Maximum Likelihood Detection for Cooperative Diversity in MIMO Relay Channels

G. V. V. Sharma, Vijay Ganwani, Uday B. Desai and S. N. Merchant
SPANN Lab, Electrical Enginnering Department
Indian Institute of Technology, Bombay
Mumbai, India 400076
Email: {gadepall,vijay,ubdesai,merchant}@ee.iitb.ac.in

***Abstract*—Antenna sharing by multiple users using cooperative diversity has been shown to mimic the performance of traditional multiple input, multiple output (MIMO) systems. While this was originally done using a single antenna at each relay, the benefits of cooperative diversity based on multiantenna relays has been a subject of considerable interest. Maximum likelihood (ML) detectors for cooperative diversity through single antenna relays are well known. In this paper, we propose to combine the benefits of cooperative diversity, and transmit and receive diversity offered by MIMO systems. This is done within the ML based decode and forward (DF) cooperative diversity framework, by using multiple antennas at each relay, through ML based processing on the source-relay link and space-time coding on the relay-destination link. Through simulations, we then show that the performance of MIMO Relay based systems is superior to those having a single antenna at each relay.**

***Index Terms*—Cooperative diversity, MIMO relay, space-time coding**

I. Introduction

A significant amount of research in recent years has been dedicated to MIMO systems, where improvement in system performance is achieved by intelligently exploiting the redundancy offered by multiple antennas at the transmitter or receiver in wireless channels. However, the impracticality of having a large number of antennas in handheld devices due to constraints on antenna separation led to the development of cooperative diversity, where wireless terminals in a network act as relays for each other. Thus, the benefits that were supposed to be offered by antenna diversity in MIMO, are now obtained by antenna sharing between users, each of them having only one antenna.

Though most of the studies on cooperative diversity have been from the information theoretic perspective, lately, there have been efforts to devise demodulation schemes for wireless communication systems employing cooperative diversity. In [1], an ML based demodulation scheme is proposed for cooperative communications, where the decode and forward (DF) protocol is used at the relay. A significant feature of this work is the detection scheme for a binary symmetric channel (BSC), which closely resembles a piecewise linear receiver for ML demodulation for frequency shift keying (FSK) and binary phase shift keying (BPSK).

The use of multiple antennas at the relays to achieve cooperative diversity has been proposed in contemporary literature [2], [3]. Again, the focus of research has primarily been on capacity and related issues. In this paper, assuming that perfect channel state information (CSI) is available both at the relay as well as the destination, we propose an ML based cooperative diversity system for a MIMO relay channel, based on the approach in [1]. In the model considered in this paper, we assume that all users, i.e. source, relays and destination are all capable of supporting multiple antennas. Space-time coding is used for transmission on all links using orthogonal designs [4]. At the relay, a decision on the symbol transmitted by the source is first made using the ML criterion, with receive diversity being provided by the multiple antennas. The relay then waits for some time to acquire a few more symbols, which are then transmitted through the same set of antennas. Such a mechanism was first proposed in [5], for a non-cooperative single relay system. The receiver, after appropriately combining the symbols received from the relays, then makes a decision based on the ML criterion. Thus, the receive diversity gain on the source-relay link is cascaded with the transmit diversity gain on the relay-destination link resulting in a significant improvement in system performance. This is demonstrated through simulations.

The rest of the paper is organized as follows. In Section II, we introduce the multiple antenna relay model. In Section III, we obtain the detection rule for the model considered in [1], when the modulation scheme is BPSK. The ML detection rule for cooperative diversity in MIMO relay channels is then derived in Section IV. Simulation results and related discussion constitute Section V. Conclusions are presented in Section VI.

II. The multiantenna relay model

We generalize the model in [1] (Fig. 1), such that the source, relays and destination may now support multiple antennas. We still have N relays in the system. The source has M_s transmit antennas, the rth relay has M_r antennas, that are used for reception on the source-relay link and transmission on relay-destination link, and the destination has M_d receive antennas. All transmissions are done on orthogonal frequency channels. This is shown in Fig 2. The DF mechanism is used, which means that the relay first demodulates the symbols transmitted by the source before retransmitting to the destination.

978-1-4244-1722-3/08/$25.00 ©2008 IEEE.

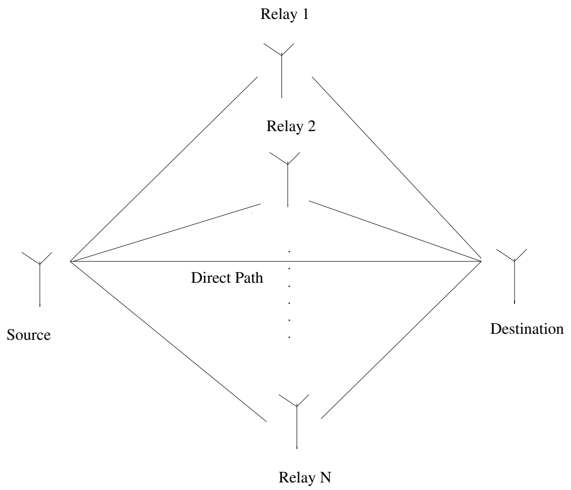

Fig. 1. Single antenna based relay model for cooperative diversity.

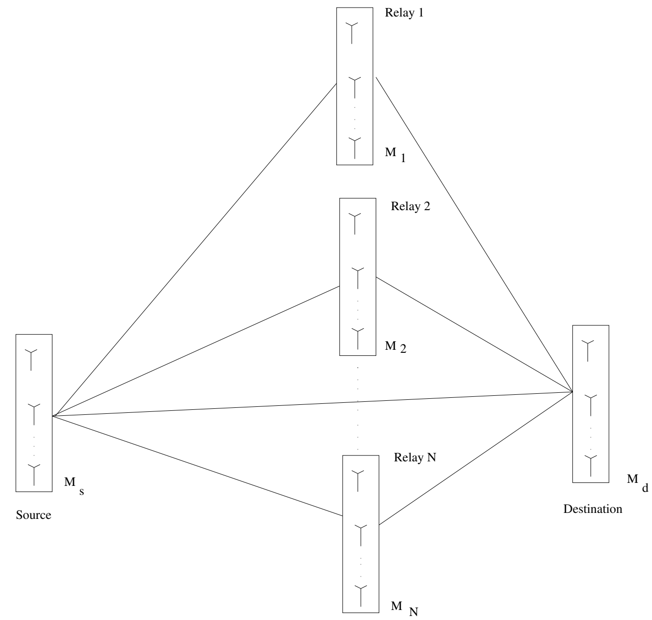

Fig. 2. Multiantenna based relay model for cooperative diversity.

A. Transmission Scheme at the Source and Relay

Orthogonal space-time block codes are used for transmission at both the source and the relay. Let $\mathbf{x}_s$ and $\mathbf{x}_r$ be the symbol vectors whose elements constitute the code word transmitted by the source and relay respectively. We assume the modulation scheme to be BPSK, so $\mathbf{x}_i^s, \mathbf{x}_j^r \in \{1, -1\}$, where $\mathbf{x}_i^s, \mathbf{x}_j^r$ are the ith and jth elements of $\mathbf{x}_s$ and $\mathbf{x}_r$ respectively. Note that $\mathbf{x}_r$ consists of symbols that are actually estimates of the elements of $\mathbf{x}_s$. Let s_t^i be the symbol transmitted at the source in the tth time slot through the ith transmit antenna. The corresponding symbol at the rth relay is represented by $c_{r,t}^i$. Further, the transmit power available with the user is assumed to be equally distributed among all the available antennas. Through the following example, we explain how transmission is done at the source and relay.

Example 1: Suppose we have a cooperative diversity system with $N = 1$, $M_d = 1$, $M_s = 2$ and $M_1 = 2$. Since there are two antennas each at the source and the relay, the Alamouti code [6] can be used for transmission. Hence,

$$\mathbf{x}_s = \begin{pmatrix} x_1^s \\ x_2^s \end{pmatrix}, \mathbf{x}_r = \begin{pmatrix} x_1^r \\ x_2^r \end{pmatrix} \tag{1}$$

and we have the designs

$$\begin{pmatrix} x_1^s & x_2^s \\ -x_2^{s*} & x_1^{s*} \end{pmatrix} \text{ and } \begin{pmatrix} x_1^r & x_2^r \\ -x_2^{r*} & x_1^{r*} \end{pmatrix}, \tag{2}$$

where rows represent the time slots and the columns represent the transmit antennas.

Then,

$$s_1^1 = x_1^s, s_1^2 = x_2^s, s_2^1 = -x_2^{s*}, s_2^2 = -x_2^{s*} \tag{3}$$

and

$$c_{1,1}^1 = x_1^r, c_{1,1}^2 = x_2^r, c_{1,2}^1 = -x_2^{r*}, c_{1,2}^2 = -x_2^{r*}. \tag{4}$$

Assuming flat fading and denoting the path gain from the ith transmit antenna of the source to the jth antenna of the rth relay as $\alpha_{i,j}^r$, we obtain the received symbol at the jth antenna of the rth relay in time slot t as

$$y_{r,t}^j = \sum_{i=1}^{M_s} \alpha_{i,j}^r s_t^i + n_{r,t}^j, \tag{5}$$

where $n_{r,t}^j$ are independent samples of additive white complex Gaussian noise (AWGN). The corresponding equation at the destination (for the source-destination link) is

$$y_{d,t}^j = \sum_{i=1}^{M_s} \alpha_{i,j}^d s_t^i + n_{d,t}^j. \tag{6}$$

For the relay-destination link, the received symbol at the destination is represented by

$$z_{r,t}^j = \sum_{i=1}^{M_r} \beta_{i,j}^r c_{r,t}^i + m_{r,t}^j, \tag{7}$$

where $\beta_{i,j}^r$ and $m_{r,t}^j$ have similar characteristics as $\alpha_{i,j}^r s_t^i$ and $n_{r,t}^j$ respectively.

B. ML Decoding at the Relay

Since perfect CSI is assumed, ML decoding at the rth relay results in minimizing the metric

$$\sum_{t=1}^{l} \sum_{j=1}^{M_r} \left| y_{r,t}^j - \sum_{i=1}^{M_s} \alpha_{i,j}^r s_t^i \right|^2 \tag{8}$$

over all the possible codewords s_t^i. Using complex orthogonal designs simplifies the decision metric resulting in reduced decoding complexity [4]. There is a tradeoff involved since full rate complex orthogonal designs of higher order do not exist. A few rate 1/2 and rate 3/4 designs are available in [4], [7].

III. ML DETECTION RULE FOR BPSK MODULATION IN SINGLE ANTENNA RELAY CHANNELS

We consider the model in Fig. 1, where there are N relays, a source and a destination, all equipped with a single antenna. Thus, we have $M_s = M_d = 1$ and $M_r = 1$. There is no antenna diversity to exploit, so the only way to improve performance is through cooperation. Using the notations defined in the previous section, the received symbols at the relays and destination are

$$\begin{aligned} y_{r,1}^1 &= \alpha_{1,1}^r s_1^1 + n_{r,1}^1 &(9)\\ y_{d,1}^1 &= \alpha_{1,1}^d s_1^1 + n_{d,1}^1 &(10)\\ z_{r,1}^1 &= \beta_{1,1}^r c_{r,1}^1 + m_{r,1}^1. &(11)\end{aligned}$$

Modeling the noise at the destination, $n_{d,1}^1$ and $m_{r,1}^1$ as zero mean complex Gaussian with $E[|n_{d,1}^1|^2] = E[|m_{r,1}^1|^2] = N_d$, where $E[\ ]$ is the expectation operator, and defining

$$\begin{aligned} l^s(y_{d,1}^1) &= \frac{4\text{Re}\{\alpha_{1,1}^{d*} y_{d,1}^1\}}{N_d}\\ l^r(z_{r,1}^1) &= \frac{4\text{Re}\{\beta_{1,1}^{r*} z_{r,1}^1\}}{N_d}, \end{aligned} \qquad (12)$$

the decision rule for symbol detection at the receiver using ML is obtained from [1] as

$$l^s(y_{d,1}^1) + \sum_{r=1}^{N} \ln \frac{\epsilon_r + (1-\epsilon_r)\exp\left(l^r(z_{r,1}^1)\right)}{(1-\epsilon_r) + \epsilon_r \exp\left(l^r(z_{r,1}^1)\right)} \underset{-1}{\overset{1}{\gtrless}} 0, \quad (13)$$

where ϵ_r is the probability of bit error at the rth relay.

IV. ML DETECTION FOR THE MIMO RELAY CHANNEL

We first show, through a simple example, that the decision rule proposed in the previous section can be used even for the MIMO relay channel.

Example 2: For simplicity, we consider the case when $M_s = M_d = 1, N = 1$ and $M_1 = 2$. Maximal ratio combining is used on the source-relay link to obtain receive diversity. Space-time coding through the Alamouti code is used on the relay-destination link. Thus, the relay waits for two symbol periods before transmitting to the destination. At the destination, we have the following equations for the received symbols

$$\begin{aligned} y_{d,1}^1 &= \alpha_{1,1}^d s_1^1 + n_{d,1}^1 &(14)\\ z_{1,1}^1 &= \sum_{i=1}^{2} \beta_{i,1}^1 c_{1,1}^i + m_{1,1}^1 &(15)\\ z_{1,2}^1 &= \sum_{i=1}^{2} \beta_{i,1}^1 c_{1,2}^i + m_{1,2}^1. &(16)\end{aligned}$$

Now, ML decoding for the Alamouti scheme leads to the following decision variables

$$\begin{aligned} \hat{x}_1^1 &= \beta_{1,1}^{1*} z_{1,1}^1 + \beta_{2,1}^1 z_{1,2}^{1*} &(17)\\ \hat{x}_2^1 &= \beta_{2,1}^{1*} z_{1,1}^1 - \beta_{1,1}^1 z_{1,1}^{1*} &(18)\end{aligned}$$

for the variables x_1^1 and x_2^1 transmitted at the relay. Since we are interested in making a decision only on one of the symbols, we choose (17) to make a decision on $x^s = s_1^1$. After some algebra, we obtain (17) as

$$\hat{x}_1^1 = \left(\sum_{i=1}^{2} |\beta_{i,1}^1|^2\right) x_1^1 + \beta_{1,1}^{1*} m_{1,1}^1 + \beta_{2,1}^1 m_{1,2}^{1*}. \quad (19)$$

If we let $z = \beta_{1,1}^{1*} m_{1,1}^1 + \beta_{2,1}^1 m_{1,2}^{1*}$, then the conditional variance

$$E[|z|^2] = \left(\sum_{i=1}^{2} |\beta_{i,1}^1|^2\right) N_d, \quad (20)$$

and (11) and (19) are similar. Also, (10) and (14) are exactly the same. Following the approach in [1], we obtain the ML decision rule for the model considered in Example 2 as

$$l^s(y_{d,1}^1) + \ln \frac{\epsilon_1 + (1-\epsilon_1)\exp\left(l^1(\hat{x}_1^1)\right)}{(1-\epsilon_1) + \epsilon_1 \exp\left(l^1(\hat{x}_1^1)\right)} \underset{-1}{\overset{1}{\gtrless}} 0, \quad (21)$$

where

$$\begin{aligned} l^s(y_{d,1}^1) &= \frac{4\text{Re}\{\alpha_{1,1}^{d*} y_{d,1}^1\}}{N_d}\\ l^1(\hat{x}_1^1) &= \frac{4\text{Re}\left\{\left(\sum_{i=1}^{2} |\beta_{i,1}^1|^2\right)^* \hat{x}_1^1\right\}}{E[|z|^2]}\\ &= \frac{4\text{Re}\left\{\hat{x}_1^1\right\}}{N_d} \end{aligned} \qquad (22)$$

and ϵ_1 is the probability of error for MRC with two receive antennas.

The key step in the above is (19), which does not contain cross terms involving other symbols. Also, the conditional noise variance is the same as the signal amplitude. This happens because of the choice of orthogonal designs in space-time coding. The $\mathcal{G}_4$ design proposed in [7] can be used in the ML cooperative diversity framework when $M_s = 4$ and/or $M_r = 4$. A justification for doing this can be provided on the lines of Example 2, by looking at [7], eqn. (9), p. 454. Thus, by choosing appropriate space-time codes, it is possible to extend the above results for any number of relays with any number of antennas.

V. RESULTS AND DISCUSSION

In this section, we present the numerical results for the bit error rate (BER) performance of the cooperative diversity model considered in this paper for various scenarios. Since we wish to compare the BER performance of this system with that of [8], we have used similar simulation parameters as in [8]. Thus, the average signal to noise ratio (SNR) on the source-relay and source-destination links are respectively given by

$$\begin{aligned} \overline{\gamma}_{i,j}^r &= \frac{E[|\alpha_{i,j}^r|^2]E_s}{N_r}\\ \overline{\gamma}_{i,j}^d &= \frac{E[|\alpha_{i,j}^d|^2]E_s}{N_d}, \end{aligned} \qquad (23)$$

where the superscript r represents the $r$the relay, E_s is the power allotted to each antenna at the source for transmission and N_r is the noise power at the rth relay. The average SNR on the relay-destination link is given by

$$\overline{\mu}_{i,j}^{r} = \frac{E[|\beta_{i,j}^{r}|^2]E_r}{N_d}. \tag{24}$$

If $d_{s,q}$ be the distance between the source and the node q, which may be either a relay or the destination, $E[|\alpha_{i,j}^{q}|^2] \propto d_{s,j}^{-4}$. Similarly, $E[|\beta_{i,j}^{r}|^2] \propto d_{r,d}^{-4}$, where $d_{r,d}$ is the distance between the rth relay and the destination.

Fig. 3 compares the performance of our system with the one considered in [8] when a single relay is located halfway between the source and the destination. This is the model discussed in Example 2. The decision relation in (22) has been used for calculating the the bit error rate (BER). The relevant expression for ϵ_1 is available in [9]. In the high SNR region (10-15 dB), as can be seen from the figure, the gains are enormous. This is because of the cascading effect of the gains in the source-relay and relay-destination link. The single hop and dual hop BER curves are also plotted for comparison. Here we considered the transmit power at the source to be $E_s = \frac{1}{2}$. The relay power is distributed equally among its two antennas so that $E_1 = \frac{1}{4}$.

In Fig. 4, numerical results for the case of two relays located at distances $0.4d$ and $0.5d$ from the source are shown, d being the distance between the source and the destination. Thus, we have $M_s = M_d = 1, N = 2$. Both relays are assumed to have two antennas each, so that $M_1 = M_2 = 2$ and $E_s = \frac{1}{2}, E_1 = E_2 = \frac{1}{8}$. We note that even at an SNR of 5 dB, our model provides a remarkable improvement over the one in [8]. This is quite significant, because this shows that even with a reduction in battery power at a handheld device in a cooperative diversity based wireless network, one may still obtain a better system performance, depending on the level of cooperation.

A comparison of the performance of single antenna based cooperative systems and multiantenna based systems with one or more relays is shown in Fig. 3. It is interesting to note that the BER curves cross over at low SNR for the multiantenna system with one and two relays. One reason for this may be that the multiple message paths may interfere with each other and prevent cooperation. However, it is quite obvious that the performance of the multiantenna relay based cooperative diversity system keeps improving with an increase in the number of cooperating relays. We have also provided the BER results for the case when $N = 1, M_d = 1$ and $M_s = M_1 = 2$. As can be seen in Fig. 5, the performance of this system is far better than those considered earlier. This is significant, because despite having a single relay (a total of four transmit antennas), this system performs better than the one with two relays, each supporting two antennas (a total of five transmit antennas, including the source).

VI. Conclusions

We considered a cooperative diversity system where each relay is equipped with more than one antenna, which can be used for both transmission as well as reception. Assuming that all transmissions are on orthogonal channels, we then developed an ML based detection scheme for cooperative diversity based on the approach in [1]. However, extra processing was done at the relay through ML decoding on the source relay link and space-time block coding using orthogonal designs on the relay destination link. Through numerical results, it was then shown that the modulation-demodulation methods proposed in this paper result in a very significant improvement in the BER performance over [8], even with reduced relay transmit power.

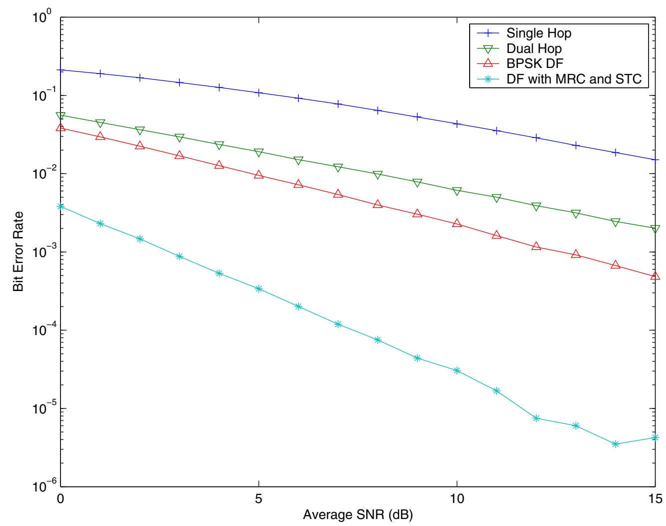

Fig. 3. BER performance for the case of a single relay located halfway between source and destination.

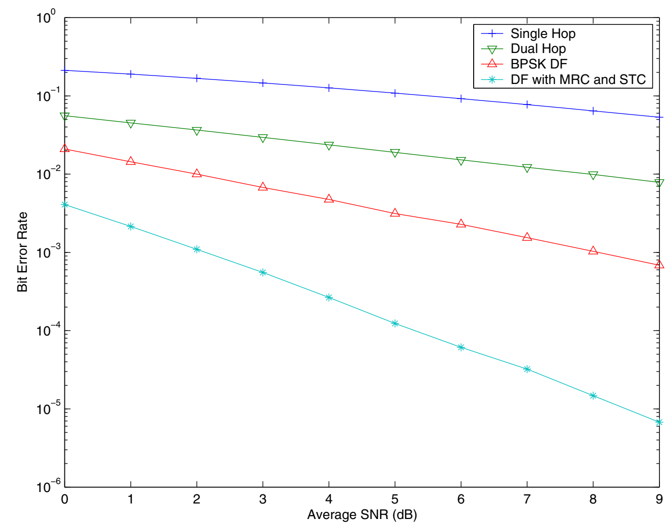

Fig. 4. BER performance for the case of two relays. The relays are located at a distances $0.4d$ and $0.5d$ from the source, d being the distance between the source and destination.

References

[1] D. Chen and J. N. Laneman, "Cooperative diversity for wireless fading channels without channel state information," *Proc. Asilomar Conf. Signals, Systems, and Computers,* Nov. 2004, pp. 1307-1312,.

[2] B. Wang and J. Zhang, "MIMO relay channel and its application for cooperative communication in adhoc networks," *Proc. Allerton Conf.*

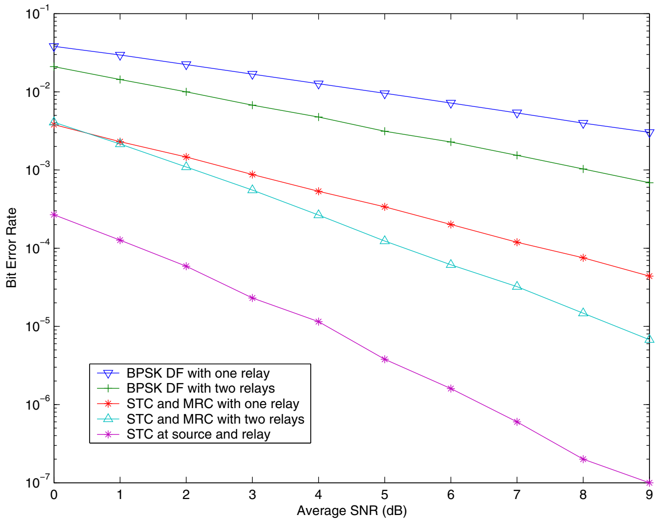

Fig. 5. A comparison of the performance of the ML based cooperative diversity systems employing a) one antenna at each relay and b) two antennas at each relay. Results are shown for up to two relays between source and destination, including the case where the source transmits using two antennas.

Communication, Control and Computing (Allerton 2003), October 2003, pp. 1556-1565.

[3] H. Bolcskei, R. U. Nabar, O. Oyman and A. J. Paulraj "Capacity scaling laws in MIMO relay networks," *IEEE Trans. Wireless. Commun.,* vol. 5, no.6, June 2006.

[4] V. Tarokh, H. Jafarkhani, and A. R. Calderbank,"Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory,* vol. 45, no. 5,pp. 1456-1467, July 1999.

[5] G. V. V. Sharma and S. H. Srinivasan, "Diversity gain using a repeater in a wireless personal area network, " *IEEE 61st Vehicular Technology Conference (VTC)*, Spring 2005, Volume 3, June 2005 pp. 1519 - 1522.

[6] S. M. Alamouti, "A simple transmitter diversity scheme for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[7] V. Tarokh, H. Jafarkhani, and A. R. Calderbank,"Space-time block coding for wireless communications: performance results," *IEEE J. Select. Areas Commun.,* vol. 17, no. 3,pp. 451-460, March 1999.

[8] D. Chen and J. N. Laneman, "Modulation and demodulation for cooperative diversity in wireless systems," *IEEE Trans. Wireless. Comm.,* vol. 5, no.7, July 2006.

[9] J. G. Proakis, *Digital Communications*, 3rd ed. New York, NY: McGraw-Hill, 1995.