Exact Error Analysis for the Piecewise Linear Combiner for Decode and Forward Cooperation with Two Relays

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***Abstract*—Exact expressions are obtained for the bit error rate (BER) for coherent and noncoherent decode and forward (DF) cooperative systems with two relays. The piecewise linear (PL) combiner, a close approximation to the maximum likelihood (ML) detector, is employed at the receiver. BER analysis is done using a contour integral approach for evaluating the Gil-Pelaez integral involving the characteristic function (CF) of the decision variable. Previous attempts relied on a direct approach, that imposed restrictions on the location of the relays. In this paper, this restriction has now been removed. Through simulation results, we verify the validity of the derived analytical expressions.**

***Index Terms*—Cooperative diversity, Gil-Pelaez, Residue theorem, PL, DF.**

I. INTRODUCTION

The performance of maximum likelihood (ML) decode and forward (DF) cooperative systems [1]–[7] is of considerable interest as it provides a benchmark for all other cooperative schemes. Exact bit error rate (BER) analysis for the piecewise linear (PL) combiner was provided in [8] and [4] for coherent and noncoherent detection respectively, for a single relay system employing DF. For noncoherent PL-DF in a two relay system, exact expressions for the BER were obtained in [9]. However, due to the multiple integration regions in the BER analysis, the results in [9] were limited to the case when both relays were located at the same distance from the source, a highly impractical scenario.

The contour integral approach for evaluating the BER for wireless communication systems in fading channels is well known [10]. Using this technique, we obtain exact expressions for the BER for BPSK and BFSK based modulation for coherent and noncoherent PL-DF cooperative systems respectively. A major advantage of this method is that it is much simpler compared to the direct approach, resulting in a general expression for the BER with no restriction on the relay location, unlike [9]. Though the results in this paper are limited to the case of two relays, the extension for multiple relays is straightforward, any ensuing complications not due to the approach, but in the multiplication of algebraic expressions. This will be evident from the later sections.

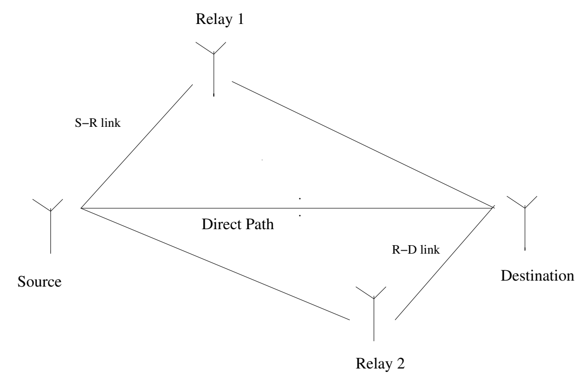

Fig. 1. Cooperative diversity system with 2 relays.

II. SYSTEM MODEL

We consider the model in [4], [8] with two relays (R_1, R_2) between the source (S) and destination (D) as shown in Figure 1. The DF method is employed at the relay followed by ML decision at the destination. The modulation technique is binary phase shift keying (BPSK) for coherent detection, and binary frequency shift keying (BFSK) for noncoherent detection; all transmissions being on orthogonal channels.

A. Coherent Detection with BPSK

let $x_s \in \{1, -1\}$ and $x_r \in \{1, -1\}$ be the bits transmitted at the source and rth relay with powers E_s and E_r respectively. Henceforth, without loss of generality, the source, relay and destination parameters will be represented by the subscripts s, r and d respectively. The received symbols on the S-D, S-R and R-D links are respectively

$$\begin{aligned} y_{d,s} &= \sqrt{E_s} h_{d,s} x_s + z_{d,s}, \\ y_{r,s} &= \sqrt{E_s} h_{r,s} x_s + z_{r,s}, \\ y_{d,r} &= \sqrt{E_r} h_{d,r} x_r + z_{d,r}, \end{aligned} \quad (1)$$

for $r = 1, 2$. Also, $z_{d,s}, z_{r,s}, z_{d,r} \sim \mathcal{CN}(0, N_0)$ represent the additive white Gaussian noise (AWGN) at the respective relays and the destination. The channel experiences Rayleigh fading, hence the fading coefficients $h_{d,s} \sim \mathcal{CN}(0, \Omega_{d,s}), h_{r,s} \sim \mathcal{CN}(0, \Omega_{r,s})$ and $h_{d,r} \sim \mathcal{CN}(0, \Omega_{d,r})$ are zero mean complex circularly Gaussian.

B. Noncoherent Detection with BFSK

For BFSK, let $x_s \in \{0, 1\}$ and $x_r \in \{0, 1\}$. Assuming flat fading, for $m \in \{0, 1\}$, the received symbols on the S-D, S-R and R-D links are respectively

$$\begin{aligned} y_{d,s}^m &= (m + (-1)^m x_s)\sqrt{E_s} h_{d,s} + z_{d,s}^m, \\ y_{r,s}^m &= (m + (-1)^m x_s)\sqrt{E_s} h_{r,s} + z_{r,s}^m, \\ y_{d,r}^m &= (m + (-1)^m x_r)\sqrt{E_r} h_{d,r} + z_{d,r}^m, \end{aligned} \tag{2}$$

where $z_{d,s}^m, z_{r,s}^m, z_{d,r}^m \sim \mathcal{CN}(0, N_0)$ represent the additive white Gaussian noise (AWGN) at the relay and the destination.

C. ML Decision

The ML decision criterion at the destination is obtained from [4], [8] as

$$X + \sum_{r=1}^{2} f(Y_r) \underset{-1}{\overset{1}{\gtrless}} 0, \tag{3}$$

$$\begin{aligned} X &= \frac{4\sqrt{E_s}\text{Re}\{h_{d,s}^* y_{d,s}\}}{N_0}, \\ Y_r &= \frac{4\sqrt{E_r}\text{Re}\{h_{d,r}^* y_{d,r}\}}{N_0}, \end{aligned} \tag{4}$$

for BPSK modulation and

$$X + \sum_{r=1}^{2} f(Y_r) \underset{0}{\overset{1}{\gtrless}} 0, \tag{5}$$

$$\begin{aligned} X &= \frac{\Omega_{d,s} E_s}{(\Omega_{d,s} E_s + N_0) N_0} \left(|y_{d,s}^0|^2 - |y_{d,s}^1|^2\right), \\ Y_r &= \frac{\Omega_{d,r} E_r}{(\Omega_{d,r} E_r + N_0) N_0} \left(|y_{d,r}^0|^2 - |y_{d,r}^1|^2\right), \end{aligned} \tag{6}$$

$$f(t) = \ln \frac{\delta + e^t}{1 + \delta e^t}, \quad 0 < \delta < 1 \tag{7}$$

for noncoherent BFSK, where $\{*\}$ denotes the complex conjugate operation. Also, $f(Y_r)$ has the parameter $\delta_r = \frac{\epsilon_r}{1-\epsilon_r}$ for the rth relay, where ϵ_r is the BER for the S-R link. Introducing suboptimality by using the average BER in place of the instantaneous BER [5], $\epsilon = \left(2 + \frac{\Omega_{r,s} E_s}{N_0}\right)^{-1}$ for noncoherent detection and $\frac{1}{2}\left[1 - \left(1 + \frac{N_0}{\Omega_{r,s} E_s}\right)^{-\frac{1}{2}}\right]$ for coherent detection.

D. PL combiner

The PL combiner for ML-DF cooperative systems is obtained by using the following piecewise linear approximation for $f(t)$ [4] in (7)

$$f(t) \approx \begin{cases} \ln \frac{1}{\delta} & t \geq \ln \frac{1}{\delta} \\ t & \ln \delta < t < \ln \frac{1}{\delta} \\ \ln \delta & t < \ln \delta \end{cases}. \tag{8}$$

E. Problem Definition

Assuming equal probability of the symbols $x_s \in \{\{1, -1\}, \{0, 1\}\}$, the average probability of error for the ML-DF cooperative diversity system in Figure 1 can be expressed as

$$P_e = \begin{cases} \sum_{\mathbf{x}} \prod_{r=1}^{2} \epsilon_r^{\frac{1-x_r}{2}} (1-\epsilon_r)^{\frac{1+x_r}{2}} \\ \quad \times P\left(X + \sum_{i=1}^{2} f(Y_i) < 0 | x_s = 1, \mathbf{x}\right) & \text{(BPSK)}, \\ \sum_{\mathbf{x}} \prod_{r=1}^{2} \epsilon_r^{1-x_r} (1-\epsilon_r)^{x_r} \\ \quad \times P\left(X + \sum_{i=1}^{2} f(Y_i) < 0 | x_s = 1, \mathbf{x}\right) & \text{(BFSK)}, \end{cases} \tag{9}$$

where $\mathbf{x} = (x_1, x_2)$ is the set of all possible 2-tuples formed by the symbols transmitted by the relays. We wish to find out a closed form expression for P_e in (9).

F. Statistics of X and Y_r

The PDF and CF of X are given by[1] [4], [8]

$$p_X(x) = \begin{cases} g e^{-\beta x} & x \geq 0 \\ g e^{\alpha x} & x < 0, \end{cases} \tag{10}$$

$$\phi_X(\jmath\omega) = \frac{g(\alpha + \beta)}{(\beta - \jmath\omega)(\alpha + \jmath\omega)}, \tag{11}$$

where, α, β are given in Table I and

$$\frac{1}{g} = \frac{1}{\alpha} + \frac{1}{\beta}. \tag{12}$$

It is obvious that even Y_r has similar statistics with parameters α_r, β_r expressed in terms of E_r and $\Omega_{d,r}$. The CF of $V = f(X)$ for the PL approximation is given by [8]

$$\phi_V(\jmath\omega) = [1 - g\phi(\alpha, \jmath\omega) - g\phi(\beta, -\jmath\omega)]. \tag{13}$$

where $\phi(x, t) = \frac{t(1-\delta^{x+t})}{x(x+t)}$.

[1]The subscript s being dropped for convenience

Modulation Scheme	ML Decision	X, Y_r	ϵ_r	α_i, β_i
BPSK	$X + \sum_{r=1}^{n} f(Y_r) \underset{-1}{\overset{1}{\gtrless}} 0$	$\frac{4\sqrt{E_i}\text{Re}\{h_{d,i}^* y_{d,i}\}}{N_0}$	$\frac{1}{2}\left[1 - \left(1 + \bar{\gamma}_{r,s}^{-1}\right)^{-\frac{1}{2}}\right]$	$\frac{1}{2}\left(\sqrt{1 + \frac{1}{\bar{\gamma}_{d,i}}} \pm 1\right)$
Coherent BFSK	$X + \sum_{r=1}^{n} f(Y_r) \underset{0}{\overset{1}{\gtrless}} 0$	$\frac{2\sqrt{E_s}}{N_0}\text{Re}\left[\left(y_{d,i}^0 - y_{d,i}^1\right) h_{d,i}^*\right]$	$\frac{1}{2}\left[1 - \left(1 + 2\bar{\gamma}_{r,s}^{-1}\right)^{-\frac{1}{2}}\right]$	$\frac{1}{2}\left(\sqrt{1 + \frac{2}{\bar{\gamma}_{d,i}}} \pm 1\right)$
Nonoherent BFSK	$X + \sum_{r=1}^{n} f(Y_r) \underset{0}{\overset{1}{\gtrless}} 0$	$\frac{\Omega_{d,i} E_i}{(\Omega_{d,i} E_i + N_0) N_0}\left(\|y_{d,i}^0\|^2 - \|y_{d,i}^1\|^2\right)$	$(2 + \bar{\gamma}_{r,s})^{-1}$	$\frac{\left(1+\bar{\gamma}_{d,i}^{-1}\right)}{\left[(m+(-1)^m x_s)\bar{\gamma}_{d,i}+1\right]}$

TABLE I
ML DECISION PARAMETERS FOR BPSK AND COHERENT AND NONCOHERENT BFSK, $i \in \{s, r\}, m \in \{0, 1\}, \bar{\gamma}_{r,s} = \frac{\Omega_{r,s} E_s}{N_0}, \bar{\gamma}_{d,i} = \frac{\Omega_{d,i} E_i}{N_0}$

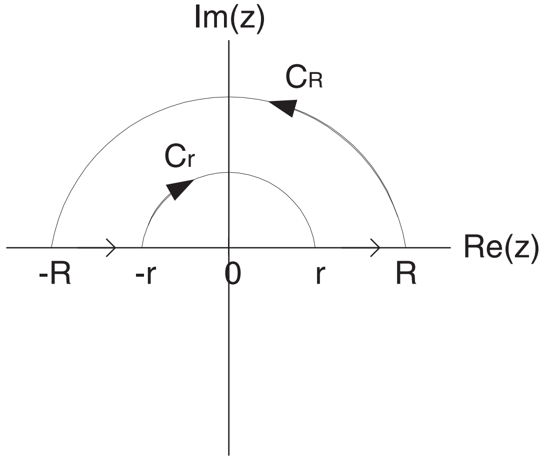

Fig. 2. Contour representing various paths.

III. BER Analysis

A. Single Relay

The conditional probability in (9) for the single relay system can be expressed using the Gil-Pelaez formula as [11]

$$P\left(X + f(Y_1) < 0 | x_s = 1, x_1\right) = \frac{1}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)\phi_{V_1}(\jmath\omega)}{\omega} d\omega, \quad (14)$$

where $V_1 = f(Y_1)$. Letting

$$\mathcal{I} = \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)\phi_{V_1}(\jmath\omega)}{\omega} d\omega \quad (15)$$

and substituting from (13),

$$\mathcal{I} = \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)}{\omega} d\omega - \frac{g_1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)\phi(\beta_1, -\jmath\omega)}{\omega} d\omega - \frac{g_1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)\phi(\alpha_1, \jmath\omega)}{\omega} d\omega, \quad (16)$$

where g_1, α_1 and β_1 are the parameters of Y_1 defined in Table I).

Remark 1. $\phi_X(\jmath\omega)$ is a rational function with degree of the denominator exceeding that of the numerator by 2. Thus the contour in Figure 2 can be readily used along with the residue theorem [12] to evaluate the first integral in (16).

Remark 2. The function $\frac{\phi(\cdot,\cdot)}{\omega}$ has a removable singularity and does not contribute any poles. On the contour $C_R : z = Re^{\jmath\theta}, 0 < \theta < \pi$,

$$|\phi(\alpha_1, \jmath z)| = \frac{|z||1 - \delta^{\alpha_1 + \jmath z}|}{\alpha_1 |\alpha_1 + \jmath z|} = \frac{R|1 - \delta^{\alpha_1 - R\sin\theta + \jmath R\cos\theta}|}{\alpha_1\sqrt{R^2 + \alpha_1^2 - 2\alpha_1 R}}, \quad (17)$$

which becomes infinite when $R \to \infty$ since $0 < \delta < 1$ and $\sin\theta > 0, 0 < \theta < \pi$. However, for the contour $C'_R : z = Re^{\jmath\theta}, -\pi < \theta < 0$, from (17), it is evident that $|\phi(\alpha_1, \jmath z)| < \infty$ as $R \to \infty$.

Thus, the poles in the lower half plane are used to evaluate the third integral in (16) while the poles from the upper half plane are used for the second integral [12, p. 277]. From the residue theorem,

$$\mathcal{I} = \frac{1}{2}\operatorname*{Res}_{z=0}\frac{\phi_X(z)}{z} + \operatorname*{Res}_{z=-\alpha}\frac{\phi_X(z)}{z} - g_1\operatorname*{Res}_{z=-\alpha}\frac{\phi_X(z)\phi(\beta_1, -z)}{z} + g_1\operatorname*{Res}_{z=\beta}\frac{\phi_X(z)\phi(\alpha_1, z)}{z}. \quad (18)$$

Note that the terms involving ϕ in the above have opposite signs due to opposing directions of the semi-circular contours of infinte radius in the upper and lower half planes. From (14), (15) and (18),

$$P\left(X + f(Y_1) < 0 | x_s = 1, x_1\right) = \frac{g}{\alpha} + gg_1\left[\frac{\phi(\alpha_1, \beta)}{\beta} - \frac{\phi(\beta_1, \alpha)}{\alpha}\right], \quad (19)$$

$$\begin{aligned} P\left(X + f(Y_1) < 0 | x_s = 1, \mathbf{x}\right) &= \frac{g}{\alpha}\left[1 - g_1\phi(\beta_1, \alpha) - g_2\phi(\beta_2, \alpha)\right] + \frac{g}{\beta}\left[g_1\phi(\alpha_1, \beta) + g_2\phi(\alpha_2, \beta)\right] \\ &- \frac{gg_1g_2\phi(\alpha_1, \beta)\phi(\alpha_2, \beta)}{\beta} + \frac{gg_1g_2\phi(\beta_1, \alpha)\phi(\beta_2, \alpha)}{\alpha} - \frac{gg_1g_2}{\beta_1(\alpha - \alpha_2)\alpha_2(\beta_1 + \alpha)}\left[\alpha(\delta_1^{\beta_1 + \alpha} - \delta_1^{\beta 1 + \alpha}\delta_2^{\alpha_2 - \alpha})\right]. \\ &+ \frac{gg_1g_2}{\alpha_1(\beta - \beta_2)}\left[\frac{(\alpha + \beta)}{(\alpha + \beta_2)(\alpha_1 + \beta_2)} - \frac{\beta(1 - \delta_1^{\alpha_1 + \beta} + \delta_1^{\alpha_1 + \beta}\delta_2^{\beta_2 - \beta})}{\beta_2(\alpha_1 + \beta)}\right] \\ &+ \frac{gg_1g_2}{\alpha_2(\beta - \beta_1)}\left[\frac{(\alpha + \beta)(1 - \delta_2^{\alpha_2 + \beta_1})}{(\alpha + \beta_1)(\alpha_2 + \beta_1)} - \frac{\beta(1 - \delta_2^{\alpha_2 + \beta})}{\beta_1(\alpha_2 + \beta)}\right] \\ &- \frac{gg_1g_2}{\beta_2(\alpha - \alpha_1)}\left[\frac{\alpha\delta_2^{\beta_2 + \alpha}}{\alpha_1(\beta_2 + \alpha)} - \frac{(\alpha + \beta)\delta_2^{\beta_2 + \alpha_1}}{(\beta + \alpha_1)(\beta_2 + \alpha_1)}\right] \end{aligned} \quad (21)$$

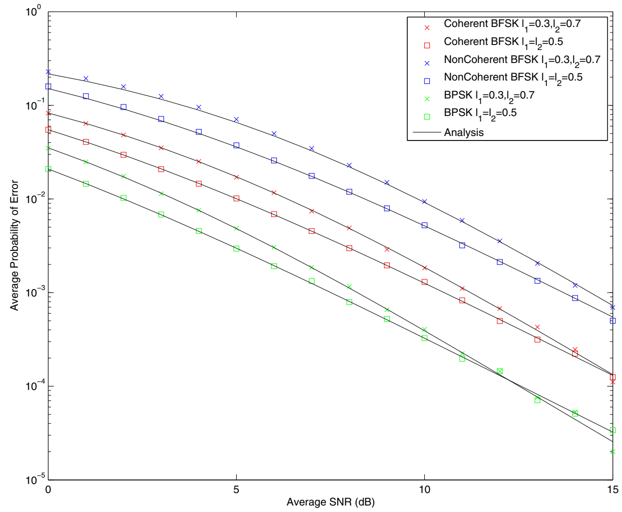

Fig. 3. Simulation and analytical results for two relay cooperation.

which is exactly equal to [8, (22)]. Note that the derivation of (19) here using residue calculus is far easier than the direct method in [4], [8].

B. Two Relays

In this case, the conditional probability in (9) can be expressed as

$$P\left(X + f(Y_1) < 0 | x_s = 1, \mathbf{x}\right) = \frac{1}{2} - \frac{1}{2\pi j}\int_{-\infty}^{\infty} \frac{\phi_X(\jmath\omega)\phi_{V_1}(\jmath\omega)\phi_{V_2}(\jmath\omega)}{\omega} d\omega, \quad (20)$$

where $V_i = f(Y_i), i = 1, 2$ with respective parameters $\delta_i, \alpha_i, \beta_i$ and g_i. Following the approach for the case of single relay, the desired expression for (20) is obtained as (21) (see Appendix).

IV. Results and Discussion

The simulation setup is similar to the one in [8]. The channel fading follows the path loss model $\Omega_{i,j} \propto \frac{1}{L_{i,j}^4}, i, j \in \{s, r, d\}$ with $L_{i,j}$ being the distance between the nodes i and j [4]. Also, $\Omega_{r,s} = \frac{\Omega_{d,s}}{l_r^4}$ and $\Omega_{d,r} = \frac{\Omega_{d,s}}{(1-l_r)^4}$, where $l_r = \frac{L_{r,s}}{L_{d,s}}$. The average system signal to noise ratio (SNR) $= \frac{\Omega_{d,s}(E_s + \sum_{r=1}^{2} E_r)}{N_0}$. A comparison of the analytical and simulation results for the BER performance of two relay systems is shown Figure 3 . We have assumed that all nodes transmit with equal power. Plots are available for both coherent and noncoherent detection for different relay positions. There is an excellent match between the theoretical and simulation results. As expected, coherent detection outperforms noncoherent detection for all cases considered.

V. Conclusions

In this paper, we have found exact expressions for the BER for a PL-DF cooperative diversity system employing two relays, for the Rayleigh fading channel. This was done through a contour integral approach for a CF based inversion formula for the probability of error, where multiple contours were intelligently employed to facilitate application of the residue theorem. We found this approach to be much simpler than other techniques in the available literature. The results obtained were quite general in nature, allowing for the evaluation of the BER for arbitrary relay locations. We believe that the techniques employed in this paper can be used for any number of relays, which is the focus of future research.

Appendix

Since

$$\begin{aligned}\phi_{V_1}(\jmath\omega)\phi_{V_2}(\jmath\omega) &= \phi_{V_1}(\jmath\omega) + \phi_{V_2}(\jmath\omega) - 1 \\ &+ g_1 g_2 \left[\phi(\alpha_1, \jmath\omega)\phi(\alpha_2, \jmath\omega) + \phi(\beta_1, -\jmath\omega)\phi(\beta_2, -\jmath\omega)\right. \\ &\left. + \phi(\alpha_1, \jmath\omega)\phi(\beta_2, -\jmath\omega) + \phi(\beta_1, -\jmath\omega)\phi(\alpha_2, \jmath\omega)\right], \quad (22)\end{aligned}$$

(15) and (18) can be used to obtain

$$\begin{aligned}\frac{1}{2} - \frac{1}{2\pi\jmath}\int_{-\infty}^{\infty} &\frac{\phi_X(\jmath\omega)\left[\phi_{V_1}(\jmath\omega) + \phi_{V_2}(\jmath\omega) - 1\right]}{\omega} d\omega \\ &= \frac{g}{\alpha}\left[1 - g_1\phi(\beta_1, \alpha) - g_2\phi(\beta_2, \alpha)\right] \\ &\quad + \frac{g}{\beta}\left[g_1\phi(\alpha_1, \beta) + g_2\phi(\alpha_2, \beta)\right]. \quad (23)\end{aligned}$$

Since ϕ is analytic,

$$\begin{aligned}\frac{1}{2\pi\jmath}\int_{-\infty}^{\infty} &\frac{\phi_X(\jmath\omega)\phi(\alpha_1, \jmath\omega)\phi(\alpha_2, \jmath\omega)}{\omega} d\omega \\ &= -\operatorname*{Res}_{z=\beta} \frac{\phi_X(z)\phi(\alpha_1, z)\phi(\alpha_2, z)}{z} \\ &= g\frac{\phi(\alpha_1, \beta)\phi(\alpha_2, \beta)}{\beta}. \quad (24)\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{1}{2\pi\jmath}\int_{-\infty}^{\infty} &\frac{\phi_X(\jmath\omega)\phi(\beta_1, -\jmath\omega)\phi(\beta_2, -\jmath\omega)}{\omega} d\omega \\ &= \operatorname*{Res}_{z=-\alpha} \frac{\phi_X(z)\phi(\beta_1, -z)\phi(\beta_2, -z)}{z} \\ &= -g\frac{\phi(\beta_1, \alpha)\phi(\beta_2, \alpha)}{z}. \quad (25)\end{aligned}$$

We have

$$\begin{aligned}\int_{-\infty}^{\infty} &\frac{\phi_X(\jmath\omega)\phi(\alpha_1, \jmath\omega)\phi(\beta_2, -\jmath\omega)}{\omega} d\omega \\ &= \int_{-\infty}^{\infty} \frac{g(\alpha+\beta)(\jmath\omega)^2 \delta_2^{\beta_2 - \jmath\omega}}{\alpha_1\beta_2\omega(\beta - \jmath\omega)(\alpha + \jmath\omega)(\alpha_1 + \jmath\omega)(\beta_2 - \jmath\omega)} d\omega \\ &- \int_{-\infty}^{\infty} \frac{g(\alpha+\beta)(\jmath\omega)^2 \left[1 - \delta_1^{\alpha_1 + \jmath\omega} + \delta_1^{\alpha_1}\delta_2^{\beta_2}\left(\frac{\delta_1}{\delta_2}\right)^{\jmath\omega}\right]}{\alpha_1\beta_2\omega(\beta - \jmath\omega)(\alpha + \jmath\omega)(\alpha_1 + \jmath\omega)(\beta_2 - \jmath\omega)} d\omega \\ &\quad (26)\end{aligned}$$

after substituting the expressions for ϕ. Without loss of generality, we can assume $\delta_1 < \delta_2$. In the above equation, choosing the poles in the upper and lower half plane for the first and second integrands respectively, we obtain

$$\frac{1}{2\pi\jmath}\int_{-\infty}^{\infty}\frac{g(\alpha+\beta)(\jmath\omega)^2\delta_2^{\beta_2-\jmath\omega}}{\alpha_1\beta_2\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)(\alpha_1+\jmath\omega)(\beta_2-\jmath\omega)}d\omega$$
$$=\sum_{\substack{\mathrm{Res}_{z=-\alpha}\\ \mathrm{Res}_{z=-\alpha_1}}}\frac{g(\alpha+\beta)z^2\delta_2^{\beta_2-z}}{\alpha_1\beta_2 z(\beta-z)(\alpha+z)(\alpha_1+\jmath\omega)(\beta_2-z)}$$
$$=\frac{g}{\beta_2(\alpha-\alpha_1)}\left[\frac{\alpha\delta_2^{\beta_2+\alpha}}{\alpha_1(\beta_2+\alpha)}-\frac{(\alpha+\beta)\delta_2^{\beta_2+\alpha_1}}{(\beta+\alpha_1)(\beta_2+\alpha_1)}\right] \quad (27)$$

and

$$\frac{g}{2\pi\jmath}\int_{-\infty}^{\infty}\frac{(\alpha+\beta)(\jmath\omega)^2\left[1-\delta_1^{\alpha_1+\jmath\omega}+\delta_1^{\alpha_1}\delta_2^{\beta_2}\left(\frac{\delta_1}{\delta_2}\right)^{\jmath\omega}\right]}{\alpha_1\beta_2\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)(\alpha_1+\jmath\omega)(\beta_2-\jmath\omega)}d\omega$$
$$=-\sum_{\substack{\mathrm{Res}_{z=\beta}\\ \mathrm{Res}_{z=\beta_2}}}\frac{g(\alpha+\beta)z^2\left[1-\delta_1^{\alpha_1+z}+\delta_1^{\alpha_1}\delta_2^{\beta_2}\left(\frac{\delta_1}{\delta_2}\right)^{z}\right]}{\alpha_1\beta_2 z(\beta-z)(\alpha+z)(\alpha_1+z)(\beta_2-z)}$$
$$=\frac{g}{\alpha_1(\beta-\beta_2)}\left[\frac{(\alpha+\beta)}{(\alpha+\beta_2)(\alpha_1+\beta_2)}-\frac{\beta\left(1-\delta_1^{\alpha_1+\beta}+\delta_1^{\alpha_1+\beta}\delta_2^{\beta_2-\beta}\right)}{\beta_2(\alpha_1+\beta)}\right]. \quad (28)$$

Similarly,

$$\int_{-\infty}^{\infty}\frac{\phi_X(\jmath\omega)\phi(\alpha_2,\jmath\omega)\phi(\beta_1,-\jmath\omega)}{\omega}$$
$$=-\int_{-\infty}^{\infty}\frac{g(\alpha+\beta)(\jmath\omega)^2(1-\delta_2^{\alpha_2+\jmath\omega})}{\alpha_2\beta_1\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)(\alpha_2+\jmath\omega)(\beta_1-\jmath\omega)}d\omega$$
$$-\int_{-\infty}^{\infty}\frac{g(\alpha+\beta)(\jmath\omega)^2(-\delta_1^{\beta_1-\jmath\omega}+\delta_1^{\beta_1}\delta_2^{\alpha_2}(\frac{\delta_1}{\delta_2})^{-\jmath\omega})}{\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)\alpha_2\beta_1(\alpha_2+\jmath\omega)(\beta_1-\jmath\omega)}d\omega \quad (29)$$

with

$$\frac{1}{2\pi\jmath}\int_{-\infty}^{\infty}\frac{g(\alpha+\beta)(\jmath\omega)^2(1-\delta_2^{\alpha_2+\jmath\omega})}{\alpha_2\beta_1\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)(\alpha_2+\jmath\omega)(\beta_1-\jmath\omega)}d\omega$$
$$=-\sum_{\substack{\mathrm{Res}_{z=\beta}\\ \mathrm{Res}_{z=\beta_1}}}\frac{g(\alpha+\beta)z^2(1-\delta_2^{\alpha_2+z})}{\alpha_2\beta_1\omega(\beta-z)(\alpha+z)(\alpha_2+z)(\beta_1-z)}$$
$$=\frac{g}{\alpha_2(\beta-\beta_1)}\left[\frac{(\alpha+\beta)(1-\delta_2^{\alpha_2+\beta_1})}{(\alpha+\beta_1)(\alpha_2+\beta_1)}-\frac{\beta(1-\delta_2^{\alpha_2+\beta})}{\beta_1(\alpha_2+\beta)}\right] \quad (30)$$

and

$$\frac{1}{2\pi\jmath}\int_{-\infty}^{\infty}\frac{g(\alpha+\beta)(\jmath\omega)^2(-\delta_1^{\beta_1-\jmath\omega}+\delta_1^{\beta_1}\delta_2^{\alpha_2}(\frac{\delta_1}{\delta_2})^{-\jmath\omega})}{\omega(\beta-\jmath\omega)(\alpha+\jmath\omega)\alpha_2\beta_1(\alpha_2+\jmath\omega)(\beta_1-\jmath\omega)}d\omega$$
$$=\sum_{\substack{\mathrm{Res}_{z=-\alpha}\\ \mathrm{Res}_{z=-\alpha_2}}}\frac{g(\alpha+\beta)z^2\left(-\delta_1^{\beta_1-z}+\delta_1^{\beta_1}\delta_2^{\alpha_2}\left(\frac{\delta_1}{\delta_2}\right)^{-z}\right)}{z(\beta-z)(\alpha+z)\alpha_2\beta_1(\alpha_2+z)(\beta_1-z)}$$
$$=\frac{g}{\beta_1(\alpha-\alpha_2)}\left[\frac{\alpha(-\delta_1^{\beta_1+\alpha}+\delta_1^{\beta_1+\alpha}\delta_2^{\alpha_2-\alpha})}{\alpha_2(\beta_1+\alpha)}\right]. \quad (31)$$

From (22)-(31), we obtain (21).

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