$ and $Re = 200$ from locations 1 and 4. The St for (a) is 0.222 and for (b) is 0.1125



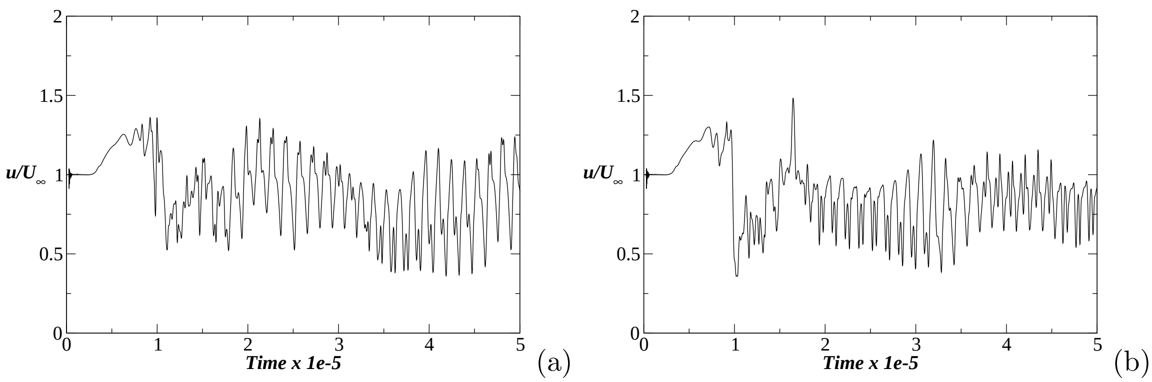


Figure 8: Time trace of total velocity downstream of the body for $A = 3$, $Fr = 5$ and $Re = 200$ from locations 3 and 4. The St for (a) and (b) is equal 0.1050

4. Conclusions

The vortex shedding behind a rectangular cylinder in a stratified medium has been studied using two-dimensional lattice Boltzmann simulations. It is shown that stratification tends to increase the critical Reynolds number at which shedding first appears. In addition, stratification promotes merger of like-signed vortices as evidenced by the local Strouhal number measured in the late wake to be reduced to half the value measured closer to the body. A top-down asymmetry is shown to exist because of stratification which is found to affect the local amplitude of the velocity, and not the frequency. This is currently being investigated.

Acknowledgements

We are grateful to NPOL, Kochi for funding this work, and to Dr. Rao Tatavarti for his keen interest.

References

Boyer, D., Davies, P., Fernando, H., and Zhang, X. (1989). Linearly stratified flow past a horizontal circular cylinder. *Phil. Trans. R. Soc. Lond. A*, 328:501–528.

Chen, S. and Doolen, G. (1998). Lattice Boltzmann method for fluid flows. *Ann. Rev. Fluid Mech.*, 30:329–364.

Davis, R., Moore, E., and Purtell, L. (1984). A numerical-experimental study of confined flow around rectangular cylinders. *Phys. Fluids*, 27(1):46–59.

Dixit, H. and Babu, V. (2006). Simulation of high Rayleigh number natural convection in a square cavity using the lattice Boltzmann method. *Intl. J. of Heat and Mass Transfer*, 49:727–739.

Inoue, O. and Yamazaki, T. (1999). Secondary vortex streets in two-dimensional cylinder wakes. *Fluid Dynamics Research*, 25:1–18.

Matsumoto, M. (1999). Vortex shedding of bluff bodies: A review. *J. of Fluids and Structures*, 13:791–811.

Mukhopadhyay, A., Biswas, G., and Sundararajan, T. (1992). Numerical investigation of confined wakes behind a square cylinder in a channel. *Intl. J. Num. Methods in Fluids*, 14(12):1473–1484.

Okajima, A. (1982). Strouhal numbers of rectangular cylinders. *J. of Fluid Mechanics*, 123:379–398.

Saha, A., Biswas, G., and Muralidhar, K. (1999). Influence of inlet shear on structure of wake behind a square cylinder. *J. Engg. Mech.*, 125(3):359–363.

Saha, A., Muralidhar, K., and Biswas, G. (2000). Vortex structures and kinetic energy budget in two-dimensional flow past a square cylinder. *Computers & Fluids*, 29(6):669–674.

Spedding, G. (1997). The evolution of initially turbulent bluff-body wakes at high internal Froude number. *J. of Fluid Mechanics*, 337:283–301.

Spedding, G. (2002). The streamwise spacing of adjacent coherent structures in stratified wakes. *Phys. Fluids*, 14(11):3820–3828.

Voropayev, S., McEachern, G., Fernando, H., and Boyer, D. (1999). Large vortex structures behind a maneuvering body in stratified fluids. *Phys. Fluids*, 11(6):1682–1684.

Williamson, C. (1996). Vortex dynamics in the cylinder wake. *Ann. Rev. of Fluid Mechanics*, 28:477–539.

Xu, Y., Fernando, H., and Boyer, D. (1995). Turbulent wakes of stratified flow past a cylinder. *Phys. Fluids*, 7(9):2243–2255.

Zdravkovich, M. M. (1996). Different modes of vortex shedding: An overview. *J. Fluids and Structures*, 10:427–437.