Minimum Conditional Probability of Error Based Multiuser Detection for Space-Time Coded CDMA Systems

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Abstract— **Space-time coding and multiuser detection are two widely used techniques that are known to substantially increase CDMA system capacity by using multiple transmit antennas and interference cancellation respectively. In this paper, we consider a space-time coded multiuser system based on real orthogonal designs. For this system, we study the performance of the minimum conditional probability of error (MCPOE) algorithm. The decision variable for MCPOE is the conditional probability of error, that is minimized to get the estimates of a set of filter coefficients. The filter coefficients are then used for symbol detection at the receiver. We find that the MCPOE algorithm scores over other adaptive multiuser detectors available in the literature, both in terms of convergence as well as bit error rate (BER) performance.**

Keywords – **Space-Time Coding, Multiuser Detection, MCPOE.**

I. Introduction

In recent years, there have been a lot of efforts toward improving system capacity to enable communication systems to provide high data rates for wireless networks without any increase in transmit power. Receive diversity schemes had been popular for quite a long time toward achieving this. However, it was shown in [1] that simple transmit diversity schemes, using two transmit antennas, with appropriate time interleaving could be used to provide diversity gain comparable to that obtained by maximal ratio receive combining (MRRC). This is achieved without any significant complexity at the receiver. The above ideas were further generalized for more than two transmit antennas, leading to the theory of generalized orthogonal designs [2].

Multiuser detection [3] has become very popular in 3G wireless communication systems due to its effectiveness in other-user interference cancellation. To combine the benefits of transmit diversity and interference cancellation, various models have been proposed for space-time coded multiuser detection. In [4], an optimal space-time multiuser receiver structure is derived for a multipath CDMA channel with receiver antenna arrays. For this model, blind adaptive multiuser detection techniques are proposed and shown to perform better than traditional single user methods. However, the adaptive techniques considered in this work are elementary. More advanced techniques are proposed for the space-time multiuser model derived in [5], where the stochastic gradient (SG), recursive least square (RLS), minimum output energy (MOE) and subspace-based MMSE (S-MMSE) algorithms are considered. Both the SG and RLS adaptive algorithms require training data while the MOE and S-MMSE algorithms do not require any training data.

A common feature of the above techniques is that they are based on some trivial function of the received vector. Recently, a few gradient descent adaptive algorithms have been proposed which, instead of using such functions, use the probability of error as the cost function for computation of the gradient.

The MCPOE algorithm is one such gradient descent algorithm proposed in [6] as a multiuser detection technique for a CDMA system using multiple receive antennas in a multipath fading environment. However, the spreading codes of the users are not used for despreading at the receiver but incorporated into the algorithm itself. In this paper, we investigate the performance of the MCPOE algorithm when multiple transmit antennas are used and focus on the system model proposed in [5].

II. System Model

We consider the system model derived in [5] where K-users transmit data bits synchronously with each user employing M antennas for transmission over Q time slots, each slot of duration T. Real orthogonal space-time block codes [2] are used. In this paper, for the sake of simplicity, we consider $M = Q$ and omit the effect of fading. Let $s_k(t), k \in \{1, 2, \ldots, K\}, t \in (0, T)$, be the signature sequence of the k^{th} user and b_{kq}, $q \in \{1, 2, \ldots, Q\}$, be the bit transmitted by user k in time slot q with amplitude A_{kq}. The transmitted signals are corrupted by AWGN with zero mean and variance σ^2.

We define the correlation coefficient between the j^{th} and k^{th} user as

$$\rho_{jk} = \int_0^T s_j(t) s_k(t) dt. \tag{1}$$

The $K \times K$ correlation matrix is then defined as [3]

$$R = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1K} \\ \rho_{21} & 1 & \cdots & \rho_{2K} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_{K1} & \rho_{K2} & \cdots & 1 \end{pmatrix}. \tag{2}$$

Also, let

$$A_q = diag\left(A_{1q} \cdots A_{Kq}\right), q = 1 \ldots Q. \tag{3}$$

For $M = Q = 8$, the received vector for the above system is given by [5]

$$\mathbf{y} = CA\mathbf{b} + \eta. \tag{4}$$

In (4), $\mathbf{b}$ is the composite bit vector comprising of the bits transmitted by all users and is given by

$$\begin{aligned} \mathbf{b}_q &= \left(b_{1q} \cdots b_{Kq}\right)^t, q = 1 \ldots Q, & (5) \\ \mathbf{b} &= \left(\mathbf{b}_1^t \cdots \mathbf{b}_Q^t\right)^t, & (6) \end{aligned}$$

where $(^t)$ is the matrix transpose operation. The amplitude matrix

$$A = \begin{pmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \\ -A_2 & A_1 & A_4 & -A_3 & A_6 & -A_5 & -A_8 & A_7 \\ -A_3 & -A_4 & A_1 & A_2 & A_7 & A_8 & -A_5 & -A_6 \\ -A_4 & A_3 & -A_2 & A_1 & A_8 & -A_7 & A_6 & -A_5 \\ -A_5 & -A_6 & -A_7 & -A_8 & A_1 & A_2 & A_3 & A_4 \\ -A_6 & A_5 & -A_8 & A_7 & -A_2 & A_1 & -A_4 & A_3 \\ -A_7 & A_8 & A_5 & -A_5 & -A_3 & A_4 & A_1 & -A_2 \\ -A_8 & -A_7 & A_6 & A_6 & -A_4 & -A_3 & A_2 & A_1 \end{pmatrix}. \tag{7}$$

For $Q < 8$, the amplitude matrix can be obtained as the upper leftmost square sub-matrix of dimension Q. The space-time correlation matrix

$$C = \begin{pmatrix} R & 0 & \cdots & 0 \\ 0 & R & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & R \end{pmatrix}. \tag{8}$$

η is a $QK \times 1$ Gaussian random vector with

$$\begin{aligned} E\left[\eta\right] &= 0, & (9) \\ E\left[\eta\eta^t\right] &= \sigma^2 C. & (10) \end{aligned}$$

The received vector $\mathbf{y}$ has a structure similar to the composite bit vector $\mathbf{b}$ and is defined as

$$\begin{aligned} \mathbf{y}_q &= \left(y_{1q} \cdots y_{Kq}\right)^t, q = 1 \ldots Q, & (11) \\ \mathbf{y} &= \left(\mathbf{y}_1^t \cdots \mathbf{y}_Q^t\right)^t. & (12) \end{aligned}$$

Since $C = diag\left(R \ldots R\right)$ is a $QK \times QK$ block diagonal matrix, where R is the correlation matrix, C is symmetric as well as positive definite. Hence there exists a matrix F such that $C = F^t F$ (Cholesky decomposition [7]) . Multiplying the vector $\mathbf{y}$ in (4) by $(F^t)^{-1}$, we have the modified equation as

$$\hat{\mathbf{y}} = FA\mathbf{b} + \sigma\mathbf{n}, \tag{13}$$

where $\mathbf{n} = (F^t)^{-1}$. Also,

$$\begin{aligned} E\left[\mathbf{n}\right] &= 0, & (14) \\ E\left[\mathbf{n}\mathbf{n}^t\right] &= \sigma^2 I, & (15) \end{aligned}$$

where I is the $QK \times QK$ identity matrix.

III. The Adaptive Algorithm

We define the decision variable for the kq^{th} bit of the vector $\mathbf{b}$ defined in the previous section as

$$\begin{aligned} D_{kq} &= \mathbf{w}_{kq}^t \hat{\mathbf{y}} & (16) \\ &= \mathbf{w}_{kq}^t FA\mathbf{b} + \sigma\mathbf{w}^t\mathbf{n}, \end{aligned}$$

where $\mathbf{w}_{kq}$ denotes the linear filter used to demodulate the kq^{th} element of $\mathbf{b}$.

The mean μ_{kq} and variance σ_{kq} of D_{kq}, conditioned on $\mathbf{b}$ are obtained from (13) and (16) as

$$\begin{aligned} \mu_{kq} &= \mathbf{w}_{kq}^t FA\mathbf{b}, & (17) \\ \sigma_{kq}^2 &= \sigma^2 \|\mathbf{w}_{kq}\|^2. \end{aligned}$$

We now refer to [6] for the derivation of the minimum conditional probability of error (MCPOE) algorithm. $P_{kq|\mathbf{b}}$ is the probability of error in the demodulation of the q^{th} bit transmitted by the k^{th} user conditioned on $\mathbf{b}$. If $\mathbf{b}^{kq-}$ and $\mathbf{b}^{kq+}$ are obtained from $\mathbf{b}$ by replacing the kq^{th} bit with -1 and $+1$ respectively, assuming that users transmit bits ($+1$ and -1) with equal probability,

$$\begin{aligned} P_{kq|\mathbf{b}} &= Pr(D_{kq} < 0|\mathbf{b}^{kq+}) + Pr(D_{kq} > 0|\mathbf{b}^{kq-}) \\ &= \frac{1}{2}Q\left(\frac{\mu_{kq+}}{\sigma_{kq}}\right) + \frac{1}{2}Q\left(-\frac{\mu_{kq-}}{\sigma_{kq}}\right), & (18) \end{aligned}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy, \quad x > 0 \tag{19}$$

and μ_{kq+} and μ_{kq-} are obtained from (17) by substituting $\mathbf{b}$ with $\mathbf{b}^{kq+}$ and $\mathbf{b}^{kq-}$ respectively.

Using the gradient descent approach, estimates for the filter $\mathbf{w}_{kq}$ can be obtained as

$$\mathbf{w}_{kq}^{(i+1)} = \mathbf{w}_{kq}^{(i)} - \lambda \nabla P_{kq|\mathbf{b}}^{(i)}, \tag{20}$$

where λ is the step size and

$$\begin{aligned} \nabla P_{kq|\mathbf{b}} = {} & \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\mu_{kq-}^2}{\sigma_{kq}^2}\right) \frac{\left(\|\mathbf{w}_{kq}\|^2 FA\mathbf{b}^{\mathbf{kq}-}\right) - \mu_{kq-}\mathbf{w}_{kq}}{2\sigma\|\mathbf{w}_{kq}\|^3} & (21) \\ & - \frac{1}{\sqrt{2\pi}} exp\left(-\frac{\mu_{kq+}^2}{\sigma_{kq}^2}\right) \frac{\left(\|\mathbf{w}_{kq}\|^2 FA\mathbf{b}^{\mathbf{kq}+}\right) - \mu_{kq+}\mathbf{w}_{kq}}{2\sigma\|\mathbf{w}_{kq}\|^3}. \end{aligned}$$

as shown in the Appendix. From (16) and (20), the estimate for the transmitted bit is

$$b_{kq} = sgn\left(D_{kq}\right). \tag{22}$$

IV. Results and Discussion

The convergence curves for the MCPOE algorithm for the system considered in this paper are shown in Fig. 1 when there are 2 users who use 2 transmit antennas with only a single antenna at the receiver. For both the signal to noise ratio (SNR) values considered (7 dB and 10 dB), we find that the filter coefficients converge to a more or less steady value within 10 iterations. Thus, the MCPOE algorithm is much faster than

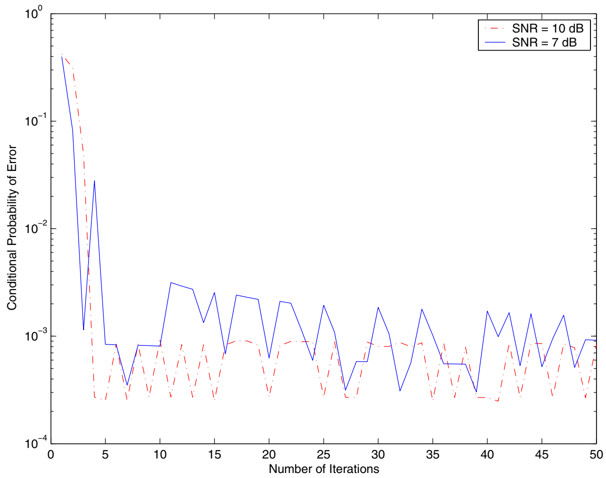

Fig. 1. Convergence curves for the MCPOE algorithm using one receive antenna. For $M = 2, K = 2$ and $\rho = 0.2$, where ρ is the correlation between the two users.

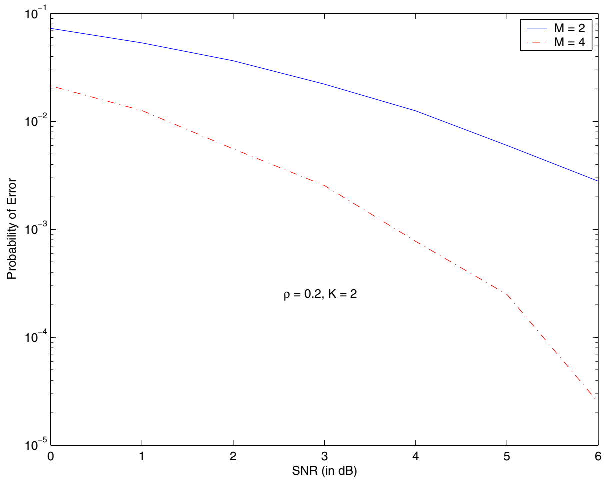

Fig. 2. The BER curves using a single receive antenna. $\mathbf{w}_{kq}$ is estimated using the MCPOE algorithm and used to compute the decision variable D_{kq}.

most adaptive multiuser detectors discussed in [5] and compares favourably to the subspace based MMSE detector.

In Fig. 2, the transmit diversity gain is obvious from the BER performance of the MCPOE algorithm in a 2 user scenario when 2 and 4 transmit antennas are used along with one receive antenna. Also, the BER performance for the 2 transmit antenna case is closer to the single user performance than the other multiuser detection techniques introduced in [5].

V. Conclusions

In this paper, we have done a performance study of the MCPOE algorithm for a system using multiple transmit antennas. We find that the algorithm converges quite fast and can be successfully applied not only in a multiple receive antenna environment [6] but also to achieve transmit diversity.

Appendix

The expression

$$\frac{\partial}{\partial \mathbf{w}_{kq}} Q\left(\frac{\mu_{kq+}}{\sigma_{kq}}\right) = Q^{'}\left(\frac{\mu_{kq+}}{\sigma_{kq}}\right) \frac{\sigma_{kq}\frac{\partial \mu_{kq+}}{\partial \mathbf{w_{kq}}} - \mu_{kq+}\frac{\partial \sigma_{kq}}{\partial \mathbf{w_{kq}}}}{\sigma_{kq}^2}. \tag{23}$$

From (19) we obtain the derivative of $Q(x)$ as

$$Q^{'}(x) = -\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}. \tag{24}$$

From (24), we have

$$Q^{'}\left(\frac{\mu_{kq+}}{\sigma_{kq}}\right) = -\frac{1}{\sqrt{2\pi}} exp\left(-\frac{{\mu_{kq+}}^2}{{\sigma_{kq}}^2}\right). \tag{25}$$

Also, from (17),

$$\frac{\partial \mu_{kq+}}{\partial \mathbf{w_{kq}}} = \frac{\partial\left(\mathbf{w}_{kq}^t FA\mathbf{b}^{kq+}\right)}{\partial \mathbf{w_{kq}}} = FA\mathbf{b}^{kq+}, \tag{26}$$

$$\begin{aligned} \frac{\partial \sigma_{kq}}{\partial \mathbf{w_{kq}}} &= \frac{\partial\left(\sigma\|\mathbf{w}_{kq}\|\right)}{\partial \mathbf{w_{kq}}} = \sigma\frac{\partial\left(\sqrt{\mathbf{w}_{kq}^t\mathbf{w}_{kq}}\right)}{\partial \mathbf{w_{kq}}} \\ &= \frac{\sigma}{\sqrt{\mathbf{w}_{kq}^t\mathbf{w}_{kq}}}\mathbf{w}_{kq} = \sigma\frac{\mathbf{w}_{kq}}{\|\mathbf{w}_{kq}\|}. \end{aligned} \tag{27}$$

From (25), (26) and (27), (23) can be written as

$$\begin{aligned} \frac{\partial}{\partial \mathbf{w}_{kq}} Q\left(\frac{\mu_{kq+}}{\sigma_{kq}}\right) &= -\frac{1}{\sqrt{2\pi}} exp\left(-\frac{{\mu_{kq+}}^2}{{\sigma_{kq}}^2}\right) \\ &\times \frac{\sigma\|\mathbf{w}_{kq}\|FA\mathbf{b}^{kq+} - \sigma\mu_{kq+}\frac{\mathbf{w}_{kq}}{\|\mathbf{w}_{kq}\|}}{\sigma^2\|\mathbf{w}_{kq}\|^2} \\ &= -\frac{1}{\sqrt{2\pi}} exp\left(-\frac{{\mu_{kq+}}^2}{{\sigma_{kq}}^2}\right) \\ &\times \frac{\|\mathbf{w}_{kq}\|^2 FA\mathbf{b}^{kq+} - \mu_{kq+}\mathbf{w}_{kq}}{\sigma\|\mathbf{w}_{kq}\|^3}. \end{aligned} \tag{28}$$

Similarly,

$$\begin{aligned} \frac{\partial}{\partial \mathbf{w}_{kq}} Q\left(-\frac{\mu_{kq-}}{\sigma_{kq}}\right) &= \frac{1}{\sqrt{2\pi}} exp\left(-\frac{{\mu_{kq-}}^2}{{\sigma_{kq}}^2}\right) \\ &\times \frac{\|\mathbf{w}_{kq}\|^2 FA\mathbf{b}^{kq-} - \mu_{kq-}\mathbf{w}_{kq}}{\sigma\|\mathbf{w}_{kq}\|^3}. \end{aligned} \tag{29}$$

(21) is then obtained from (18), (28) and (29).

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