Performance Analysis of Maximum Likelihood Detection for Decode and Forward MIMO Relay Channels in Rayleigh Fading

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***Abstract*—Maximum-likelihood (ML) detectors for single-input-single-output (SISO) relay systems are well known. In this paper, closed form expressions for the bit error rate (BER) for multiple-input-multiple -output (MIMO) relay systems employing ML based decode and forward (DF) cooperative diversity are obtained. The DF operation at the relay intelligently employs the multiple antennas available at the relay for receive diversity on the source-relay link and space-time coding (STC) on the relay-destination link resulting in an extension of the ML detection rule for SISO systems to MIMO systems as well. For the piecewise linear (PL) approximation to the ML detector, exact expressions for the BER are obtained for single-relay systems with both the source and relay supporting multiple antennas. For a multirelay system, each relay having multiple antennas, approximate expressions for the BER are obtained. This is done by finding the statistics of conditionally Gaussian (CG) random variables, that appear in the decision variable. Through numerical results obtained from the BER expressions, it is shown that using multiple antennas at the source as well as the relay leads to significant improvement in BER performance.**

***Index Terms*—MIMO relay, Decode and forward, conditionally Gaussian, Space-time coding**

I. I

Practical constraints on the deployment of traditional multiple input multiple output (MIMO) systems in wireless communication networks have led to considerable interest in cooperative communication systems, where diversity gain is obtained by transmission through relays distributed across the wireless network. Thus, it is no longer necessary for the mobile station to support multiple antennas, a requirement that is difficult to satisfy given the constraints on antenna separation at a node. Consequently, the benefits that were supposed to be offered by antenna diversity in MIMO systems, are now obtained through cooperative diversity by antenna sharing between users.

Several practical protocols for relay based transmission are discussed in [1]. Of these, one of the most popular relaying techniques employed in cooperative diversity systems is the decode and forward (DF) protocol, where the relay makes a decision on the symbol transmitted by the source before retransmitting to the destination. Maximum-likelihood (ML) detection for DF cooperative diversity [1]–[7] was first proposed in [2] followed by a detailed derivation in [3] for coherent and noncoherent single-input-single-output (SISO) relay systems. For practical applications, a piecewise linear (PL) approximation to the ML detector, known as the PL combiner, was also provided in [3], [4]. While DF based transmission has been a subject of considerable interest among researchers, the focus has rarely been on bit error rate (BER) analysis [8]–[12].

A. Motivation

The BER for the PL combiner for a single relay cooperative system with noncoherent DF was derived in [4]. This expression was subsequently used to derive the diversity order of the corresponding multirelay system. Error rate analysis for the optimum coherent DF receiver, considered to be a challenging proposition due to the nonlinearity of the ML detector [4], was addressed in [13], [14]. A similar attempt was made in [5] for the Gaussian fading channel, but no closed form expressions could be obtained. While these contributions are quite significant, they also lead to certain interesting problems. Firstly, the bounds on the diversity order derived in [4] are not supported by numerical results. Secondly, because of the loss in diversity order due to DF for SISO systems, there is considerable interest in the design and performance of MIMO relay systems that are known to have a higher diversity order [15]. Closed form expressions for the BER for MIMO relay systems employing two antennas at each relay [6] were obtained in [16].

B. Main Contributions

In this paper, we obtain

1) a general detection rule for ML-DF MIMO relay systems. This is done by exploiting receive diversity on the source-relay link and transmit diversity on the relay-destination link through space-time coding (STC) [6].
2) exact closed form expression for the BER for the PL combiner for a single relay with the source as well as the relay supporting multiple antennas. A novel concept of conditionally Gaussian (CG) random variables is developed in the process.
3) closed form approximation for the BER for the PL combiner for the general MIMO multirelay system.
4) simulation results validating the closed form BER expressions.
5) numerical results confirming the loss in diversity order due to DF [4].

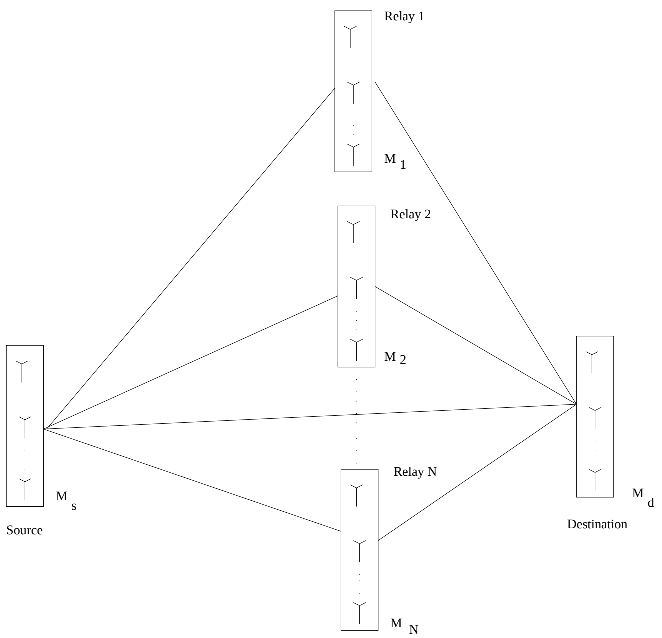

Fig. 1. Multiantenna based relay model for cooperative diversity.

II. S M

We consider the general MIMO relay model in [6] with N relays (R) between the source (S) and destination (D) as shown in Figure 1. The source can support M_s transmit antennas, the rth relay M_r antennas that are used in DF mode, and the destination has $M_d = 1$ receive antennas. The modulation is BPSK and all transmissions are on mutually orthogonal frequencies to avoid interference at the relays as well as the destination [4]. Further, the channel is assumed to be quasi-static with flat fading so that the channel gains remain constant over a few symbols. Perfect channel state information (CSI) is assumed both at the relay as well as the destination for the S-R and R-D links across all paths. For $M_s, M_r > 1$, the source and relays use full rate real square orthogonal space-time block codes (STBCs) for transmission [17].

A. Transmission Scheme

Example 2.1: Let $M_s = M_r = 2$. The source transmits the symbols $x_s \in \{1, -1\}, x'_s \in \{1, -1\}$ on consecutive time slots using the Alamouti code [18] on the S-R link according to the following scheme

$$\begin{pmatrix} x_s & x'_s \\ -x'_s & x_s \end{pmatrix}, \tag{1}$$

where the rows represent time slots and columns represent transmit antennas. The relay waits for two time slots and then makes the decisions x_r and x'_r corresponding to the respective transmitted symbols x_s and x'_s according to the combining scheme in [18]. In this case, the S-R link performs as well as four branch maximal ratio combining (MRC). The relay then transmits x_r and x'_r on the R-D link, again using the Alamouti code.

The transmission technique in Example 2.1 can be generalized for cases where the source and relay nodes support higher number of antennas employing higher order orthogonal STBCs. In this paper, since we employ full rate real orthogonal designs, $M_s, M_r = 1, 2, 4$ or 8. For $M_s = 1$, MRC is performed at the relays on the S-R link. We use the alphabet x to represent a transmitted symbol, h to represent the channel gain on a given link, n for additive white Gaussian noise, y for the received symbol at a node and E for the transmit power at a given node. For i, j denoting the antenna indices, in a given time slot[1], the received symbols on the S-D, S-R and R-D links are respectively

$$\begin{aligned} y_{d,s} &= \sum_{i=1}^{M_s} h_{d,s}^{(i)} x_{d,s}^{(i)} + n_{d,s}^{(i)}, \\ y_{r,s}^{(i)} &= \sum_{j=1}^{M_s} h_{r,s}^{(ij)} x_{d,s}^{(j)} + n_{r,s}^{(i)}, \quad r = 1, 2, \dots, N, \\ y_{d,r} &= \sum_{i=1}^{M_r} h_{d,r}^{(i)} x_{d,r}^{(i)} + n_{d,r}^{(i)}, \end{aligned} \tag{2}$$

where $x_{d,s}^{(i)} \in \{1, -1\}$ is the symbol transmitted by ith antenna of the source and $x_{d,r}^{(i)} \in \{1, -1\}$ is the symbol transmitted by the ith antenna of the rth relay. Thus, for Example 2.1, in the first time slot $x_{d,s}^{(1)} = x_s, x_{d,s}^{(2)} = x'_s$. In the second time slot, $x_{d,s}^{(1)} = -x'_s, x_{d,s}^{(2)} = x_s$. The channel experiences Rayleigh fading, hence the fading coefficients $h_{d,s}^{(i)} \sim \mathcal{CN}(0, \Omega_{d,s} E_s^i), h_{r,s}^{ij} \sim \mathcal{CN}(0, \Omega_{r,s} E_s^j)$ and $h_{d,r}^{(i)} \sim \mathcal{CN}(0, \Omega_{d,r} E_r^i)$ are zero mean complex circularly Gaussian, with E_s^i and E_r^i denoting the source and relay powers at their respective ith transmit antennas. Also, $n_{d,s}^{(i)}, n_{r,s}^{(i)}, n_{d,r}^{(i)} \sim \mathcal{CN}(0, N_0)$.

B. ML Decision at the Destination

At the destination, the ML decision criterion for the symbol transmitted by the source in the first time slot through the first antenna may be expressed as (see Appendix A)

$$X + \sum_{r=1}^{N} f(Y_r) \underset{-1}{\overset{1}{\gtrless}} 0, \tag{3}$$

where

$$\begin{aligned} X &= \frac{4\mathrm{Re}\left\{\sum_{i=1}^{M_s} h_{d,s}^{(i)*} y_{d,s}^{(i)}\right\}}{N_0}, \\ Y_r &= \frac{4\mathrm{Re}\left\{\sum_{i=1}^{M_r} h_{d,r}^{(i)*} y_{d,r}^{(i)}\right\}}{N_0}, \end{aligned} \tag{4}$$

$$f(x) = \ln \frac{\nu + e^x}{1 + \nu e^x}, \quad 0 < \nu < 1 \tag{5}$$

with $y_{d,s}^{(i)}, y_{d,r}^{(i)}$ being the respective received symbols on the S-D and R-D links in the ith time slot. Also, $f(Y_r)$ now has the parameter $\nu_r = \frac{\epsilon_r}{1-\epsilon_r}$ for the rth relay, where ϵ_r is the BER for the S-R link. To simplify the analysis, we use the suboptimal ML scheme proposed in [5] by considering ϵ_r to be the average BER instead of the instantaneous BER. Expressions for ϵ_r can be easily obtained from [19, 14.4-15, p.825]. In the rest of the paper, the fading characteristics in the S-R link have been

[1] The time domain notation has been dropped to simplify the representation.

assumed to be similar for all the relays. As a consequence, r can be ignored and $\nu = \frac{\epsilon}{1-\epsilon}$. We now state a couple of facts related to the ML decision rule in (3).

1) The decision variables X and Y_r for other symbols transmitted by the source can be expressed as [17, (8)]

$$X = \frac{4\mathrm{Re}\left\{\sum_{i=1}^{M_s} \lambda_i h_{d,s}^{(i)*} y_{d,s}^{(\varepsilon(i))}\right\}}{N_0},$$
$$Y_r = \frac{4\mathrm{Re}\left\{\sum_{i=1}^{M_r} \lambda_i h_{d,r}^{(i)*} y_{d,r}^{(\varepsilon(i))}\right\}}{N_0}, \quad (6)$$

where $\lambda_i \in \{1, -1\}$ denotes a sign change in the linear combination and $\varepsilon(i)$ represents a permutation for the index i. From (36) in Appendix A, we find that for $M_s = 2, \lambda_1 = -1, \lambda_2 = 1, \varepsilon(1) = 2, \varepsilon(2) = 1$, for the second symbol transmitted by the source.

2) Real orthogonal designs with full rate allow linear processing at the receiver, which results in (3) for BPSK modulation. However, even for $M_s, M_r \notin \{1, 2, 4, 8\}$ it is possible to use orthogonal designs with linear processing but reduced rate [20]–[22]. Thus, (3) with (6) holds for square orthogonal designs with any number of transmit antennas at the source or relay. The extension to rectangular orthogonal designs is straightforward.

C. PL Combiner

Lemma 2.1: The function $f(x)$ in (5) has the following piecewise linear approximation [3], [4]

$$f(x) \approx \begin{cases} \ln\frac{1}{\nu} & x \geq \ln\frac{1}{\nu} \\ t & \ln\nu < x < \ln\frac{1}{\nu} \\ \ln\nu & x < \ln\nu \end{cases}. \quad (7)$$

Using the above approximation in (3), we obtain the PL combiner for ML-DF systems.

D. Problem Definition

Let $x_s \in \{1, -1\}$ be the symbol transmitted by the source in the first time slot through the first antenna. Assuming equal probability of the symbols $\{1, -1\}$, the average probability of error for the ML-DF cooperative diversity system for BPSK can be expressed as

$$P_e = \sum_{\mathbf{x}} \prod_{r=1}^{N} \epsilon_r^{\frac{1-x_r}{2}} (1-\epsilon_r)^{\frac{1+x_r}{2}} \Pr\left(X + \sum_{i=1}^{N} f(Y_i) < 0 \mid x_s = 1, \mathbf{x}\right), \quad (8)$$

where $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is the set of all possible decisions made at the N relays corresponding to the transmitted symbol x_s. We wish to find a closed form expression for P_e in (8).

E. Solution Strategy

Knowledge of the statistics of X and Y_r is essential for evaluating P_e in (8). A simple approach for finding the distribution of X and Y_r is outlined through the following example.

Example 2.2: Let $M_s = 1$ and $M_r = 2$. From (2) and (4), X can be expressed as

$$X = \frac{4x_s}{N_0}|h_{d,s}|^2 + \frac{4}{N_0}\mathrm{Re}\{h_{d,s}^* n_{d,s}\}, \quad (9)$$

where x_s is the symbol transmitted by the source. The conditional mean and variance of X can be expressed as

$$E[X \mid h_{d,s}] = \frac{4}{N_0} x_s |h_{d,s}|^2,$$
$$var(X \mid h_{d,s}) = \frac{8}{N_0}|h_{d,s}|^2. \quad (10)$$

From (9) and (10), we find that $X \mid h_{d,s} \sim \mathcal{N}\left(\frac{4x_s}{N_0}|h_{d,s}|^2, \frac{8}{N_0}|h_{d,s}|^2\right)$. Significantly, both the conditional mean and variance of X are proportional to $|h_{d,s}|^2$. It is easy to show that even $Y_r \mid h_{d,r}^{(1)}, h_{d,r}^{(2)} \sim \mathcal{N}\left(\frac{4x_r}{N_0}\sum_{i=1}^{2}|h_{d,r}^{(i)}|^2, \frac{8}{N_0}\sum_{i=1}^{2}|h_{d,r}^{(i)}|^2\right)$ has a similar property. In general, from (4), we find that

$$X \mid \mathbf{h}_{d,s} \sim \mathcal{N}\left(\frac{4x_s}{N_0}||\mathbf{h}_{d,s}||^2, \frac{8}{N_0}||\mathbf{h}_{d,s}||^2\right),$$
$$Y \mid \mathbf{h}_{d,r} \sim \mathcal{N}\left(\frac{4x_r}{N_0}||\mathbf{h}_{d,r}||^2, \frac{8}{N_0}||\mathbf{h}_{d,r}||^2\right), \quad (11)$$

where $\mathbf{h}_{d,s} = \{h_{d,s}^{(i)}\}_{i=1}^{M_s}$, $\mathbf{h}_{d,r} = \{h_{d,r}^{(i)}\}_{i=1}^{M_r}$ and $||\cdot||$ represents the Euclidean norm. We refer to such distributions as being *conditionally Gaussian* (CG). In the following sections, by exploiting their conditionally Gaussian nature, we find the statistics of CG random variables like X, Y_r and their functions $f(Y_r)$. These are then used to find closed form expressions for P_e in (8) in the subsequent sections.

III. S C G D

Without loss of generality, we use the symbols p_X for the probability density function (PDF), F_X for the cumulative distribution function (CDF) and Φ_X for the characteristic function (CF) of a random variable X.

Lemma 3.1: Let $X \mid \mathbf{h} \sim \mathcal{N}\left(a||\mathbf{h}||^2, b||\mathbf{h}||^2\right), b > 0$, where the $M \times 1$ random vector $\mathbf{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Omega})$, with $\mathbf{\Omega} = E\left[\mathbf{h}\mathbf{h}^H\right]$ being a diagonal matrix, $\{\cdot\}^H$ and $E[\cdot]$ denoting the Hermitian and expectation operations respectively. Also, let $\Omega_l, l = 1, \ldots, L$ be the distinct diagonal elements of $\mathbf{\Omega}$ with multiplicities m_l so that $\sum_{l=1}^{L} m_l = M$. The CF of X is then given by

$$\Phi_X(\jmath\omega) = \frac{1}{\prod_{l=1}^{L}\left(1 - \jmath a\omega\Omega_l + \frac{b}{2}\omega^2\Omega_l\right)^{m_l}}. \quad (12)$$

Corollary 3.1: The CDF of X is given by

$$F_X(x) = \begin{cases} \mathcal{F}(\alpha_1, \alpha_2, x) & x < 0 \\ 1 - \mathcal{F}(\alpha_2, \alpha_1, x) & x \geq 0, \end{cases} \quad (13)$$

where

$$\mathcal{F}(\alpha_1, \alpha_2, x) = \sum_{l=1}^{L} \frac{(-\alpha_{l2})^{2M-m_l} e^{-\frac{x}{\alpha_{l2}}}}{\prod_{k=1}^{L}(\alpha_{k1} - \alpha_{l2})^{m_k} \prod_{\substack{k=1 \\ k\neq l}}^{L}(\alpha_{k2} - \alpha_{l2})^{m_k}} \sum_{\mathbf{q}.\mathbf{v}_{m_l-1} = m_l - 1} \prod_{n=1}^{m_l-1} \frac{1}{q_n!}$$
$$\times \left(\frac{1}{n} + \frac{1}{n}\sum_{k=1}^{L}\frac{m_k\alpha_{k1}^n}{(\alpha_{k1} - \alpha_{l2})^n} + \frac{1}{n}\sum_{\substack{k=1 \\ k\neq l}}^{L}\frac{m_k\alpha_{k2}^n}{(\alpha_{k2} - \alpha_{l2})^n} + \frac{x\delta_{n-1}}{\alpha_{l2}}\right)^{q_n} \quad (14)$$

with $\mathbf{q} = \{q_i\}_{i=1}^{m_l-1}, 0 \leq q_n \leq m_l - 1, \mathbf{v}_{m_l-1} = \{i\}_{i=1}^{m_l-1}, \alpha_j = \{\alpha_{lj}\}_{l=1}^{L}, j = 1,2$, (.) denoting the inner product, $\delta_{(\cdot)}$ is the Kronecker delta function and

$$\alpha_{l1}, \alpha_{l2} = \frac{a\Omega_l}{2}\left(1 \pm \frac{a}{|a|}\sqrt{1 + \frac{2b}{a^2\Omega_l}}\right), \quad l = 1, \dots, L. \quad (15)$$

Corollary 3.2: The Nth moment of $f(X)$ is

$$E\left[\{f(X)\}^N\right] = \mathcal{E}_N\left(\alpha_2, \alpha_1, \ln\frac{1}{\nu}\right) + \mathcal{E}_N\left(\alpha_1, \alpha_2, \ln\nu\right), \quad (16)$$

where

$$\begin{aligned}
&\mathcal{E}_N\left(\alpha_2, \alpha_1, \ln\frac{1}{\nu}\right) = \\
&\sum_{l=1}^{L} \frac{N(-1)^N(-\alpha_{l1})^{2M+N-m_l}}{\prod_{k=1}^{L}(\alpha_{k2}-\alpha_{l1})^{m_k}\prod_{\substack{k=1\\k\neq l}}^{L}(\alpha_{k1}-\alpha_{l1})^{m_k}} \sum_{\mathbf{q}.\mathbf{v}_{m_l-1}=m_l-1} \frac{1}{q_1!} \\
&\times \prod_{n=2}^{m_l-1}\frac{1}{q_n!}\left(\frac{1}{n} + \frac{1}{n}\sum_{k=1}^{L}\frac{m_k\alpha_{k2}^n}{(\alpha_{k2}-\alpha_{l1})^n} + \frac{1}{n}\sum_{\substack{k=1\\k\neq l}}^{L}\frac{m_k\alpha_{k1}^n}{(\alpha_{k1}-\alpha_{l1})^n}\right)^{q_n} \\
&\times \sum_{\kappa=0}^{q_1}\binom{q_1}{\kappa}\left(1 + \sum_{k=1}^{L}\frac{m_k\alpha_{k2}}{(\alpha_{k2}-\alpha_{l1})} + \sum_{\substack{k=1\\k\neq l}}^{L}\frac{m_k\alpha_{k1}}{(\alpha_{k1}-\alpha_{l1})}\right)^{q_1-\kappa} \\
&\times \Gamma\left(\kappa + N, \frac{1}{\alpha_{l1}}\ln\frac{1}{\nu}\right) \quad (17)
\end{aligned}$$

and $\Gamma(\cdot,\cdot)$ is the incomplete Gamma function [23, (4.4.5), p.197].

Proof: See Appendix B.

IV. BER A

A. Single Relay System

For $N = 1$, (8) reduces to

$$P_e = \sum_{x\in\{1,-1\}} \epsilon^{\frac{1-x}{2}}(1-\epsilon)^{\frac{1+x}{2}} \Pr\left(X + f(Y) < 0 \mid x_s = 1, x\right), \quad (18)$$

where the subscript r has been dropped for convenience. The conditional probability in (18) can then be expressed using the Steiltjes form as

$$\begin{aligned}
\Pr\left(X + f(Y) < 0 \mid x_s = 1, x\right) &= \Pr\left(X < -f(Y) <\mid x_s = 1, x\right) \\
&= \int_{-\infty}^{\infty} F_X(-f(x))dF_Y(x). \quad (19)
\end{aligned}$$

Using the PL approximation in the above,

$$\begin{aligned}
&\int_{-\infty}^{\infty} F_X(-f(x))dF_Y(x) \\
&\approx \int_{-\infty}^{\ln\nu} F_X(-\ln\nu)dF_Y(x) + \int_{\ln\nu}^{0} F_X(x)dF_Y(x) \\
&\quad + \int_{0}^{\ln\frac{1}{\nu}} F_X(-x)dF_Y(x) + \int_{\ln\frac{1}{\nu}}^{\infty} F_X(\ln\nu)dF_Y(x) \\
&= F_X(-\ln\nu)F_Y(\ln\nu) + F_X(\ln\nu)\left[1 - F_Y(-\ln\nu)\right] \\
&\quad + \int_{\ln\nu}^{\ln\frac{1}{\nu}} F_X(-x)dF_Y(x) \quad (20)
\end{aligned}$$

after rearranging. Integrating the above by parts and simplifying results in

$$\Pr\left(X + f(Y) < 0 \mid x_s = 1, x\right) = F_X(\ln\nu) + \int_{\ln\nu}^{\ln\frac{1}{\nu}} F_Y(-x)dF_X(x) \quad (21)$$

Since X and Y are known to be CG from (11), their respective CDFs F_X and F_Y are given by (13). Let F_X and F_Y have the parameters (α_1, α_2) and (β_1, β_2) respectively. Substituting the respective CDFs followed by a change of variables, (21) can be expressed as

$$\begin{aligned}
\Pr\left(X + f(Y) < 0 \mid x_s = 1, x\right) &= \mathcal{F}\left(\alpha_1, \alpha_2, 0\right) \\
&+ \int_{0}^{\ln\frac{1}{\nu}} \mathcal{F}\left(\beta_2, \beta_1, x\right) d\mathcal{F}\left(\alpha_1, \alpha_2, -x\right) \\
&- \int_{0}^{\ln\frac{1}{\nu}} \mathcal{F}\left(\beta_1, \beta_2, -x\right) d\mathcal{F}\left(\alpha_2, \alpha_1, x\right). \quad (22)
\end{aligned}$$

Evaluating the above integrals and simplifying, we obtain (see Appendix C)

$$\begin{aligned}
&\Pr\left(X + f(Y) < 0 \mid x_s = 1, x\right) = \\
&\quad \mathcal{F}\left(\alpha_1, \alpha_2, 0\right) + \mathcal{F}\left(\beta_1, \beta_2, 0\right)\mathcal{F}\left(\alpha_2, \alpha_1, 0\right) \\
&\quad - \mathcal{F}\left(\beta_1, \beta_2, \ln\nu\right)\mathcal{F}\left(\alpha_2, \alpha_1, -\ln\nu\right) \\
&\quad + \mathcal{G}\left(\alpha_1, \alpha_2; \beta_2, \beta_1\right) + \mathcal{G}\left(\beta_1, \beta_2; \alpha_2, \alpha_1\right), \quad (23)
\end{aligned}$$

where

$$\begin{aligned}
&\mathcal{G}\left(\beta_1, \beta_2; \alpha_2, \alpha_1\right) = \\
&\sum_{i=1}^{L_r} \frac{(-\beta_{i2})^{2M_r-m_i}}{\prod_{j=1}^{L_r}\left(\beta_{j1}-\beta_{i2}\right)^{m_j}\prod_{\substack{j=1\\j\neq i}}^{L_r}\left(\beta_{j2}-\beta_{i2}\right)^{m_j}} \sum_{\mathbf{p}.\mathbf{v}_{m_i-1}=m_i-1} \frac{1}{p_1!} \\
&\times \prod_{t=2}^{m_i-1}\frac{1}{p_t!}\left(\frac{1}{t} + \frac{1}{t}\sum_{j=1}^{L_r}\frac{m_j\beta_{j1}^t}{(\beta_{j1}-\beta_{i2})^t} + \frac{1}{t}\sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}^t}{(\beta_{j2}-\beta_{i2})^t}\right)^{p_t} \\
&\times \sum_{\rho=0}^{p_1}\binom{p_1}{\rho}\left(1 + \sum_{j=1}^{L_r}\frac{m_j\beta_{j1}}{(\beta_{j1}-\beta_{i2})} + \sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}}{(\beta_{j2}-\beta_{i2})}\right)^{p_1-\rho}\left(-\frac{1}{\beta_{i2}}\right)^{\rho} \\
&\times \sum_{l=1}^{L_s} \frac{(-\alpha_{l1})^{2M_s-m_l}}{\prod_{k=1}^{L}(\alpha_{k2}-\alpha_{l1})^{m_k}\prod_{\substack{k=1\\k\neq l}}^{L_s}(\alpha_{k1}-\alpha_{l1})^{m_k}} \sum_{\mathbf{q}.\mathbf{v}_{m_l-1}=m_l-1} \frac{1}{q_1!} \\
&\times \prod_{n=2}^{m_l-1}\frac{1}{q_n!}\left(\frac{1}{n} + \frac{1}{n}\sum_{k=1}^{L_s}\frac{m_k\alpha_{k2}^n}{(\alpha_{k2}-\alpha_{l1})^n} + \frac{1}{n}\sum_{\substack{k=1\\k\neq l}}^{L_s}\frac{m_k\alpha_{k1}^n}{(\alpha_{k1}-\alpha_{l1})^n}\right)^{q_n} \\
&\times \sum_{\kappa=0}^{q_1}\left(\frac{1}{\alpha_{l1}}\right)^{\kappa}\left(1 + \sum_{k=1}^{L_s}\frac{m_k\alpha_{k2}}{(\alpha_{k2}-\alpha_{l1})} + \sum_{\substack{k=1\\k\neq l}}^{L_s}\frac{m_k\alpha_{k1}}{(\alpha_{k1}-\alpha_{l1})^n}\right)^{q_1-\kappa} \\
&\times \left(\frac{\alpha_{l1}\beta_{i2}}{\beta_{i2}-\alpha_{l1}}\right)^{\rho+\kappa}\left[\left(\frac{\alpha_{l1}}{\beta_{i2}-\alpha_{l1}}\right)\Gamma\left(\rho+\kappa+1, \left(\frac{1}{\alpha_{l1}} - \frac{1}{\beta_{i2}}\right)\ln\frac{1}{\nu}\right)\right. \\
&\left. + \rho\Gamma\left(\rho+\kappa, \left(\frac{1}{\alpha_{l1}} - \frac{1}{\beta_{i2}}\right)\ln\frac{1}{\nu}\right)\right], \quad (24)
\end{aligned}$$

the subscripts s and r used to represent the source and relay parameters.

B. Central Limit Theorem approximation for multiple relays (large N)

For large N, using the central limit theorem (CLT) [24], (8) can be expressed as

$$P_e = \sum_{\mathbf{x}} \prod_{r=1}^{N} \epsilon_r^{\frac{1-x_r}{2}} (1-\epsilon_r)^{\frac{1+x_r}{2}} \Pr\left(X + Y < 0 \mid x_s = 1, \mathbf{x}\right), \quad (25)$$

where $Y = \mathcal{N}\left(\mu, \sigma^2\right)$ for $\mu = \sum_{r=1}^{N} E\left[f(Y_r)\right], \sigma^2 = \sum_{r=1}^{N} \mathrm{var}\left[f(Y_r)\right]$ as Y_r in (8) are independent, with var[·] denoting the variance. The first and second moments of $f(Y_r)$ may then be obtained from Corollary 3.2 to compute μ and σ^2. Thus, the statistics of Y are now completely known. Note that the PL approximation is implied through Corollary 3.2. Using the approach outlined in Section IV,

$$\begin{aligned}
\Pr(X + Y < 0 \mid x_s = 1, \mathbf{x}) &= \int_{-\infty}^{\infty} F_X(-x) p_Y(x) dx \\
&= \int_{-\infty}^{0} \left[1 - \mathcal{F}\left(\alpha_2, \alpha_1, -x\right)\right] p_Y(x) dx \\
&\quad + \int_{0}^{\infty} \mathcal{F}\left(\alpha_1, \alpha_2, -x\right) p_Y(x) dx \\
&= F_Y(0) - \int_{0}^{\infty} \mathcal{F}\left(\alpha_2, \alpha_1, x\right) p_Y(-x) dx \\
&\quad + \int_{0}^{\infty} \mathcal{F}\left(\alpha_1, \alpha_2, -x\right) p_Y(x) dx \qquad (26)
\end{aligned}$$

after substituting for $F_X(x)$ from (13). Evaluating the above integrals and substituting, we obtain (see Appendix D)

$$\begin{aligned}
\Pr\left(X + Y < 0 \mid x_s = 1, \mathbf{x}\right) &= Q\left(\frac{\mu}{\sigma}\right) \\
&\quad - \mathcal{P}\left(\alpha_2, \alpha_1, \gamma_1, -1\right) + \mathcal{P}\left(\alpha_1, \alpha_2, -\gamma_2, 1\right), \quad (27)
\end{aligned}$$

where $\gamma_i = \left(\frac{1}{\alpha_{li}} + \frac{\mu}{\sigma^2}\right), i = 1, 2$ and

$$\begin{aligned}
&\mathcal{P}\left(\alpha_2, \alpha_1, \gamma, t\right) = \sigma e^{-\frac{\mu^2}{2\sigma^2}} \\
&\times \sum_{l=1}^{L} \frac{(-\alpha_{l1})^{2M-m_l}}{\prod_{k=1}^{L} \left(\alpha_{k2} - \alpha_{l1}\right)^{m_k} \prod_{\substack{k=1 \\ k \neq l}}^{L} \left(\alpha_{k1} - \alpha_{l1}\right)^{m_k}} \sum_{\mathbf{q}.\mathbf{v}_{m_l-1} = m_l - 1} \frac{1}{q_1!} \\
&\times \prod_{n=2}^{m_l-1} \frac{1}{q_n!} \left(\frac{1}{n} + \frac{1}{n} \sum_{k=1}^{L} \frac{m_k \alpha_{k2}^n}{\left(\alpha_{k2} - \alpha_{l1}\right)^n} + \frac{1}{n} \sum_{\substack{k=1 \\ k \neq l}}^{L} \frac{m_k \alpha_{k1}^n}{\left(\alpha_{k1} - \alpha_{l1}\right)^n} \right)^{q_n} \\
&\times \sum_{\kappa=0}^{q_1} \binom{q_1}{\kappa} \left(\frac{t}{\alpha_{l1}}\right)^{\kappa} \left(1 + \sum_{k=1}^{L} \frac{m_k \alpha_{k2}}{\left(\alpha_{k2} - \alpha_{l1}\right)} + \sum_{\substack{k=1 \\ k \neq l}}^{L} \frac{m_k \alpha_{k1}}{\left(\alpha_{k1} - \alpha_{l1}\right)} \right)^{q_1 - \kappa} \\
&\times \left(\gamma\sigma^2\right)^{\kappa-1} \left[(\gamma\sigma)\, Q(\gamma\sigma) e^{\gamma^2\sigma^2/2} \sum_{k=0}^{\lfloor \frac{\kappa}{2} \rfloor} \frac{\kappa!}{k!(\kappa - 2k)!(2\gamma^2\sigma^2)^k} + \frac{1}{\sqrt{2\pi}} \right. \\
&\left. \times \sum_{j=1}^{\kappa} \sum_{k=0}^{\lfloor \frac{j-1}{2} \rfloor} \sum_{i=0}^{\lfloor \frac{\kappa-j}{2} \rfloor} \binom{\kappa}{j} \frac{(-1)^{k+j}(j-1)!(\kappa-j)!}{k!(j-1-2k)!i!(\kappa-j-2i)!(2\gamma^2\sigma^2)^{k+i}} \right],
\end{aligned} \quad (28)$$

with $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy$. Substituting (27) in (25) results in a closed form expression for the BER for multiple relays.

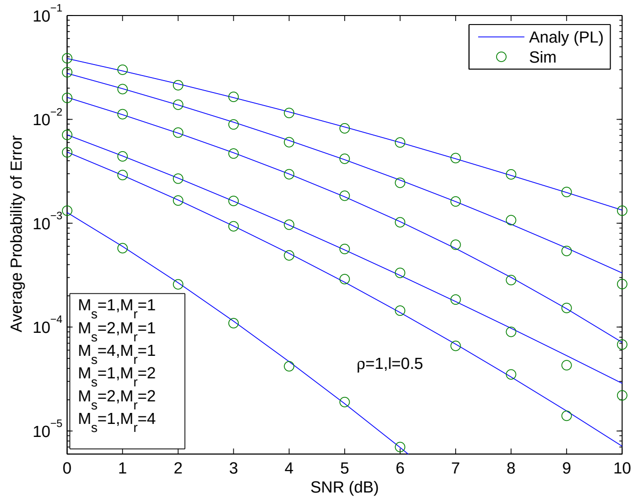

Fig. 2. Theoretical and simulation results for single-relay cooperation. Curves plotted according to the source and relay antenna specifications listed in the box in the same order.

V. R D

A. Simulation Parameters

The channel fading follows the following path loss model [4]. $\Omega_{i,j} \propto \frac{1}{L_{i,j}^4}, i, j \in \{s, r, d\}$ with $L_{i,j}$ being the distance between the nodes i and j. Also, $\Omega_{r,s} = \frac{\Omega_{d,s}}{l^4}$ and $\Omega_{d,r} = \frac{\Omega_{d,s}}{(1-l)^4}$, where $l = \frac{L_{r,s}}{L_{d,s}}$. The average system signal to noise ratio (SNR) $= \frac{\Omega_{d,s}(E_s+E_r)}{N_0}$ and the relay to source power $\rho = \frac{E_r}{E_s}$. Also, when a node (source or relay) supports multiple antennas, the power allocated to each antenna is $\frac{E_i}{M_i}, i \in \{s, r\}$. This particular model is used for the following reasons. Firstly, it allows some insight into the effect of the relay location on the BER. Secondly, by keeping the total power distributed across all the antennas at the source as well as relays constant, a fair comparison of the performance of MIMO relay systems for different configurations is possible.

To compute the BER using (23) and (27), the parameters $(a, b, M, \mathbf{\Omega})$ for the source and relay decision variables X and Y_r are required. For the above simulation setup, the respective parameters are $\left(\frac{4x_s}{N_0}, \frac{8E_s}{M_s N_0}, M_s, \frac{1}{l^4}\mathbf{I}_{M_s}\right)$ and $\left(\frac{4x_r}{N_0}, \frac{8E_r}{M_r N_0}, M_r, \frac{1}{(1-l)^4}\mathbf{I}_{M_r}\right)$, where $\mathbf{I}_M$ is the identity matrix of size M.

B. Single Relay Performance

A comparison of the theoretical and simulation results are provided in Figure 2 for various values of M_s and M_r, with $N = 1$. The variation of the BER P_e with respect to the system SNR is shown for scenarios where the source and relay support upto 4 antennas. For simulations, the Alamouti code was employed at nodes that supported 2 antennas. For nodes supporting 4 antennas, the 4×4 real orthogonal design [17] was used. For all cases, $l = 0.5$, which means that the relay is located halfway between the source and the destination. Also, the source and relay powers are considered to be equal. The theoretical results were obtained from (23) and (18). From

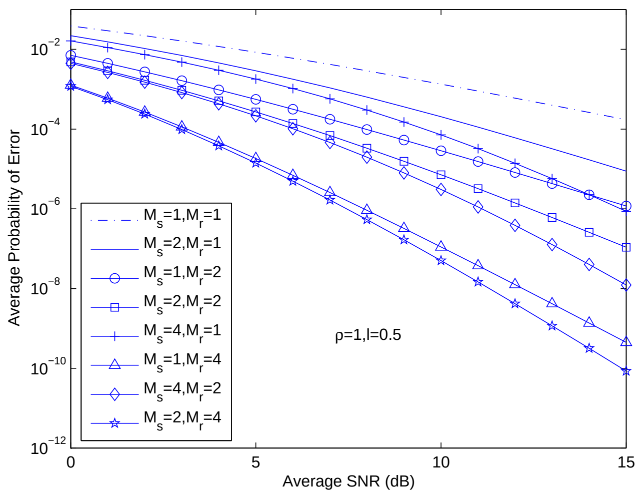

Fig. 3. BER plots for various values of M_s and M_r for the single relay system. Performance improves with increasing M_r.

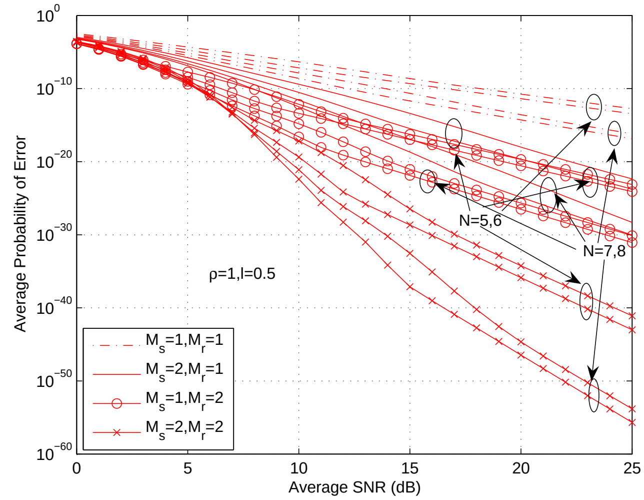

Fig. 5. BER plots for different values of M_s and M_r for the multirelay system. Loss in diversity due to DF.

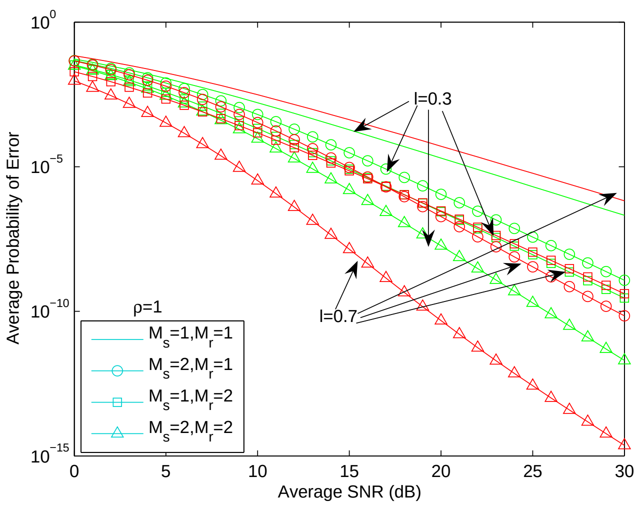

Fig. 4. Effect of relay location on the BER. S-R link dominates when $M_r = 1$, R-D link dominates when $M_r = 2$.

Figure 2, we can observe that there is an excellent match between the theoretical and simulation results.

A more detailed plot of the BER with respect to the average system SNR for the single relay system is available in Figure 3 for various values of M_s and M_r. From the figure, it is evident that using multiple antennas at the source and/or the relay leads to a significant improvement in system performance.

The effect of relay location on the BER performance is shown in Figure 4. In the high SNR region, the BER performance of systems for which $M_s = 1$ is much better when $l = 0.3$, i.e. when the relay is closer to the source. However, systems with $M_s = 2$ perform better when the relay is located closer to the destination at $l = 0.7$. This behaviour is explained as follows.

The performance of cooperative systems is dependent on the strength of the S-R, R-D and S-D links. The S-R link is stronger when the relay is located closer to the source than the destination. The behaviour of the R-D link is just the opposite. For $M_s = 1$, the BER performance of the system is ultimately influenced by the strength of the S-R link, which explains the improved performance for $l = 0.3$. On the other hand, for $M_s = 2$, due to transmit diversity, the effect of relay location on the S-R link is relatively very less. Therefore, system performance is heavily influenced by the R-D link resulting in a better BER performance for $l = 0.7$.

C. *Multirelay performance*

The high SNR performance of cooperative diversity systems for multiple antennas as well as multiple relays is shown in Figure 5 for $M_s, M_r = 1, 2$ and $N = 5, 6, 7, 8$. The BER curves plotted in the figures are obtained using the CLT approximation from (25) and (27). For simplicity, we have considered $\rho = 1$ and $l = 0.5$. The loss in diversity order due to DF by a factor of 2 is obvious from Figure 5. For $M_s = M_r = 1$, this has been rigorously proved in [4]. The plots for $M_s, M_r > 1$, indicate that this property may hold even for MIMO relay systems. To the best of our knowledge, such numerical results based on the error rate are not available elsewhere. Using the notation (M_s, M_r, N) to denote the source and relay antenna parameters of a MIMO relay system, from Figure 6, we find that a (1,1,8) system has the same BER performance as (2,1,4), (1,2,4) and (2,2,2) systems. However, the (1,1,8) system in Figure 6 has a total (adding source and relay antennas) number of 9 antennas, (2,1,4) with 6, (1,2,4) with 9 antennas and a (2,2,2) system with 6 antennas. Thus, systems with multiple antennas at both the source as well as relays are more resource optimal and may be preferred in the design of fixed relay systems.

VI. C

In this paper, we have obtained closed form expressions for the BER for ML-DF MIMO relay systems, by extending the ML decision rule for primitive cooperative systems. Exact

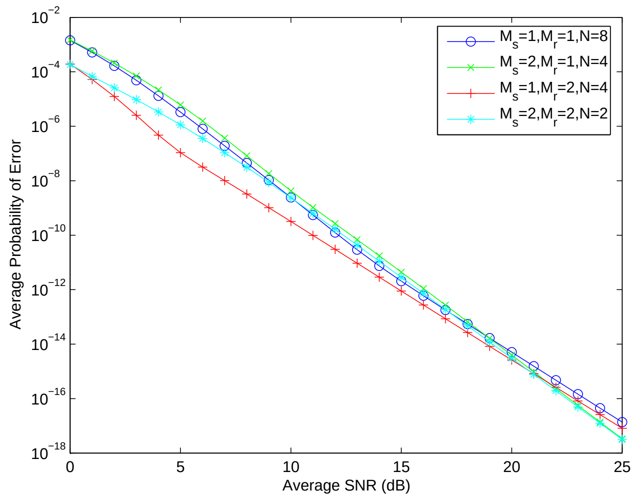

Fig. 6. MIMO compensates for reduced cooperation between relays.

expressions were obtained for single relay systems employing the PL combiner, a practical alternative to the ML detector for DF cooperative systems. Using the central limit theorem, a closed form approximation was obtained for the BER when multiple relays were employed. From the BER plots, multirelay systems were found to suffer from a loss in diversity order due to DF. On the basis of our numerical results, we conclude that the performance of cooperative diversity systems can be significantly enhanced by using multiple antennas at the relays as well as the source.

A A

The derivation of the decision rule is explained through the following cases [3], [6].
$M_s = M_r = 1$: Dropping the superscripts for the transmit antennas in (2), the ML decision rule at the destination is given by

$$\hat{x}_s = \arg\max_{x_s} \ln p_{y_{d,s}|x_s}(y_{d,s} \mid x_s) + \sum_{r=1}^{N} \ln p_{y_{d,r}|x_s}(y_{d,r} \mid x_s), \quad (29)$$

where x_s and x_r are the symbols transmitted by the source and relay respectively and $p_{(\cdot|\cdot)}(\cdot \mid \cdot)$ represents the conditional PDF. Since $\Pr(x_r \neq x_s \mid x_s) = \epsilon_r$ is the probability of error on the S-R link,

$$\begin{aligned}\ln p_{y_{d,r}|x_s}(y_{d,r} \mid x_s) = \ln \Big\{ & p_{y_{d,r}|x_r}(y_{d,r} \mid x_r = x_s)(1-\epsilon_r) \\ & + p_{y_{d,r}|x_r}(y_{d,r} \mid x_r \neq x_s)\epsilon_r \Big\}. \end{aligned} \quad (30)$$

Since $y_{ds} \mid h_{d,s} \sim \mathcal{CN}\left(h_{d,s}x_s, N_0\right), y_{dr} \mid h_{d,r} \sim \mathcal{CN}\left(h_{d,r}x_r, N_0\right)$, for BPSK modulation, from (2), (29) and (30), the decision rule is then obtained as [3], [4]

$$X + \sum_{r=1}^{N} \ln \frac{\epsilon_r + (1-\epsilon_r)\exp(Y_r)}{(1-\epsilon_r) + \epsilon_r \exp(Y_r)} \underset{-1}{\overset{1}{\gtrless}} 0 \quad (31)$$

where

$$\begin{aligned} X &= \frac{4\mathrm{Re}\{h^*_{d,s} y_{d,s}\}}{N_0}, \\ Y_r &= \frac{4\mathrm{Re}\{h^*_{d,r} y_{d,r}\}}{N_0} \end{aligned} \quad (32)$$

with $\{*\}$ representing the complex conjugate operation.
$M_s = M_r = 2$: For the transmission scheme in Example (2.1), the received symbols for the R-D link on consecutive time slots are obtained from (2) as

$$\begin{aligned} y^{(1)}_{d,r} &= h^{(1)}_{d,r}x_r + h^{(2)}_{d,r}x'_r + n^{(1)}_{d,r}, \\ y^{(2)}_{d,r} &= h^{(2)}_{d,r}x^*_r - h^{(1)}_{d,r}x'^*_r + n^{(2)}_{d,r} \quad r = 1, \ldots, N. \end{aligned} \quad (33)$$

After combining appropriately at the destination using the Alamouti scheme [18] for the R-D link, we obtain

$$\begin{aligned} \tilde{y}^{(1)}_{d,r} &= h^{(1)*}_{d,r} y^{(1)}_{d,r} + h^{(2)}_{d,r} y^{(2)*}_{d,r} \\ &= \left(|h^{(1)}_{d,r}|^2 + |h^{(2)}_{d,r}|^2\right) x_r + h^{(1)*}_{d,r} n^{(1)}_{d,r} + h^{(2)}_{d,r} n^{(2)*}_{d,r}, \end{aligned} \quad (34)$$

$$\begin{aligned} \tilde{y}^{(2)}_{d,r} &= h^{(2)*}_{d,r} y^{(1)}_{d,r} - h^{(1)}_{d,r} y^{(2)*}_{d,r} \\ &= \left(|h^{(1)}_{d,r}|^2 + |h^{(2)}_{d,r}|^2\right) x'_r - h^{(1)}_{d,r} n^{(2)*}_{d,r} + h^{(2)*}_{d,r} n^{(1)}_{d,r}. \end{aligned} \quad (35)$$

From the above, it is obvious that the decisions for x_r and x'_r on the R-D link can be made independently. This is possible due to the orthogonality of the Alamouti code [17], [20]. Similarly, for the S-D link, we have

$$\begin{aligned} \tilde{y}^{(1)}_{d,s} &= h^{(1)*}_{d,s} y^{(1)}_{d,s} + h^{(2)}_{d,s} y^{(2)*}_{d,s} \\ &= \left(|h^{(1)}_{d,s}|^2 + |h^{(2)}_{d,s}|^2\right) x_s + h^{(1)*}_{d,s} n^{(1)}_{d,s} + h^{(2)}_{d,s} n^{(2)*}_{d,s}. \end{aligned} \quad (36)$$

Noting that

$$\begin{aligned} \tilde{y}^{(1)}_{ds} \mid h^{(1)}_{d,s}, h^{(2)}_{d,s} &\sim \mathcal{CN}\left(\left(|h^{(1)}_{d,s}|^2 + |h^{(2)}_{d,s}|^2\right) x_s, \left(|h^{(1)}_{d,s}|^2 + |h^{(2)}_{d,s}|^2\right) N_0\right) \\ \tilde{y}^{(1)}_{dr} \mid h^{(1)}_{d,r}, h^{(2)}_{d,r} &\sim \mathcal{CN}\left(\left(|h^{(1)}_{d,r}|^2 + |h^{(2)}_{d,r}|^2\right) x_r, \left(|h^{(1)}_{d,r}|^2 + |h^{(2)}_{d,r}|^2\right) N_0\right), \end{aligned} \quad (37)$$

it is possible to directly use the decision rule in (31) for $M_s = M_r = 2$ with

$$\begin{aligned} X &= \frac{4\mathrm{Re}\{\tilde{y}^{(1)}_{d,s}\}}{N_0} = \frac{4\mathrm{Re}\{h^{(1)*}_{d,s} y^{(1)}_{d,s} + h^{(2)*}_{d,s} y^{(2)}_{d,s}\}}{N_0}, \\ Y_r &= \frac{4\mathrm{Re}\{\tilde{y}^{(1)}_{d,r}\}}{N_0} = \frac{4\mathrm{Re}\{h^{(1)*}_{d,r} y^{(1)}_{d,r} + h^{(2)*}_{d,r} y^{(2)}_{d,r}\}}{N_0} \end{aligned} \quad (38)$$

and ϵ_r equivalent to the BER for MRC with diversity order 4 [18]. The above ML decision scheme is valid for $M_s, M_r > 2$ provided linear processing at the relay and/or destination results in conditionally complex Gaussian decision variables like (34) and (36). For $M_r = 4$, the appropriate orthogonal design is available in [17], [20], [21].

A B

The CF of $||\mathbf{h}||^2$ can be expressed as [25]

$$\Phi_{||\mathbf{h}||^2}(\jmath\omega) = \frac{1}{\prod_{l=1}^{L}(1 - \jmath\omega\Omega_l)^{m_l}}. \quad (39)$$

The CF of $X \mid ||\mathbf{h}||^2$ is given by

$$\Phi_{X|||\mathbf{h}||^2}(\jmath\omega) = \exp\left\{\left(\jmath a\omega - \frac{b}{2}\omega^2\right)||\mathbf{h}||^2\right\}. \quad (40)$$

Averaging the above over $||\mathbf{h}||^2$, we have [26]

$$\begin{aligned}\Phi_X(\jmath\omega) &= \int_{-\infty}^{\infty} \exp\left\{\jmath\left(a\omega + \jmath\frac{b}{2}\omega^2\right)y\right\} p_{||\mathbf{h}||^2}(y)dy \\ &= \Phi_{||\mathbf{h}||^2}\left\{\jmath\left(a\omega + \jmath\frac{b}{2}\omega^2\right)\right\}, \end{aligned} \tag{41}$$

which, from (39), can be expressed as (12). Since the CF of X can be further simplified as

$$\Phi_X(\jmath\omega) = \frac{1}{\prod_{l=1}^{L}(1-\jmath\omega\alpha_{l1})^{m_l}(1-\jmath\omega\alpha_{l2})^{m_l}}, \tag{42}$$

using the Gil-Pelaez inversion formula [27] and the residue theorem from complex analysis [28], [29], the CDF of X can be expressed as

$$F_X(x) = \begin{cases} \operatorname*{Res}_{z=\frac{1}{\alpha_{l2}}} -\frac{\Phi_X(z)}{z}e^{-xz} & x<0 \\ 1 + \operatorname*{Res}_{z=\frac{1}{\alpha_{l1}}} \sum \frac{\Phi_X(z)}{z}e^{-xz} & x \geq 0. \end{cases} \tag{43}$$

The residues in (43) can be directly computed using the Fàa Di Bruno formula [30], [31] to obtain (13). Using the Steiltjes integral,

$$\begin{aligned} E\left[\{f(X)\}^N\right] &= \int_{-\infty}^{\infty}\{f(x)\}^N dF_X(x) \\ &= \int_{-\infty}^{\ln\nu}(\ln\nu)^N dF_X(x) + \int_{\ln\nu}^{\ln\frac{1}{\nu}} x^n dF_X(x) \\ &\quad + \int_{\ln\frac{1}{\nu}}^{\infty}\left(\ln\frac{1}{\nu}\right)^N dF_X(x) \\ &= (\ln\nu)^N F_X(\ln\nu) + \left(\ln\frac{1}{\nu}\right)^N\left[1 - F_X\left(\ln\frac{1}{\nu}\right)\right] \\ &\quad + \int_{\ln\nu}^{\ln\frac{1}{\nu}} x^N dF_X(x). \end{aligned} \tag{44}$$

Substituting the expression for $F_X(x)$ from (13) in (44), integrating by parts and simplifying, we obtain

$$\begin{aligned} E\left[\{f(X)\}^N\right] &= N\int_0^{\ln\frac{1}{\nu}} x^{N-1}\mathcal{F}(\alpha_2,\alpha_1,x)\,dx \\ &\quad - N\int_{\ln\nu}^{0} x^{N-1}\mathcal{F}(\alpha_1,\alpha_2,x)\,dx. \end{aligned} \tag{45}$$

Substituting the expression for $\mathcal{F}(\alpha_2,\alpha_1,x)$ from (14) in the first integral in (45) results in

$$\begin{aligned} &\int_0^{\ln\frac{1}{\nu}} x^{N-1}\mathcal{F}(\alpha_2,\alpha_1,x)\,dx = \\ &\sum_{l=1}^{L}\frac{(-\alpha_{l1})^{2M-m_l}}{\prod_{k=1}^{L}(\alpha_{k2}-\alpha_{l1})^{m_k}\prod_{\substack{k=1\\k\neq l}}^{L}(\alpha_{k1}-\alpha_{l1})^{m_k}}\sum_{\mathbf{q}.\mathbf{v}_{m_l-1}=m_l-1}\frac{1}{q_1!} \\ &\times\prod_{n=2}^{m_l-1}\frac{1}{q_n!}\left(\frac{1}{n}+\frac{1}{n}\sum_{k=1}^{L}\frac{m_k\alpha_{k2}^n}{(\alpha_{k2}-\alpha_{l1})^n}+\frac{1}{n}\sum_{\substack{k=1\\k\neq l}}^{L}\frac{m_k\alpha_{k1}^n}{(\alpha_{k1}-\alpha_{l1})^n}\right)^{q_n} \\ &\times\sum_{\kappa=0}^{q_1}\binom{q_1}{\kappa}\left(\frac{1}{\alpha_{l1}}\right)^{\kappa}\left(1+\sum_{k=1}^{L}\frac{m_k\alpha_{k2}}{(\alpha_{k2}-\alpha_{l1})}+\sum_{\substack{k=1\\k\neq l}}^{L}\frac{m_k\alpha_{k1}}{(\alpha_{k1}-\alpha_{l1})}\right)^{q_1-\kappa} \\ &\times\int_0^{\ln\frac{1}{\nu}} x^{\kappa+N-1}e^{-\frac{x}{\alpha_{l1}}}dx, \end{aligned} \tag{46}$$

which, upon evaluating the integral yields

$$N\int_0^{\ln\frac{1}{\nu}} x^{N-1}\mathcal{F}(\alpha_2,\alpha_1,x)\,dx = \mathcal{E}\left(\alpha_2,\alpha_1,\ln\frac{1}{\nu}\right) \tag{47}$$

for $\mathcal{E}(\cdot,\cdot,\cdot)$ defined in (17). Similarly, it can be shown that

$$N\int_{\ln\nu}^{0} x^{N-1}\mathcal{F}(\alpha_1,\alpha_2,x)\,dx = -\mathcal{E}(\alpha_1,\alpha_2,\ln\nu). \tag{48}$$

From (45), (47) and (48), we obtain (16).

A C

From (14), the second integral in (22) can be expressed as

$$\begin{aligned} &\int_0^{\ln\frac{1}{\nu}}\mathcal{F}(\beta_1,\beta_2,-x)\,d\mathcal{F}(\alpha_2,\alpha_1,x) = \\ &\sum_{i=1}^{L_r}\frac{(-\beta_{i2})^{2M_r-m_i}}{\prod_{j=1}^{L_r}\left(\beta_{j1}-\beta_{i2}\right)^{m_j}\prod_{\substack{j=1\\j\neq i}}^{L_r}\left(\beta_{j2}-\beta_{i2}\right)^{m_j}}\sum_{\mathbf{p}.\mathbf{v}_{m_i-1}=m_i-1}\frac{1}{p_1!} \\ &\times\prod_{t=2}^{m_i-1}\frac{1}{p_t!}\left(\frac{1}{t}+\frac{1}{t}\sum_{j=1}^{L_r}\frac{m_j\beta_{j1}^t}{(\beta_{j1}-\beta_{i2})^t}+\frac{1}{t}\sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}^t}{(\beta_{j2}-\beta_{i2})^t}\right)^{p_t} \\ &\times\sum_{\rho=0}^{p_1}\binom{p_1}{\rho}\left(1+\sum_{j=1}^{L_r}\frac{m_j\beta_{j1}}{(\beta_{j1}-\beta_{i2})}+\sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}}{(\beta_{j2}-\beta_{i2})}\right)^{p_1-\rho}\left(-\frac{1}{\beta_{i2}}\right)^{\rho} \\ &\times\int_0^{\ln\frac{1}{\nu}} x^{\rho}e^{\frac{x}{\beta_{i2}}}\,d\mathcal{F}(\alpha_2,\alpha_1,x). \end{aligned} \tag{49}$$

Integrating by parts,

$$\begin{aligned} \int_0^{\ln\frac{1}{\nu}} x^{\rho}e^{\frac{x}{\beta_{i2}}}\,d\mathcal{F}(\alpha_2,\alpha_1,x) &= x^{\rho}e^{\frac{x}{\beta_{i2}}}\mathcal{F}(\alpha_2,\alpha_1,x)\Big|_0^{\ln\frac{1}{\nu}} \\ &- \int_0^{\ln\frac{1}{\nu}}\left(\frac{1}{\beta_{i2}}x^{\rho}e^{\frac{x}{\beta_{i2}}}+\rho x^{\rho-1}e^{\frac{x}{\beta_{i2}}}\right)\mathcal{F}(\alpha_2,\alpha_1,x)\,dx. \end{aligned} \tag{50}$$

From (49) and (50), substituting the expression for $\mathcal{F}(\alpha_2,\alpha_1,x)$,

$$\begin{aligned} &\int_0^{\ln\frac{1}{\nu}}\mathcal{F}(\beta_1,\beta_2,-x)\,d\mathcal{F}(\alpha_2,\alpha_1,x) = \\ &\qquad\mathcal{F}(\beta_1,\beta_2,-x)\,\mathcal{F}(\alpha_2,\alpha_1,x)\Big|_0^{\ln\frac{1}{\nu}} \\ &-\sum_{i=1}^{L_r}\frac{(-\beta_{i2})^{2M_r-m_i}}{\prod_{j=1}^{L_r}\left(\beta_{j1}-\beta_{i2}\right)^{m_j}\prod_{\substack{j=1\\j\neq i}}^{L_r}\left(\beta_{j2}-\beta_{i2}\right)^{m_j}}\sum_{\mathbf{p}.\mathbf{v}_{m_i-1}=m_i-1}\frac{1}{p_1!} \\ &\times\prod_{t=2}^{m_i-1}\frac{1}{p_t!}\left(\frac{1}{t}+\frac{1}{t}\sum_{j=1}^{L_r}\frac{m_j\beta_{j1}^t}{(\beta_{j1}-\beta_{i2})^t}+\frac{1}{t}\sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}^t}{(\beta_{j2}-\beta_{i2})^t}\right)^{p_t} \\ &\times\sum_{\rho=0}^{p_1}\binom{p_1}{\rho}\left(1+\sum_{j=1}^{L_r}\frac{m_j\beta_{j1}}{(\beta_{j1}-\beta_{i2})}+\sum_{\substack{j=1\\j\neq i}}^{L_r}\frac{m_j\beta_{j2}}{(\beta_{j2}-\beta_{i2})}\right)^{p_1-\rho}\left(-\frac{1}{\beta_{i2}}\right)^{\rho} \\ &\times\sum_{l=1}^{L_s}\frac{(-\alpha_{l1})^{2M_s-m_l}}{\prod_{k=1}^{L_s}(\alpha_{k2}-\alpha_{l1})^{m_k}\prod_{\substack{k=1\\k\neq l}}^{L}(\alpha_{k1}-\alpha_{l1})^{m_k}}\sum_{\mathbf{q}.\mathbf{v}_{m_l-1}=m_l-1}\frac{1}{q_1!} \end{aligned}$$

$$\times \prod_{n=2}^{m_l-1} \frac{1}{q_n!} \left(\frac{1}{n} + \frac{1}{n} \sum_{k=1}^{L_s} \frac{m_k \alpha_{k2}^n}{(\alpha_{k2}-\alpha_{l1})^n} + \frac{1}{n} \sum_{\substack{k=1\\k\neq l}}^{L_s} \frac{m_k \alpha_{k1}^n}{(\alpha_{k1}-\alpha_{l1})^n} \right)^{q_n}$$

$$\times \sum_{\kappa=0}^{q_1} \left(\frac{1}{\alpha_{l1}}\right)^{\kappa} \left(1 + \sum_{k=1}^{L_s} \frac{m_k \alpha_{k2}}{(\alpha_{k2}-\alpha_{l1})} + \sum_{\substack{k=1\\k\neq l}}^{L_s} \frac{m_k \alpha_{k1}}{(\alpha_{k1}-\alpha_{l1})^n} \right)^{q_1-\kappa}$$

$$\times \int_0^{\ln\frac{1}{\nu}} \left[\frac{1}{\beta_{i2}} x^{\rho+\kappa} e^{-x\left(\frac{1}{\alpha_{l1}}-\frac{1}{\beta_{i2}}\right)} + \rho x^{\rho+\kappa-1} e^{-x\left(\frac{1}{\alpha_{l1}}-\frac{1}{\beta_{i2}}\right)} \right] dx,$$

which can be simplified to obtain

$$\int_0^{\ln\frac{1}{\nu}} \mathcal{F}(\beta_1,\beta_2,-x)\, d\mathcal{F}(\alpha_2,\alpha_1,x) = \mathcal{F}(\beta_1,\beta_2,-x)\,\mathcal{F}(\alpha_2,\alpha_1,x)\big|_0^{\ln\frac{1}{\nu}} - \mathcal{G}(\beta_1,\beta_2;\alpha_2,\alpha_1). \quad (51)$$

Integrating by parts, the first integral in (22) can be expressed using (51) as

$$\int_0^{\ln\frac{1}{\nu}} \mathcal{F}(\beta_2,\beta_1,x)\, d\mathcal{F}(\alpha_1,\alpha_2,-x) = \mathcal{G}(\alpha_1,\alpha_2;\beta_2,\beta_1). \quad (52)$$

From (22), (51) and (52), we obtain (23).

A D

Substituting the expression for $p_Y(-x)$,

$$\int_0^{\infty} \mathcal{F}(\alpha_2,\alpha_1,x)\, p_Y(-x) dx =$$

$$\sum_{l=1}^{L} \frac{(-\alpha_{l1})^{2M-m_l}}{\prod_{k=1}^{L}(\alpha_{k2}-\alpha_{l1})^{m_k} \prod_{\substack{k=1\\k\neq l}}^{L}(\alpha_{k1}-\alpha_{l1})^{m_k}} \sum_{\mathbf{q}.\mathbf{v}_{m_l-1}=m_l-1} \frac{1}{q_1!}$$

$$\times \prod_{n=2}^{m_l-1} \frac{1}{q_n!} \left(\frac{1}{n} + \frac{1}{n} \sum_{k=1}^{L} \frac{m_k \alpha_{k2}^n}{(\alpha_{k2}-\alpha_{l1})^n} + \frac{1}{n} \sum_{\substack{k=1\\k\neq l}}^{L} \frac{m_k \alpha_{k1}^n}{(\alpha_{k1}-\alpha_{l1})^n} \right)^{q_n}$$

$$\times \sum_{\kappa=0}^{q_1} \binom{q_1}{\kappa} \left(\frac{1}{\alpha_{l1}}\right)^{\kappa} \left(1 + \sum_{k=1}^{L} \frac{m_k \alpha_{k2}}{(\alpha_{k2}-\alpha_{l1})} + \sum_{\substack{k=1\\k\neq l}}^{L} \frac{m_k \alpha_{k1}}{(\alpha_{k1}-\alpha_{l1})} \right)^{q_1-\kappa}$$

$$\times \frac{1}{\sqrt{2\pi}\sigma} \int_0^{\infty} x^{\kappa} e^{-\frac{x}{\alpha_{l1}}} e^{-\frac{(x+\mu)^2}{2\sigma^2}} dx. \quad (53)$$

Since

$$\exp\left(-\frac{x}{\alpha_{l1}}\right) \exp\left(-\frac{(x+\mu)^2}{2\sigma^2}\right) = \exp\left(-\frac{\mu^2}{2\sigma^2}\right) \exp\left[-\left(\frac{1}{\alpha_{l1}} + \frac{\mu}{\sigma^2}\right) x - \frac{x^2}{2\sigma^2} \right], \quad (54)$$

letting $\gamma = \left(\frac{1}{\alpha_{l1}} + \frac{\mu}{\sigma^2}\right)$, from [32, (3.322.2)],

$$\frac{e^{\mu^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \int_0^{\infty} e^{-\frac{x}{\alpha_{l1}}} e^{-\frac{(x+\mu)^2}{2\sigma^2}} dx = e^{\gamma^2\sigma^2/2} Q(\gamma\sigma). \quad (55)$$

Using the Leibnitz rule, the κth order derivative of the above with respect to γ yields

$$\frac{1}{\sqrt{2\pi}\sigma} \int_0^{\infty} x^{\kappa} \exp\left(-\gamma x - \frac{x^2}{2\sigma^2}\right) dx = (-1)^{\kappa} \sum_{j=0}^{\kappa} \binom{\kappa}{j} \left[\frac{d^j}{d\gamma^j} Q(\gamma\sigma) \right] \left[\frac{d^{\kappa-j}}{d\gamma^{\kappa-j}} e^{\gamma^2\sigma^2/2} \right]. \quad (56)$$

Since [32, (0.432.2)]

$$\frac{d^j}{d\gamma^j} Q(\gamma\sigma) = \begin{cases} Q(\gamma\sigma) & j=0 \\ -\frac{\sigma}{\sqrt{2\pi}} \left(-\gamma\sigma^2\right)^{j-1} e^{-\gamma^2\sigma^2/2} & \\ \quad \times \sum_{k=0}^{\lfloor \frac{j-1}{2} \rfloor} \frac{(j-1)!}{k!(j-1-2k)!(-2\gamma^2\sigma^2)^k} & \text{otherwise} \end{cases}$$

and

$$\frac{d^{\kappa-j}}{d\gamma^{\kappa-j}} e^{\gamma^2\sigma^2/2} = \left(\gamma\sigma^2\right)^{\kappa-j} e^{\gamma^2\sigma^2/2} \sum_{k=0}^{\lfloor \frac{\kappa-j}{2} \rfloor} \frac{(\kappa-j)!}{k!(\kappa-j-2k)!(2\gamma^2\sigma^2)^k}, \quad (57)$$

from (53), (56) and (57), we obtain

$$\int_0^{\infty} \mathcal{F}(\alpha_2,\alpha_1,x)\, p_Y(-x) dx = \mathcal{P}(\alpha_2,\alpha_1,\gamma_1,-1). \quad (58)$$

Similarly, the second integral in (26) can be obtained, resulting in (28).

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