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Performance Analysis of Maximum Likelihood Detection for Decode and Forward MIMO Relay Channels in Rayleigh Fading

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Abstract— **Closed form expressions for the bit error rate (BER) for a multiple input multiple output (MIMO) relay system employing maximum likelihood (ML) based decode and forward (DF) cooperative diversity are obtained. The DF operation at the relay involves maximal ratio combining (MRC) on the source-relay link and space-time coding (STC) on the relay-destination link. Exact expressions of the BER are obtained for the case of a single relay supporting two antennas. For a system employing a large number of relays, each having two antennas, approximate expressions for the BER are obtained using the piecewise linear (PL) approximation for the ML detector. This is done by finding the statistics of conditionally Gaussian random variables, that appear in the decision variable. The validity of the analytical expressions is then verified through simulations. Through numerical results obtained from the BER expressions for large number of relays, it is then shown that the loss in diversity order due to DF can be compensated by using multiple antennas at the relay.**

I. INTRODUCTION

Cooperative diversity has been found to be an efficient substitute for traditional multiple input multiple output (MIMO) systems, with diversity gain being obtained by transmission through relays distributed across the wireless network. This removes the necessity of having multiple transmit antennas at a node, a requirement that is difficult to satisfy at the mobile station due to size constraints, thus affecting the performance on the reverse link.

Maximum likelihood (ML) detection for decode and forward (DF) cooperative diversity was proposed in [1], [2] for single input single output (SISO) relay systems. A piecewise linear (PL) approximation to the ML detector, known as the PL combiner, was also provided in [2], [3] for practical applications. For MIMO relay systems [4], [5], the ML decision rule in [3] was shown to be valid even while using space-time coding (STC) on the relay-destination link in [6]. However, the only valid full rate STC for such a system is the Alamouti code [7] for a two antenna relay system. Further, only simulation results were available in [6] for the bit error rate (BER) for a two antenna relay system employing maximal ratio combining (MRC) on the source-relay link and the Alamouti code on the relay-destination link.

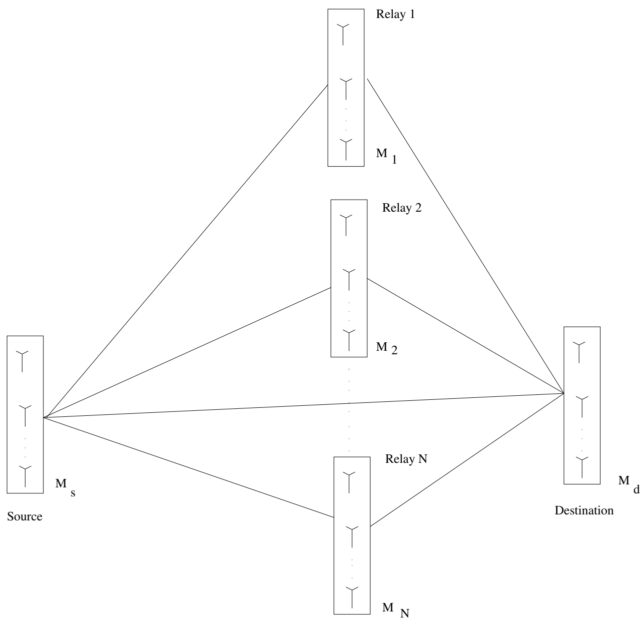

Fig. 1. Multiantenna based relay model for cooperative diversity.

In this paper, using a novel concept of conditionally Gaussian random variables, we obtain i) exact expressions for the BER for ML detector and PL combiner for a single relay with two antennas and ii) close approximation to the BER for PL combiner with multiple relays, each having two antennas, for the coherent MRC-STC based ML-DF cooperative system in [6] for a Rayleigh fading channel. The numerical results obtained from the closed form BER expressions in our paper exactly match the simulations in [6].

Also, the ML-DF scheme for SISO relay channels has been shown to suffer a loss in the diversity order by a factor of two [3]. A significant contribution of this paper is that, through numerical results, it is shown that this loss can be compensated using two antennas at the relay with MRC-STC based ML-DF.

II. System Model

We consider a specific case of the general MIMO relay model in [6] with N relays (R) between the source (S) and destination (D) as shown in Fig. 1. The source has $M_s = 1$ transmit antennas, the rth relay has $M_r = 2$ antennas, that are used for reception on the source-relay link and transmission on relay-destination link, and the destination has $M_d = 1$ receive antennas. The DF mechanism is used, which means that the relay first demodulates the symbols transmitted by the source before retransmitting to the destination. Further, we assume that the modulation is BPSK and all transmissions are on orthogonal flat fading channels. Without loss of generality, we use the alphabet x to represent a transmitted symbol, h to represent the channel gain on a given link, n for additive white Gaussian noise, y for the received symbol at a node and E for the transmit power at a given node. With $i = 1, 2$ denoting the antenna index, the received symbols on the S-D, S-R and R-D links are respectively

$$\begin{aligned} y_{d,s} &= \sqrt{E_s} h_{d,s} x_s + n_{d,s}, \\ y^i_{r,s} &= \sqrt{E_s} h^i_{r,s} x_s + n^i_{r,s}, \\ y_{d,r} &= \sqrt{\frac{E_r}{2}} \sum_{i=1}^{2} h^i_{d,r} x^i_{d,r} + n^i_{d,r}, \end{aligned} \tag{1}$$

where $x_s \in \{1, -1\}$ is the symbol transmitted by the source and $x^i_{d,r} \in \{1, -1\}$ is the symbol transmitted by the ith antenna of the rth relay. The channel experiences Rayleigh fading, hence the fading coefficients $h_{d,s} \sim \mathcal{CN}(0, \Omega_{d,s}), h^i_{r,s} \sim \mathcal{CN}(0, \Omega_{r,s})$ and $h^i_{d,r} \sim \mathcal{CN}(0, \Omega_{d,r})$ are zero mean complex circularly Gaussian. Channel knowledge on each link is assumed to be perfect. Also, $n_{d,s}, n^i_{r,s}, n^i_{d,r} \sim \mathcal{CN}(0, N_0)$. E_s is the source power and E_r is the relay power distributed equally among the two antennas during transmission.

A. MRC and STC at the Relay

Let x_r and x'_r be the decision made at the rth relay in consecutive time slots using MRC with the following rule

$$\sum_{i=1}^{2} h^{i*}_{r,s} y^i_{r,s} \quad \underset{-1}{\overset{1}{\gtrless}} \quad 0. \tag{2}$$

Note that this gives us receive diversity on the S-R link. The average BER on this link is denoted by ϵ_r, with a closed form expression available in [8]. The symbols x_r and x'_r are now transmitted using the Alamouti code [7] on the R-D link according to the following scheme

$$\begin{pmatrix} x_r & x'_r \\ -x'_r & x_r \end{pmatrix} \tag{3}$$

where the rows represent the time slots and columns represent the transmit antennas. Thus, in the first time slot, $x^1_{d,r} = x_r$ and $x^2_{d,r} = x'_r$.

B. ML Decision

The ML decision criterion at the destination for BPSK modulation may be obtained from [2], [6] as

$$X + \sum_{r=1}^{N} f(Y_r) \quad \underset{-1}{\overset{1}{\gtrless}} \quad 0, \tag{4}$$

where

$$\begin{aligned} X &= \frac{4\text{Re}\{h^*_{d,s} y_{d,s}\}\sqrt{E_s}}{N_0}, \\ Y_r &= \sqrt{\frac{E_r}{2}} \frac{4\text{Re}\{h^{1*}_{d,r} y^1_{d,r} + h^2_{d,r} y^{2*}_{d,r}\}}{N_0}, \end{aligned} \tag{5}$$

$$f(t) = \ln \frac{\delta + e^t}{1 + \delta e^t}, \quad 0 < \delta < 1 \tag{6}$$

with $y^1_{d,r}$ and $y^2_{d,r}$ being the received symbols on the R-D link over consecutive time slots and $\{^*\}$ denoting the complex conjugate operation. We would like to mention that $f(Y_r)$ now has the parameter $\delta_r = \frac{\epsilon_r}{1-\epsilon_r}$ for the rth relay, where ϵ_r is the BER for the S-R link. To simplify the analysis, we use the suboptimal ML scheme proposed in [9] by considering ϵ_r to be the average BER instead of the instantaneous BER.

C. PL Combiner

Lemma 2.1: The function $f(t)$ in (6) has the following piecewise linear approximation [2], [3]

$$f(t) \approx \begin{cases} \ln \frac{1}{\delta} & t \geq \ln \frac{1}{\delta} \\ t & \ln \delta < t < \ln \frac{1}{\delta} \\ \ln \delta & t < \ln \delta \end{cases} . \tag{7}$$

Using the above approximation in (4), we obtain the PL combiner for ML-DF cooperative systems.

D. Problem Definition

Assuming equal probability of the symbols $x_s = \{1, -1\}$, the average probability of error for the ML-DF cooperative diversity system can be expressed as

$$\begin{aligned} P_e &= \sum_{\mathbf{x}} \prod_{r=1}^{N} \epsilon_r^{\frac{1-x_r}{2}} (1 - \epsilon_r)^{\frac{1+x_r}{2}} \\ &\quad \times P\left(X + \sum_{i=1}^{N} f(Y_i) < 0 | x_s = 1, \mathbf{x}\right), \end{aligned} \tag{8}$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_M)$ is the set of all possible $M-$tuples formed by the symbols transmitted by the relays. We wish to find out a closed form expression for P_e in (8).

E. Solution Strategy

From (8), it is clear that the knowledge of the statistics of X and $f(Y_r)$ defined in (5) and (6) would greatly simplify the analysis. The variable X can be expressed using (1) as

$$X = \frac{4E_s}{N_0} x_s |h_{d,s}|^2 + \frac{4\sqrt{E_s}}{N_0} Re(h^*_{d,s} n_{d,s}) \tag{9}$$

The conditional mean and variance of X can be expressed as

$$E[X|h_{d,s}] = \frac{4E_s}{N_0} x_s |h_{d,s}|^2$$
$$var(X|h_{d,s}) = \frac{8E_s}{N_0} |h_{d,s}|^2 \quad (10)$$

From (9) and (10), we find that the variable $X \sim \mathcal{N}\left(\frac{4E_s}{N_0} x_s |h_{d,s}|^2, \frac{8E_s}{N_0}|h_{d,s}|^2\right)$ It is easy to show that even $Y_r \sim \mathcal{N}\left(\frac{2E_r}{N_0} x_s \sum_{i=1}^2 |h_{d,r}^i|^2, \frac{4E_r}{N_0}\sum_{i=1}^2 |h_{d,r}^i|^2\right)$ has a similar distribution We call such distributions as conditionally Gaussian. In the following section, we find the cumulative distribution function (CDF), probability density function (PDF) and higher order moments of conditionally Gaussian random variables like X, Y_r and their functions $f(Y_r)$. Using these, we find a closed form expression for P_e in (8) for both the ML detector as well as the PL combiner in the subsequent sections.

III. Conditionally Gaussian Distributions

A. Preliminaries

Definition 3.1: (The Gamma distribution) A gamma distributed random variable A, with scale parameter $c > 0$ and order $n > 0$ has the PDF [10]

$$p_A(x) = \frac{1}{\Gamma(n)} c^n x^{n-1} e^{-cx}, \quad x > 0 \quad (11)$$

For $n = 1$, A is *exponential* with parameter c.

Definition 3.2: A conditionally Gaussian random variable with parameters $a, b > 0$ is defined as $Z \sim \mathcal{N}(aA, bA)$ where A is a random variable. In this paper, we assume that A is gamma distributed with parameter c and order n. We refer to such variables as *gamma conditionally Gaussian random variables*. For $n = 1$, we obtain *exponential conditionally Gaussian random variables* with parameter c.

Definition 3.3: For $0 < \delta < 1$, we define the function

$$s(t) = \frac{e^{-t} - \delta}{1 - \delta e^{-t}}. \quad (12)$$

It directly follows from the above that $s(t) \le e^{-t}, t > 0$. In the rest of the paper, we use the symbols f and s to represent the functions defined in (6) and (12) respectively.

Lemma 3.1: For any constants $a, b > 0$ and $c > 0$,

$$\int_0^\infty Q\left(\frac{ax+b}{\sqrt{x}}\right) e^{-cx} dx = \frac{\exp\left(-b\left(a + \sqrt{a^2+2c}\right)\right)}{\sqrt{a^2+2c}\left(a+\sqrt{a^2+2c}\right)}, \quad (13)$$

$$\int_0^\infty x Q\left(\frac{ax+b}{\sqrt{x}}\right) e^{-cx} dx = \frac{e^{-b\left(a+\sqrt{a^2+2c}\right)}}{(a^2+2c)\left(a+\sqrt{a^2+2c}\right)} \times \left[b + \frac{a + 2\sqrt{a^2+2c}}{(a^2+2c)^{\frac{1}{2}}\left(a+\sqrt{a^2+2c}\right)}\right] \quad (14)$$

Proof: Defining the error function $\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ and noting that $Q(x) = \frac{1}{2}\left[1 - \Phi\left(\frac{x}{\sqrt{2}}\right)\right]$, we obtain (13) using the result in [11, 6.297.1] Differentiating (13) with respect to c in the integral, we obtain (14).

Lemma 3.2: For any constants $a > 0, b$ and $c > 0$,

$$\frac{1}{\sqrt{2\pi}c}\int_0^\infty e^{-ax} e^{-\frac{(x-b)^2}{2c^2}} dx = e^{-ab + \frac{b^2c^2}{2}} Q\left(ac - \frac{b}{c}\right) \quad (15)$$

Proof: Completing squares and using [11, 3.322.2], we obtain (15).

Lemma 3.3: For any constants $a > 0, b, n \ge 0$, the integral defined by

$$I(n,a,b) = \int_0^{\ln\frac{1}{\delta}} x^n e^{-ax}(b+x) dx \quad (16)$$

is obtained as

$$I(n,a,b) = \frac{\Gamma\left(n+2, a\ln\frac{1}{\delta}\right) + ab\Gamma\left(n+1, a\ln\frac{1}{\delta}\right)}{a^{n+2}} \quad (17)$$

where $\Gamma(.,.)$ is the incomplete Gamma function defined in [12, p.197].

Proof: The integral $I(n,a,b)$ can be expressed as

$$I(n,a,b) = \int_0^{\ln\frac{1}{\delta}} x^{n+1} e^{-ax} dx + b\int_0^{\ln\frac{1}{\delta}} x^n e^{-ax} dx \quad (18)$$

which is obtained as (17) using [12, (4.4.5)]

Definition 3.4: For $a > 0, b > 0$ and $c > 0$, we define the integral

$$\Lambda(a,b,c) = \int_0^{\ln\frac{1}{\delta}} (s(x))^a e^{-cx}(b - \ln s(x)) dx. \quad (19)$$

Lemma 3.4: For $\delta << 1$,

$$\Lambda(a,b,c) \approx I(0, a+c, b). \quad (20)$$

Proof: For $\delta << 1$, $s(t) \approx e^{-t}$. Substituting in (19),

$$\Lambda(a,b,c) = \int_0^{\ln\frac{1}{\delta}} e^{-(a+c)x}(b+x) dx, \quad (21)$$

which can be expressed using Definition 3.3 as (20).

B. Closed Form Expressions for the Statistical Functions of Conditionally Gaussian and Related Distributions

Theorem 3.1: If Z be *gamma conditionally Gaussian* with parameters a, b, c and order $n = 2$, the CDF and PDF of Z can be expressed as

$CDF:$

$$F_Z(z) = \begin{cases} 1 - \beta(-1) e^{-z\alpha(-1)}(z + \gamma(-1)) & z \ge 0 \\ \beta(1) e^{z\alpha(1)}(-z + \gamma(1)) & z < 0, \end{cases} \quad (22)$$

$PDF:$

$$p_Z(z) = p_{Z+}(z) + p_{Z-}(z), \text{ where}$$
$$p_{Z-}(z) = \beta(1)\left[\alpha(1)(-z+\gamma(1)) - 1\right] e^{z\alpha(1)} u(-z), \quad (23)$$
$$p_{Z+}(z) = \beta(-1)\left[\alpha(-1)(z+\gamma(-1)) - 1\right] e^{-z\alpha(-1)} u(z),$$

u(.) is the unit step function and for $m \in \{1, -1\}$,

$$\alpha(m) = \frac{\sqrt{a^2+2bc} + ma}{b},$$
$$\beta(m) = \frac{bc^2}{(a^2+2bc)\left(\sqrt{a^2+2bc}+ma\right)},$$
$$\gamma(m) = \frac{b\left(2\sqrt{a^2+2bc}+ma\right)}{\sqrt{a^2+2bc}\left(\sqrt{a^2+2bc}+ma\right)},$$
$$\beta(1)\gamma(1) + \beta(-1)\gamma(-1) = 1. \quad (24)$$

Proof: See Appendix.

Theorem 3.2: Let X be *exponential conditionally Gaussian* with parameters a, b and c, as in Definition 3.2. We then have the following expressions for the CDF and PDF of X.

$$CDF: \quad F_X(x)= \begin{cases} 1-\frac{ge^{-xh(-1)}}{h(-1)} & x\geq 0 \\ \frac{ge^{xh(1)}}{h(1)} & x<0, \end{cases} \tag{25}$$

$$PDF: \quad p_X(x)= \begin{cases} ge^{-xh(-1)} & x\geq 0 \\ ge^{xh(1)} & x<0, \end{cases} \tag{26}$$

where

$$h(m)= \frac{\sqrt{a^2+2bc}+ma}{b}, \quad m\in\{1,-1\}$$
$$g= \frac{c}{\sqrt{a^2+2bc}} \quad \text{and} \quad \frac{1}{g}=\frac{1}{h(1)}+\frac{1}{h(-1)}. \tag{27}$$

Proof: See Appendix.

Theorem 3.3: The CDF of $V = f(Z)$, for Z defined in Theorem 3.1 is given by

$$F_V(v) = \begin{cases} 1 & v>\ln\frac{1}{\delta} \\ 1-\beta(-1)s(v)^{\alpha(-1)}\left(-\ln s(v)+\gamma(-1)\right)) & 0\leq v<\ln\frac{1}{\delta} \\ \beta(1)s(v)^{-\alpha(1)}\left(\ln s(v)+\gamma(1)\right)) & \ln\delta<v<0 \\ 0 & v<\ln\delta \end{cases} \tag{28}$$

Corollary: Using the PL approximation, the nth moment of V is given by

$$E[V^n] = \sum_{m\in\{1,-1\}}\beta(m)\left[(-1)^{\frac{n(1-m)}{2}}(\ln\delta)^n\delta^{\alpha(m)}\left(\gamma(m)-\ln\delta\right)\right.$$
$$\left. + (-1)^{\frac{n(1+m)}{2}}\alpha(m)I\left(n,\alpha(m),\gamma(m)-\frac{1}{\alpha(m)}\right)\right]. \tag{29}$$

The variance of V can be obtained from the first and second moment as $\text{var}(V) = E[V^2] - (E[V])^2$.

Proof: See Appendix.

C. Gamma Conditionally Gaussian distributions in ML-DF MIMO Relay Systems

From the discussion in section II.E and Definition 3.2, we have the following lemma.

Lemma 3.5: Let X and $\{Y_r\}_{r=1}^N$ be as defined in (5). Then X is *exponential conditionally Gaussian* with parameters $a = \frac{4E_s}{N_0}x_s, b = \frac{8E_s}{N_0}, c = \frac{1}{\Omega_{d,s}}$ and Y_r are *gamma conditionally Gaussian* of order $n = 2$, with parameters $a_r = \frac{2E_r}{N_0}x_r, b_r = \frac{4E_r}{N_0}, c_r = \frac{1}{\Omega_{d,r}}$ respectively.

IV. BER FOR MRC-STC ML-DF MIMO RELAY SYSTEMS

A. Exact Analysis for Single Relay

Theorem 4.1: (ML Detector) For a single relay between the source and destination, the closed form expression for the conditional probability in (8) is given by

$$P(X+f(Y_1)<0|x_s=1,x_1) = \frac{g}{h(1)}+g \times\sum_{m\in\{1,-1\}}m\beta_1(m)\Lambda\left(\alpha_1(m),\gamma_1(m),h(-m)\right) \tag{30}$$

for Λ in Definition 3.4 and g, h and α_1, β_1 and γ_1 defined in (27) and (24) being the respective parameters for X and Y_1.

Corollary: Using Lemma 3.4 in (30) results in the following simple closed form approximation for the BER.

$$P(X+f(Y_1)<0|x_s=1,x_1) = \frac{g}{h(1)}+g \times\sum_{m\in\{1,-1\}}m\beta_1(m)I\left(0,\alpha_1(m)+h(-m),\gamma_1(m)\right). \tag{31}$$

Proof: From (8), we have

$$P\left(X+f(Y_1)<0|x_s=1,x_1\right) = F_{f(Y_1)}(-X). \tag{32}$$

From Theorem 3.3, we obtain

$$F_{f(Y_1)}(-X) = \begin{cases} 1 & X<\ln\delta \\ 1-\beta_1(-1)s(X)^{-\alpha_1(-1)}\left(\ln s(X)+\gamma_1(-1)\right)) & \ln\delta<X<0 \\ \beta_1(1)s(X)^{\alpha_1(1)}\left(-\ln s(X)+\gamma_1(1)\right)) & 0\leq X<\ln\frac{1}{\delta} \\ 0 & X>\ln\frac{1}{\delta} \end{cases} \tag{33}$$

The required probability may now be obtained by averaging the above over X. This results in $\int_{-\infty}^{\infty}F_{f(Y_1)}(-x)p_X(x)dx$, which upon substituting for $p_X(x)$ from Theorem 3.2, and using (19) from Definition 3.4 gives (30). From Lemma 3.4, for $\delta << 1$, we obtain a closed form approximation for (30) in (31).

Theorem 4.2: (PL Combiner) The closed form expression for the conditional probability in (8) for the PL-DF system is given by

$$P(X+f(Y_1)<0|x_s=1,x_1) = \beta_1(1)\gamma_1(1) - g\sum_{m\in\{1,-1\}}\frac{m\beta_1(m)}{h(-m)}\left[(\gamma_1(m)-\ln\delta)\delta^{h(-m)+\alpha_1(m)}\right.$$
$$\left. + \alpha_1(m)I\left(0,\alpha_1(m)+h(-m),\gamma_1(m)-\frac{1}{\alpha_1(m)}\right)\right] \tag{34}$$

Proof: We have

$$P\left(X+f(Y_1)<0|x_s=1,x_1\right) = F_X(-f(Y_1)). \tag{35}$$

From (35), using Theorem 3.1, we obtain

$$F_X(-f(Y_1)) = \begin{cases} 1-\frac{g\exp[f(Y_1)h(-1)]}{h(-1)} & f(Y_1)<0 \\ \frac{g\exp[-f(Y_1)h(1)]}{h(1)} & f(Y_1)\geq 0 \end{cases}. \tag{36}$$

Using the PL approximation from Lemma 2.1 in the above, we obtain

$$F_X(-f(Y_1)) = \begin{cases} 1-\frac{g\delta^{h(-1)}}{h(-1)} & Y_1<\ln\delta \\ 1-\frac{g\exp[Y_1h(-1)]}{h(-1)} & \ln\delta<Y_1<0 \\ \frac{g\exp[-Y_1h(1)]}{h(1)} & 0\leq Y_1<\ln\frac{1}{\delta} \\ \frac{g\delta^{h(1)}}{h(1)} & Y_1>\ln\frac{1}{\delta} \end{cases} \tag{37}$$

where we have made use of the fact that $f(t) \lesseqqgtr 0 \Leftrightarrow t \lesseqqgtr 0$ for $0 < \delta < 1$. Averaging (37) over Y_1, the required probability may be expressed as

$$\int_{-\infty}^{\infty} F_X(-f(x))p_{Y_1}(x)dx = \left[1 - \frac{g\delta^{h(-1)}}{h(-1)}\right] F_{Y_1}(\ln\delta)$$
$$F_{Y_1}(0) - F_{Y_1}(\ln\delta) - \frac{g}{h(-1)}\int_{\ln\delta}^{0} e^{xh(-1)}p_{Y_1}(x)dx$$
$$\frac{g\delta^{h(1)}}{h(1)}\left[1 - F_{Y_1}\left(\ln\frac{1}{\delta}\right)\right] + \frac{g}{h(1)}\int_0^{\ln\frac{1}{\delta}} e^{-xh(1)}p_{Y_1}(x)dx$$

which, after substituting for $p_{Y_1}(x)$ from Theorem 3.1 and evaluating the integral and simplifying results in (34). Using Lemma 3.5 and substituting the expressions obtained from Theorems 4.1 and 4.2 in (8), a closed form expression for the BER for the DF cooperative diversity system is obtained for the ML detector and PL combiner respectively.

B. Central Limit Theorem Approximation for Multiple Relays

We use the central limit theorem (CLT) [10] to find an approximate expression for the BER when a large number of relays are used.

Theorem 4.3: (CLT approximation) For large N and μ_r, σ_r^2 defined to be the mean and variance of $V_r = f(Y_r)$ respectively, the conditional probability defined in (8) can be expressed using the CLT as

$$P(X + Y < 0|x_s = 1, \mathbf{x}) = Q\left(\frac{\mu}{\sigma}\right) + J(1) - J(-1) \quad (38)$$

where $Y = \sum_{r=1}^{N} f(Y_r)$ and for $m \in \{1, -1\}$

$$J(m) = \frac{g}{h(m)} e^{-mh(m)\mu + \frac{[h(m)]^2\sigma^2}{2}} Q\left(\sigma h(m) - m\frac{\mu}{\sigma}\right)$$

for $\mu = \sum_{r=1}^{N} \mu_r, \sigma^2 = \sum_{r=1}^{N} \sigma_r^2$.

Proof: From Corollary 3.1, expressions for μ_r and σ_r can be obtained. Then, using the central limit theorem, for large M, we have the distribution $Y \sim \mathcal{N}(\mu, \sigma^2)$. Since $P(X + Y < 0|x_s = 1|\mathbf{x}) = F_X(-Y)$, this probability may be expressed, using Theorem 3.1 as

$$F_X(-Y) = \begin{cases} 1 - \frac{g}{h(-1)} e^{h(-1)Y} & Y \le 0 \\ \frac{g}{h(1)} e^{-h(1)Y} & Y > 0 \end{cases} \quad (39)$$

Unconditioning with respect to Y, the desired probability may be expressed as $\int_{-\infty}^{\infty} F_X(-y)p_Y(y)dy$, where $p_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{(y-\mu)^2}{2\sigma^2}}, y \in (-\infty, \infty)$. Using Lemma 3.2 to evaluate this integral and simplifying, we obtain (38).

V. Results and Discussion

Following the simulation setup in [6], the total power used by the system is assumed to be one unit, shared by the source and relays, each relay supporting exactly 2 antennas. This may be represented as $E_s + \sum_{r=1}^{N} E_r = 1$. We define $L_r = \frac{\text{distance between source and } r\text{th relay}}{\text{distance between source and destination}}$ with the fading power on the source-relay link $\Omega_{r,s} \propto \frac{1}{L_r^4}$. In Fig. 2, a comparison of the simulation and analytical results for the case of a single relay ($N = 1$) is provided for $L = 0.3, 0.5$ and 0.8 and $E_s = E_r = 0.5$. For the analytical results,

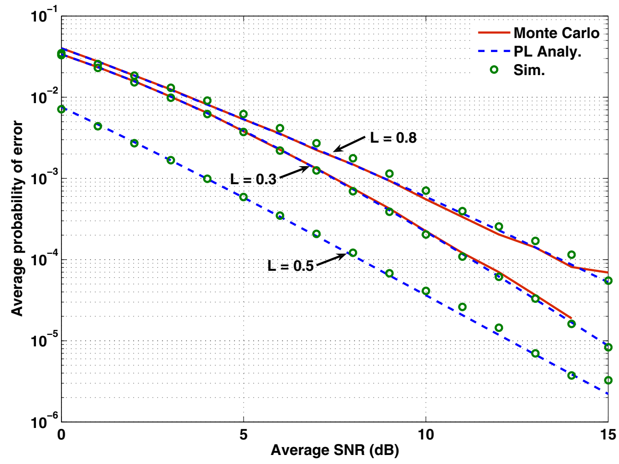

Fig. 2. Comparison of analytical results with simulations for single relay ($N = 1$). Relay supports two antennas ($M_1 = 2$).

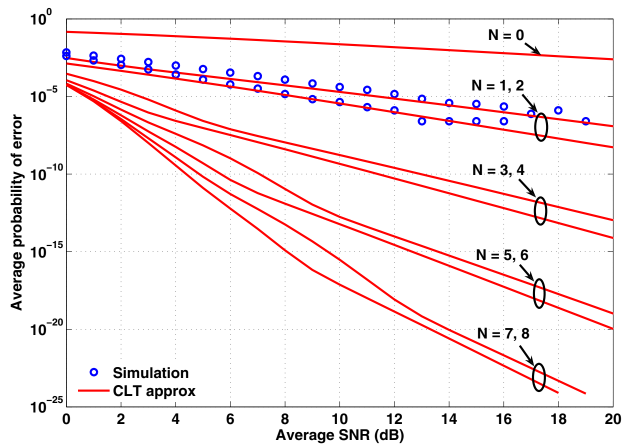

Fig. 3. CLT approximation for MIMO relay. $N = 1, \ldots, 8, M_r = 2$. Loss in total diversity order due to DF.

the closed form BER expression in (8) is used along with Theorem 4.1 and Theorem 4.2 for the ML detector and PL combiner respectively. Monte Carlo simulations have been used to evaluate the definite integral that appears in the conditional BER expression for the ML detector. As we can see from Fig. 2, the simulation results obtained in [6] exactly match the the closed form BER results obtained in this paper for $N = 1$.

In Fig. 3, CLT based BER results are plotted for upto $N = 8$ relays using Theorem 4.3. A comparison of the CLT approximation with the simulation results for $N = 1, 2$ shows that the CLT approximation acts as a lower bound on the BER for low SNR, but closely follows the simulations in the high SNR regions. We have considered $E_s = 0.5, E_r = \frac{0.5}{N}$ and $L_r = 0.5$. Representing the diversity order and the corresponding number of relays as (d, N), we graphically obtain [8, p.829], for even number of relays, the values $(2.78, 2), (4.76, 4), (6.9, 6)$

and $(8.74, 8)$. For odd number of relays, the diversity order is obtained as $(2.33, 1), (4.35, 3), (6.45, 5)$ and $(8.34, 7)$. We sum up our results as

$$\begin{aligned} N < d \leq N+1, &\quad N \text{ even} \\ N+1 < d \leq N+2, &\quad N \text{ odd}. \end{aligned} \tag{40}$$

The total number of antennas used in our MIMO relay system for transmission is $2N+1$, but from (40), we find the diversity order to be $N+1$. This suggests a loss in the overall diversity order by a factor of 2, as seen in Fig. 3. However, full relay diversity $(N+1)$ is obtained. Thus, using multiple antennas at the relay can offset the loss in relay diversity due to DF [3].

VI. Conclusions

In this paper, we have found exact expressions for the BER of an MRC-STC based ML-DF MIMO relay system employing a single relay with two antennas, for the Rayleigh fading channel. A closed form approximation for BER for the ML detector is also obtained. An alternative measure of the performance of ML is obtained by finding a closed form expression for BER for the PL combiner, again for the single relay system. In the process, we have derived expressions for the CDF, PDF and moments of *gamma conditionally Gaussian variables*. Using the central limit theorem, we then obtain an approximate expression for the BER when multiple relays are employed. The diversity order of the system was obtained graphically from this expression. This was found to be exactly equal to the achievable relay diversity for the equivalent SISO system. On the basis of our numerical results, we conclude that the MRC-STC based technique proposed in [6] for ML-DF MIMO relay systems can compensate for the loss in the diversity order for the ML-DF based SISO relay system in [3].

Appendix

A. Proof of Theorems 3.1 and 3.2

The conditional CDF of Z can be expressed as

$$P(Z < z|A) = \begin{cases} 1 - Q\left(\frac{-aA+z}{\sqrt{bA}}\right) & z \geq 0 \\ Q\left(\frac{aA-z}{\sqrt{bA}}\right) & z < 0 \end{cases}. \tag{41}$$

Unconditioning (41) with respect to A, we have

$$F_Z(z) = \begin{cases} 1 - c\int_0^\infty Q\left(\frac{-ax+z}{\sqrt{bx}}\right) e^{-cx} dx & z \geq 0 \\ c\int_0^\infty Q\left(\frac{ax-z}{\sqrt{bx}}\right) e^{-cx} & z < 0 \end{cases}. \tag{42}$$

Now, substituting (14), Lemma 3.1 in (42), we obtain (22). Differentiating $F_Z(z)$ in (22) results in the expression for $p_Z(z)$ in (23). Repeating the above analysis for X, (25) and (26) can be obtained from (42) and (13), Lemma 3.1.

B. Proof of Theorem 3.3

Since $V = f(Z)$, the CDF of V can be expressed as

$$\begin{aligned} F_V(v) &= P(V < v) = P\left(f(Z) < v\right) \\ &= P\left(e^Z < s(-v)\right). \end{aligned} \tag{43}$$

Since $e^Z > 0$,

$$\begin{aligned} F_V(v) &= P\left(Z < \ln(s(-v))\right) \\ &= \begin{cases} 1 & v > \ln\frac{1}{\delta} \\ F_Z(\ln(s(-v)) & \ln\delta < v < \ln\frac{1}{\delta} \\ 0 & v < \ln\delta \end{cases}. \end{aligned} \tag{44}$$

Using the expression for the CDF of Z from Theorem 3.1 in (44) and noting that $\ln(s(v))$ is odd, we obtain (33). Now, using the PL approximation from Lemma 2.1 and the expression for $p_Z(z)$ from Theorem 3.1,

$$\begin{aligned} E[V^n] &= \int_{-\infty}^{\infty} [f(z)]^n p_Z(z) dz \\ &= \left(\ln\frac{1}{\delta}\right)^n \int_{\ln\frac{1}{\delta}}^{\infty} p_Z(z) dz + \int_0^{\ln\frac{1}{\delta}} z^n p_{Z+}(z) dz \\ &\quad + (-1)^n \int_0^{\ln\frac{1}{\delta}} z^n p_{Z-}(-z) dz + (\ln\delta)^n \int_{-\infty}^{\ln\delta} p_Z(z) dz. \end{aligned}$$

From the above, noting that the first and last integrals can be expressed in terms of $F_Z\left(\ln\frac{1}{\delta}\right)$ and $F_Z(\ln\delta)$ and substituting the expressions for $p_{Z+}(z)$ and $p_{Z-}(z)$ from (23), using Lemma 3.3 we obtain (29).

References

[1] J. N. Laneman and G. W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," *IEEE Wireless Communications and Networking Conference (WCNC)*, vol. 1, pp. 7–12, 2000.

[2] D. Chen and J. N. Laneman, "Cooperative diversity for wireless fading channels without channel state information," *Proc. Asilomar Conf. Signals, Systems, and Computers*, November.

[3] ——, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 7, pp. 1785–1794, July 2006.

[4] B. Wang and J. Zhang, "Mimo relay channel and its application for cooperative communication in adhoc networks," *Proc. Allerton Conf. Communication, Control and Computing (Allerton 2003)*, pp. 1556–1565, October 2003.

[5] H. Bolcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, "Capacity scaling laws in mimo relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 6, June 2006.

[6] G. V. V. Sharma, V. Ganwani, U. B. Desai, and S. N. Merchant, "Maximum likelihood detection for cooperative diversity in mimo relay channels," *to appear in IEEE Vehicular Technology Conference (VTC), Fall*, September 2008.

[7] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451–1458, October 1998.

[8] J. G. Proakis, *Digital Communications*, 3rd ed. McGraw-Hill, 1995.

[9] W. Su, "Performance analysis for a suboptimum ml receiver in decode-and-forward communications," *IEEE Global Telecommunications Conference (GLOBECOM)*, pp. 2962–2966, November 2007.

[10] W. Feller, *An introduction to Probability Theory and Its Applications, Vol II*, 2nd ed. John Wiley and Sons, 1970.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, 5th ed. Academic Press, 1994.

[12] G. E. Andrews, R. Askey, and R. Roy, *Special Functions*, 1st ed. Cambridge University Press, 1999.