Let's push

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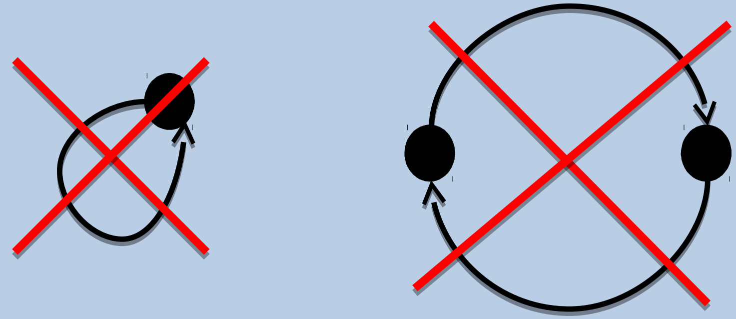
Oriented coloring

oriented graphs are directed graphs without



Oriented coloring

partition into independent sets such that all arcs between two independent sets are from one to another.

Oriented coloring

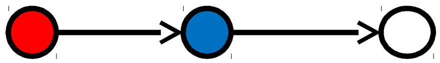
partition into independent sets such that all arcs between two independent sets are from one to another.

$\chi_o(G) = \min\{\#\text{independent sets}\}$

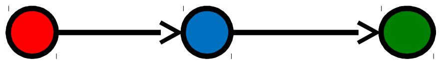
Examples

Examples

Examples

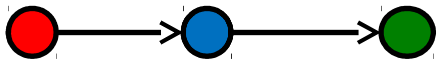


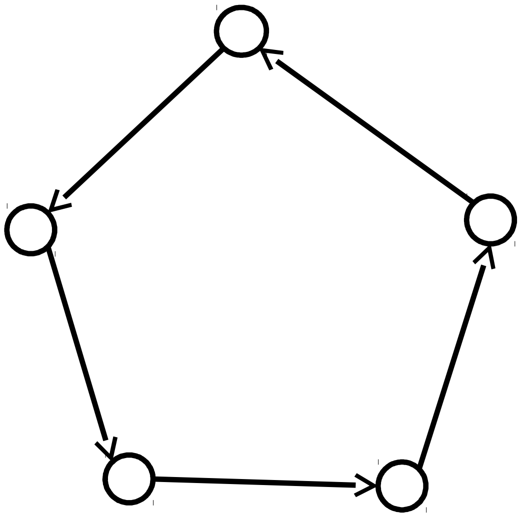
Examples



$\chi_o(G) = 3$

Examples





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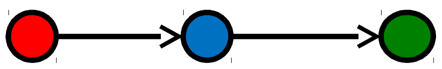

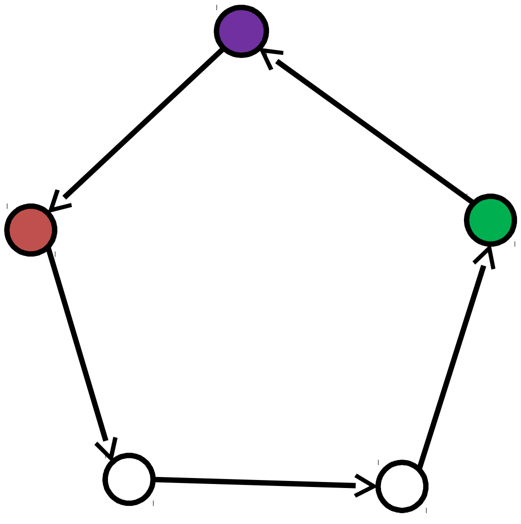

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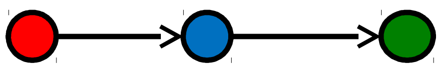

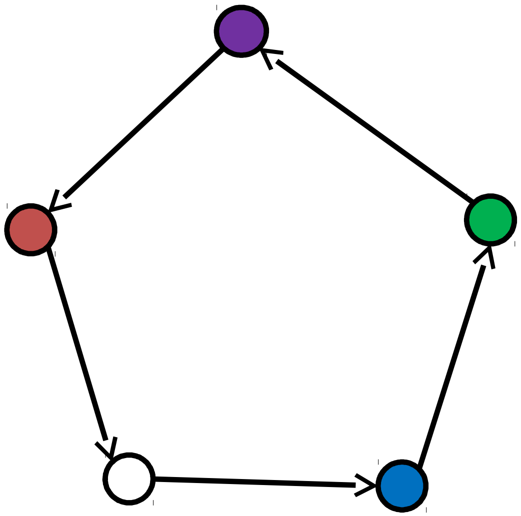

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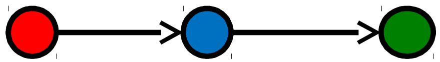

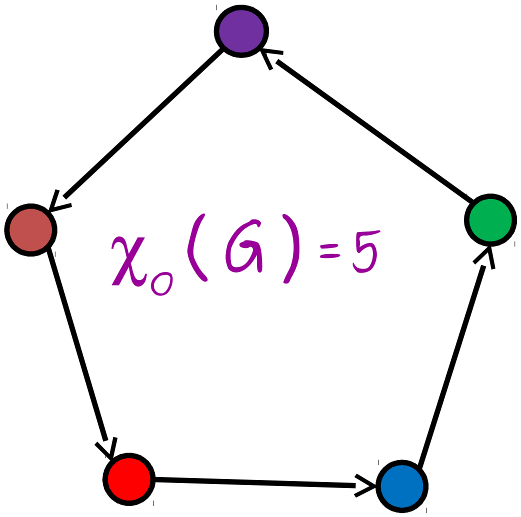

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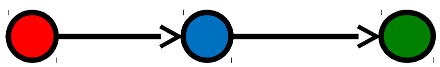

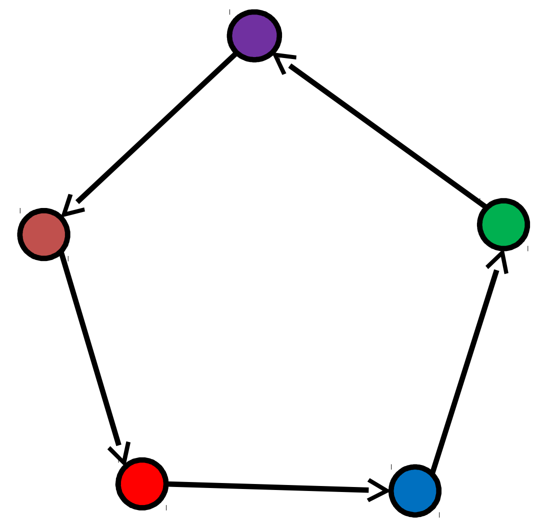

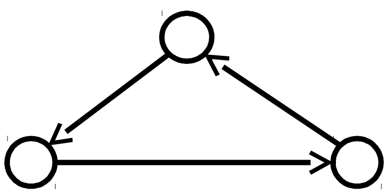

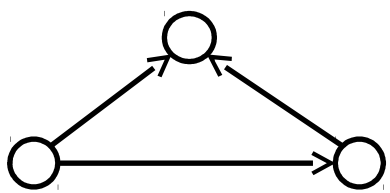

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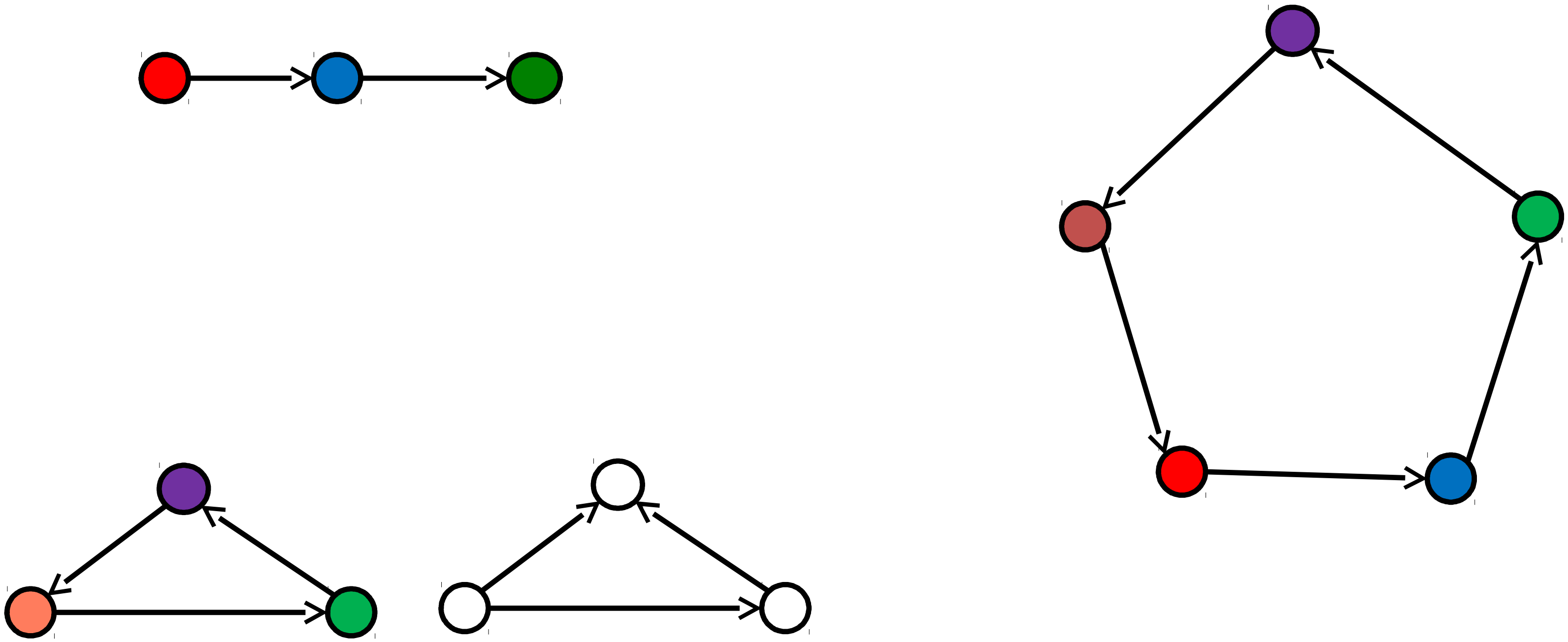

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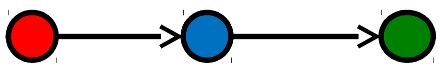

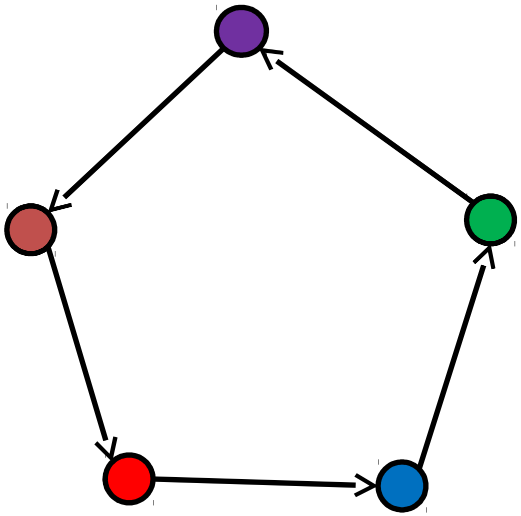

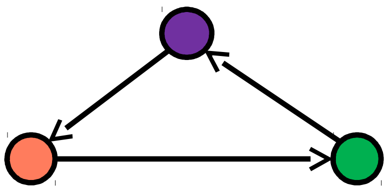

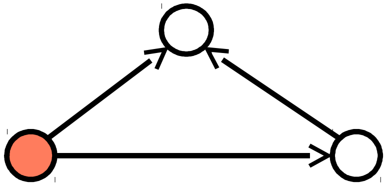

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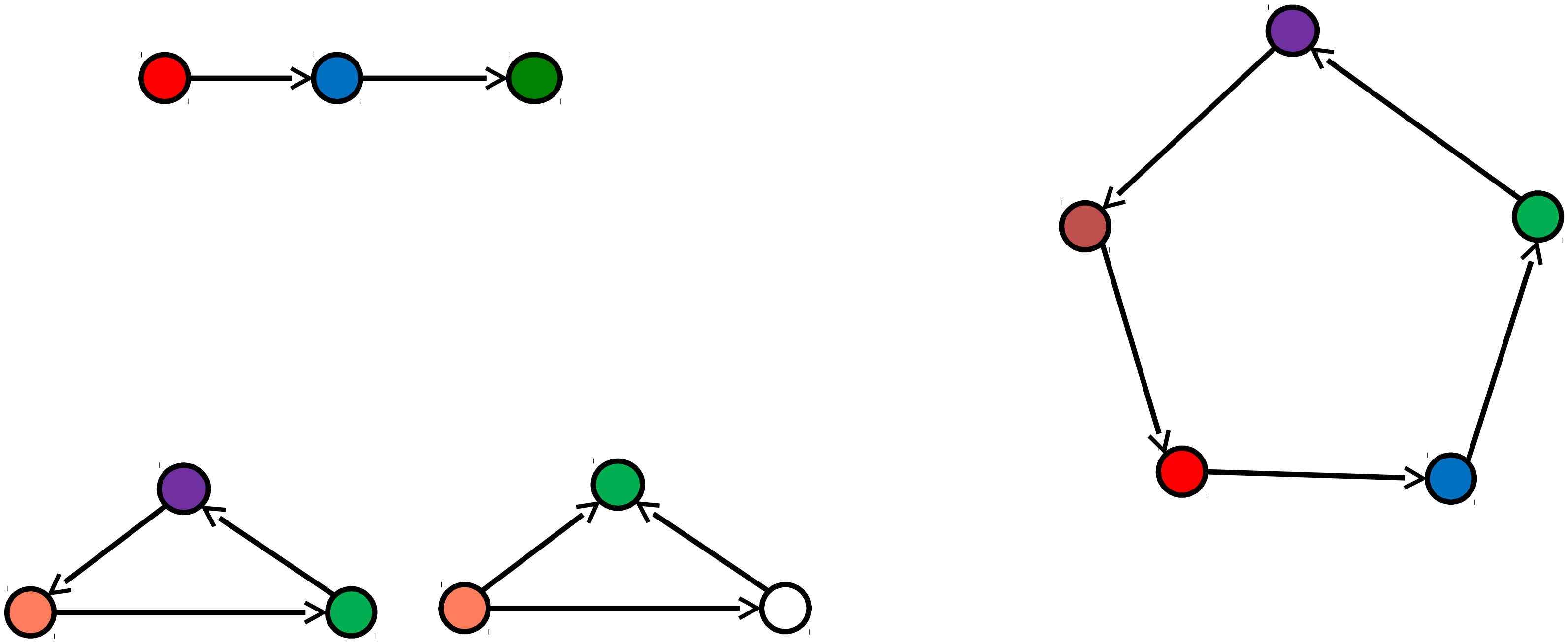

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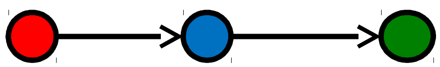
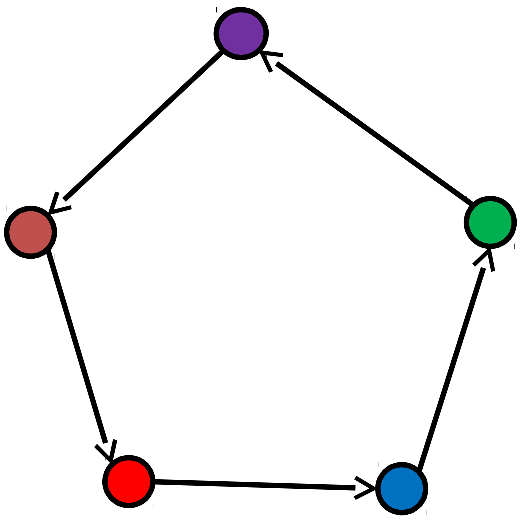
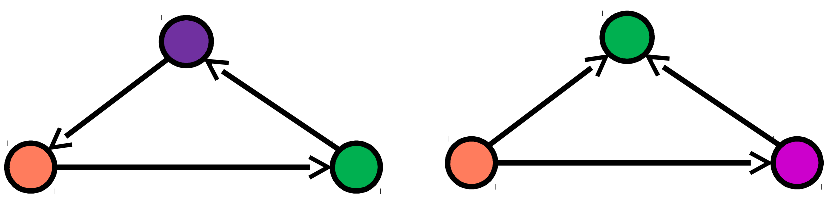
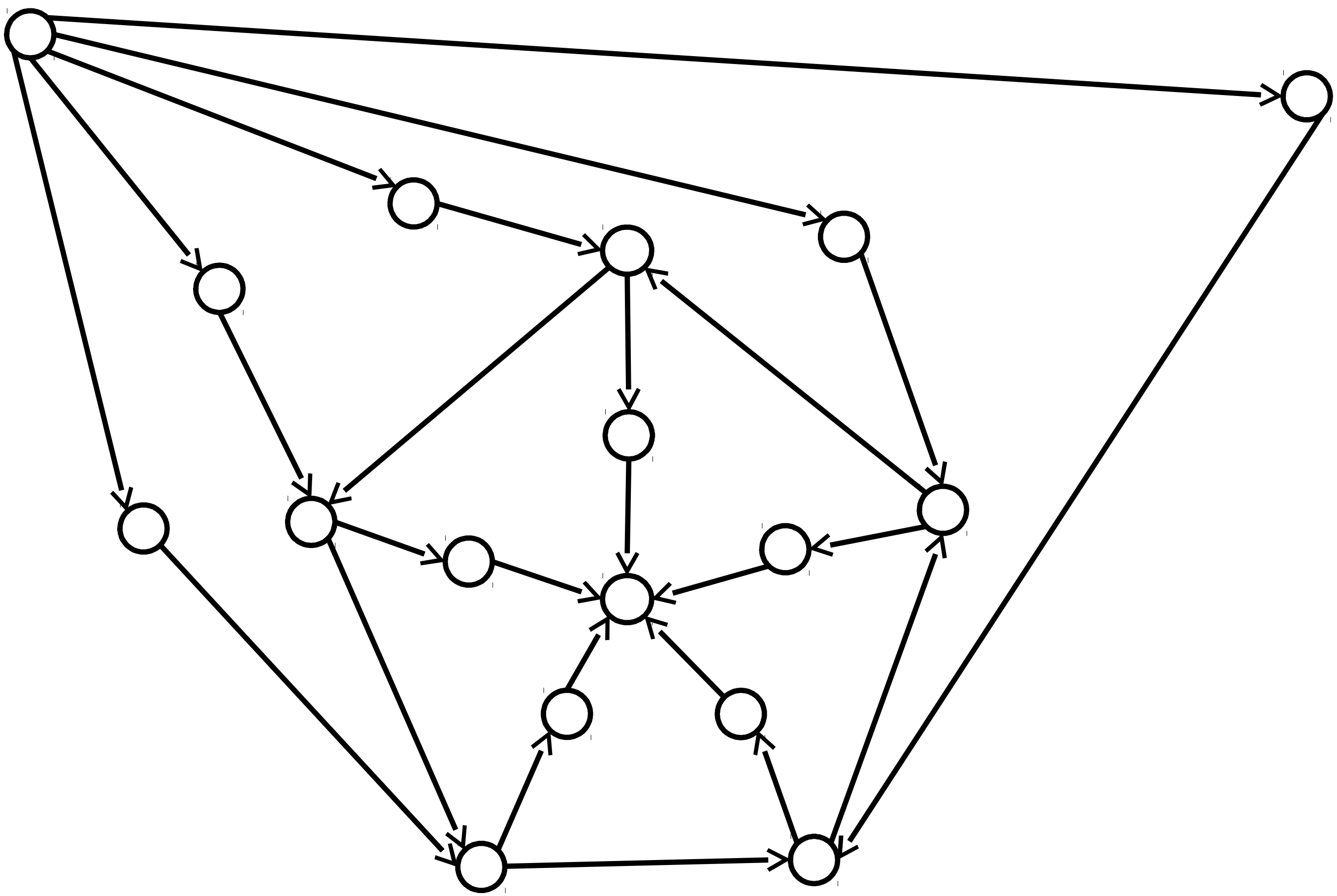






$\chi_o(G) = 4$

Examples



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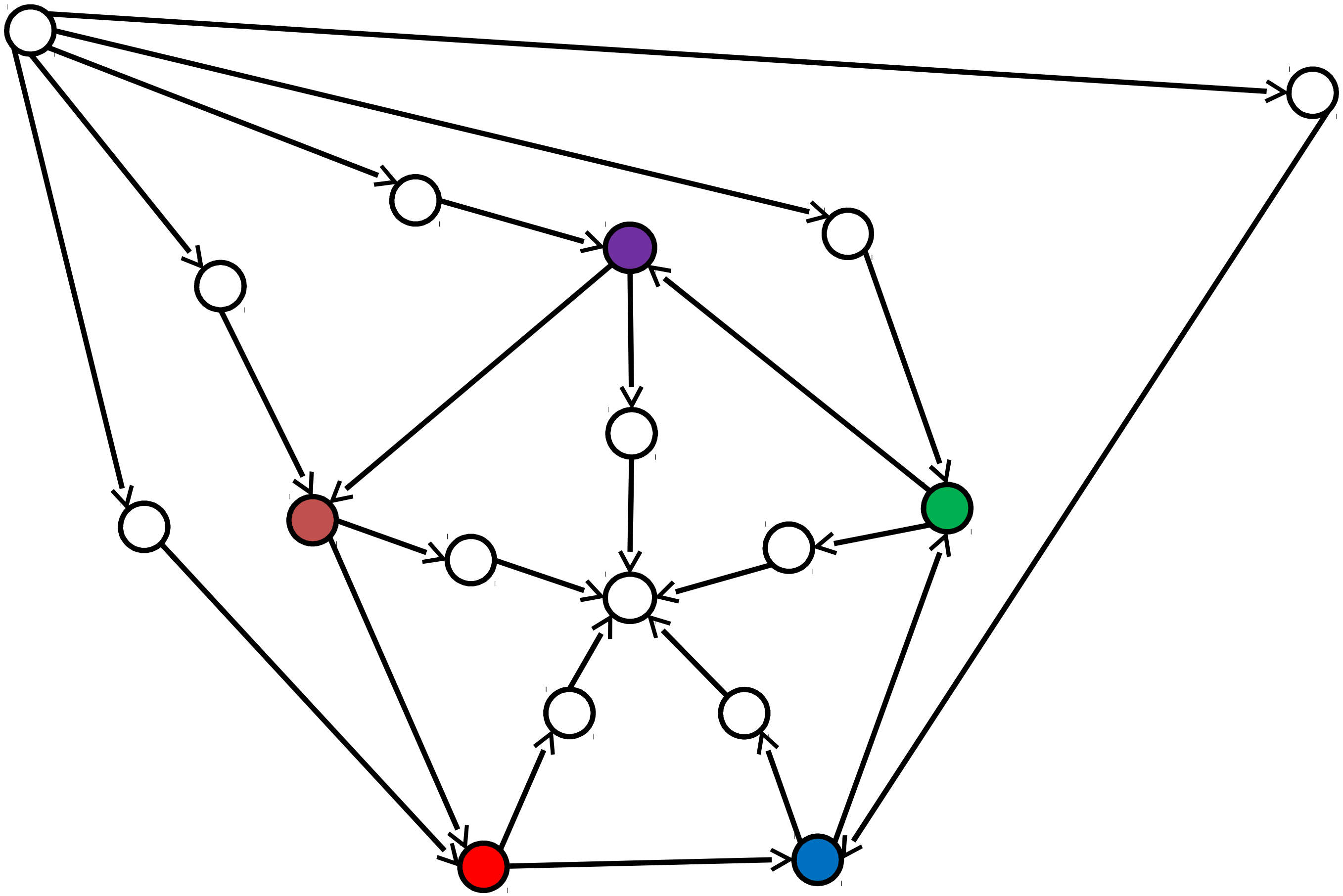

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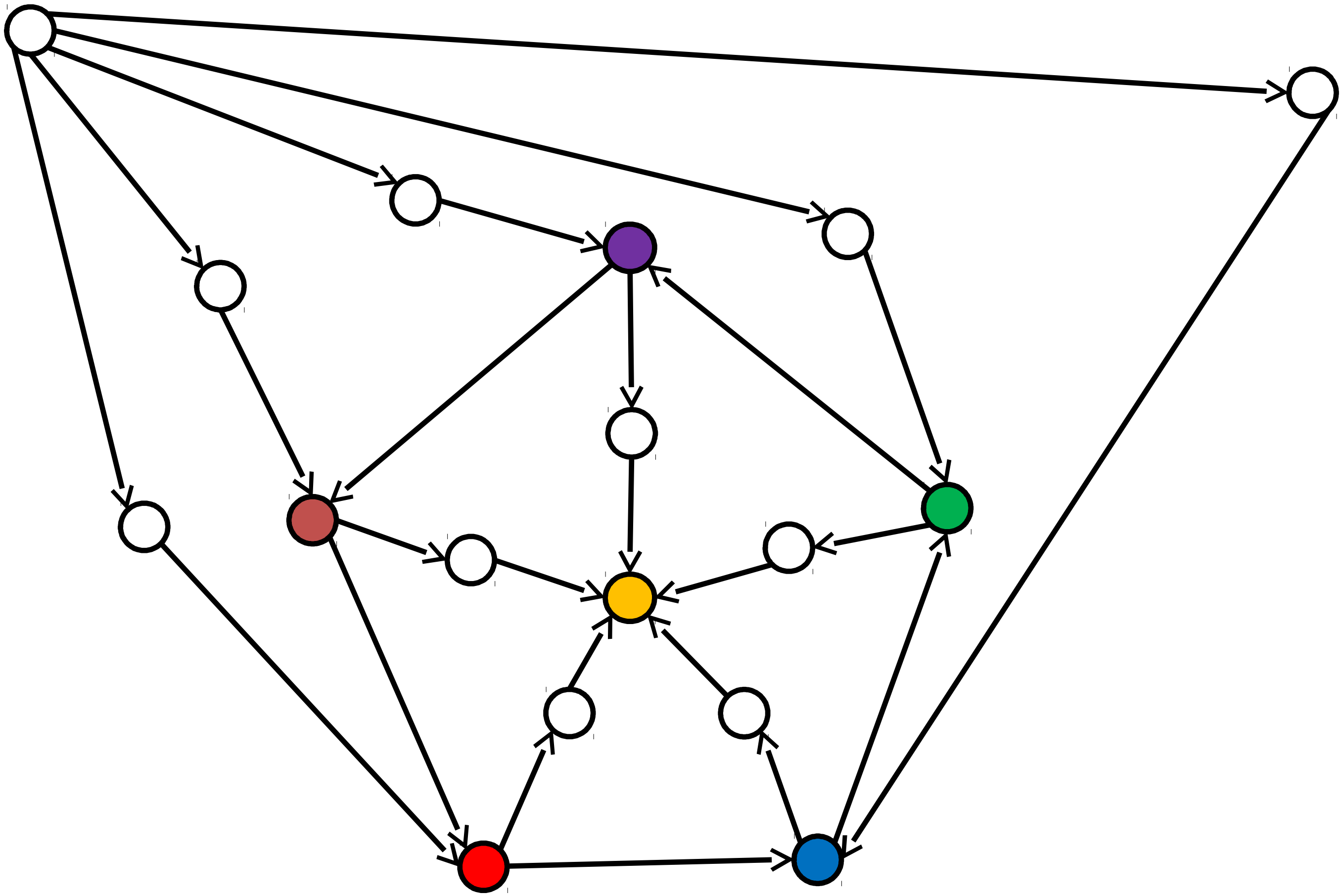

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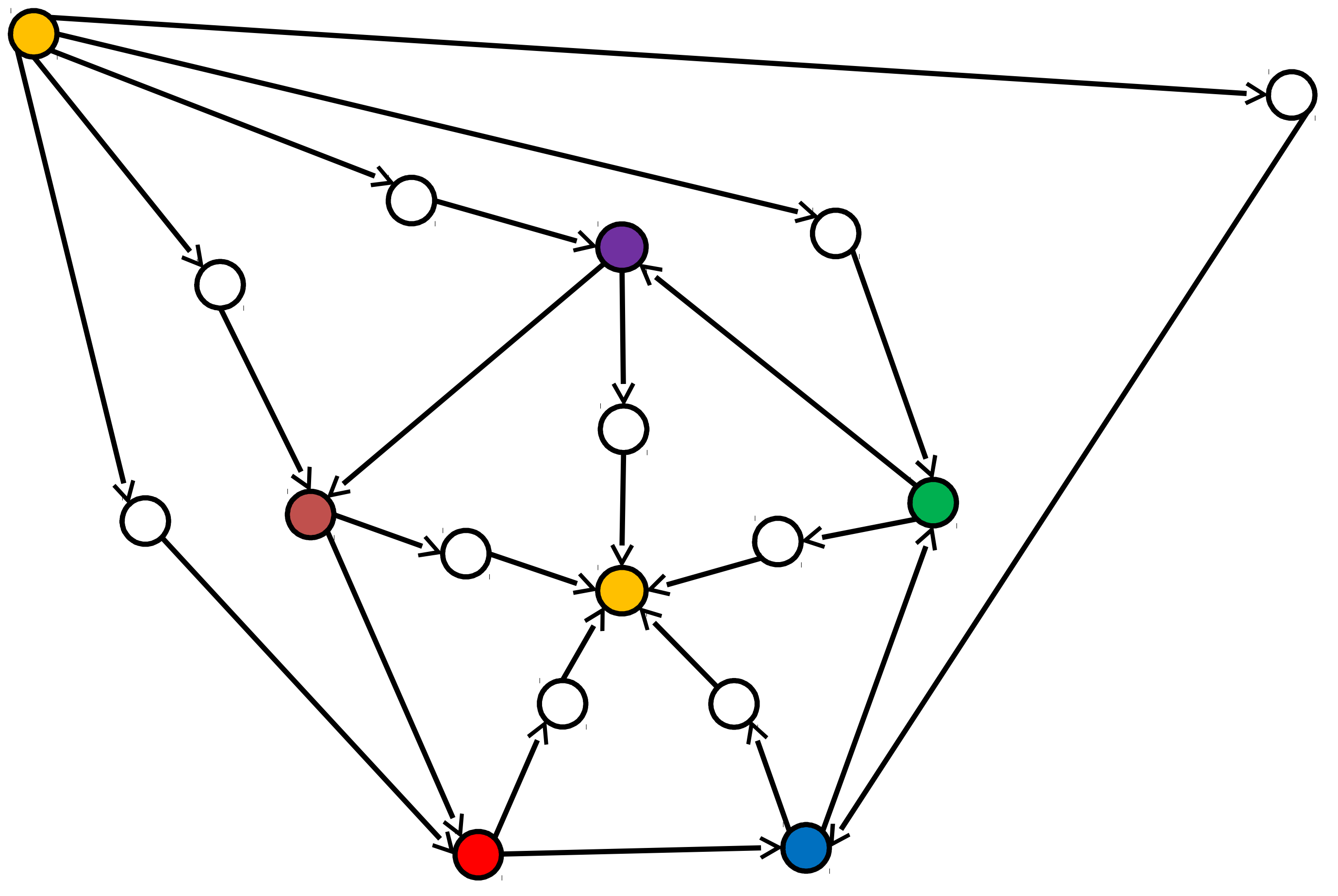

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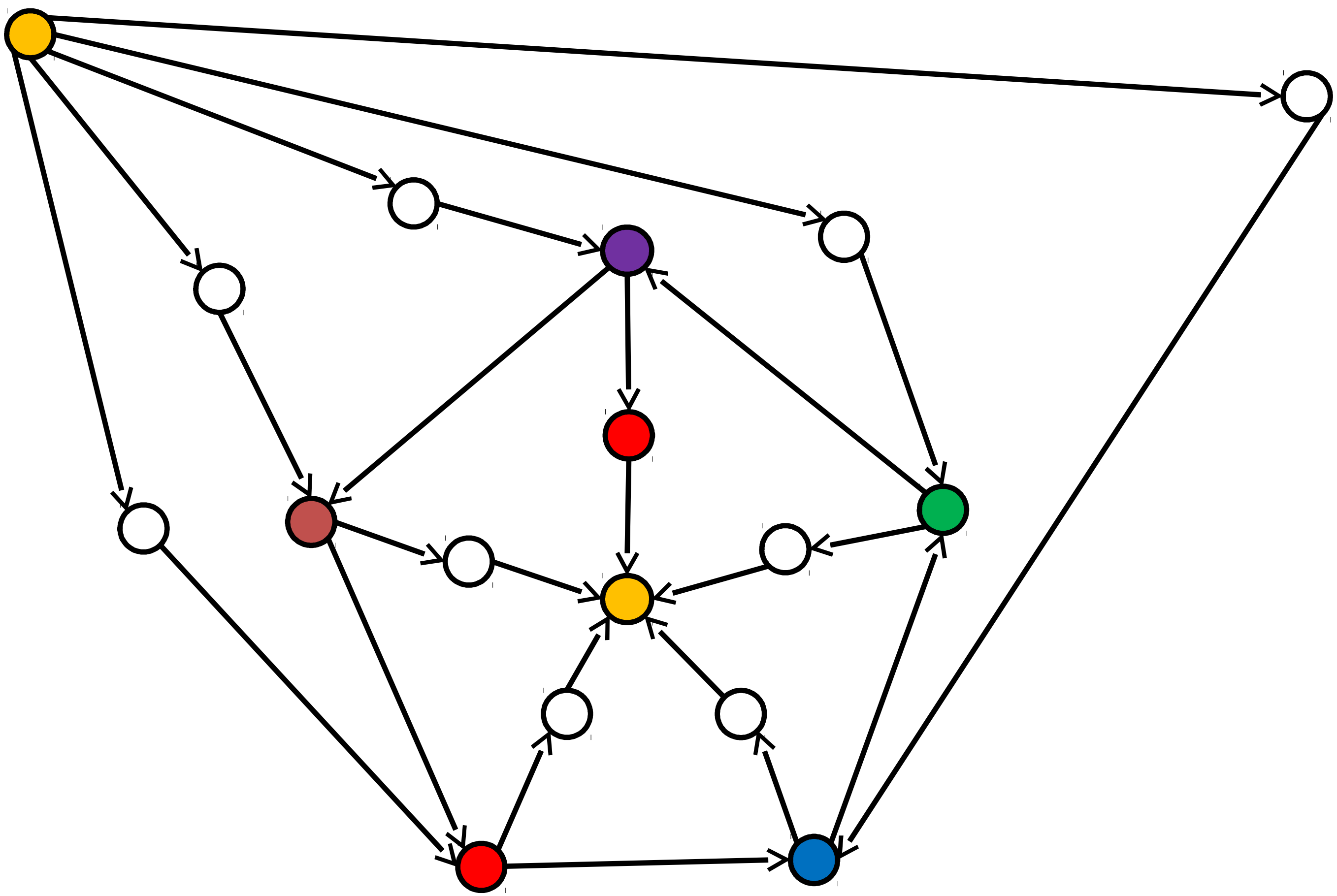

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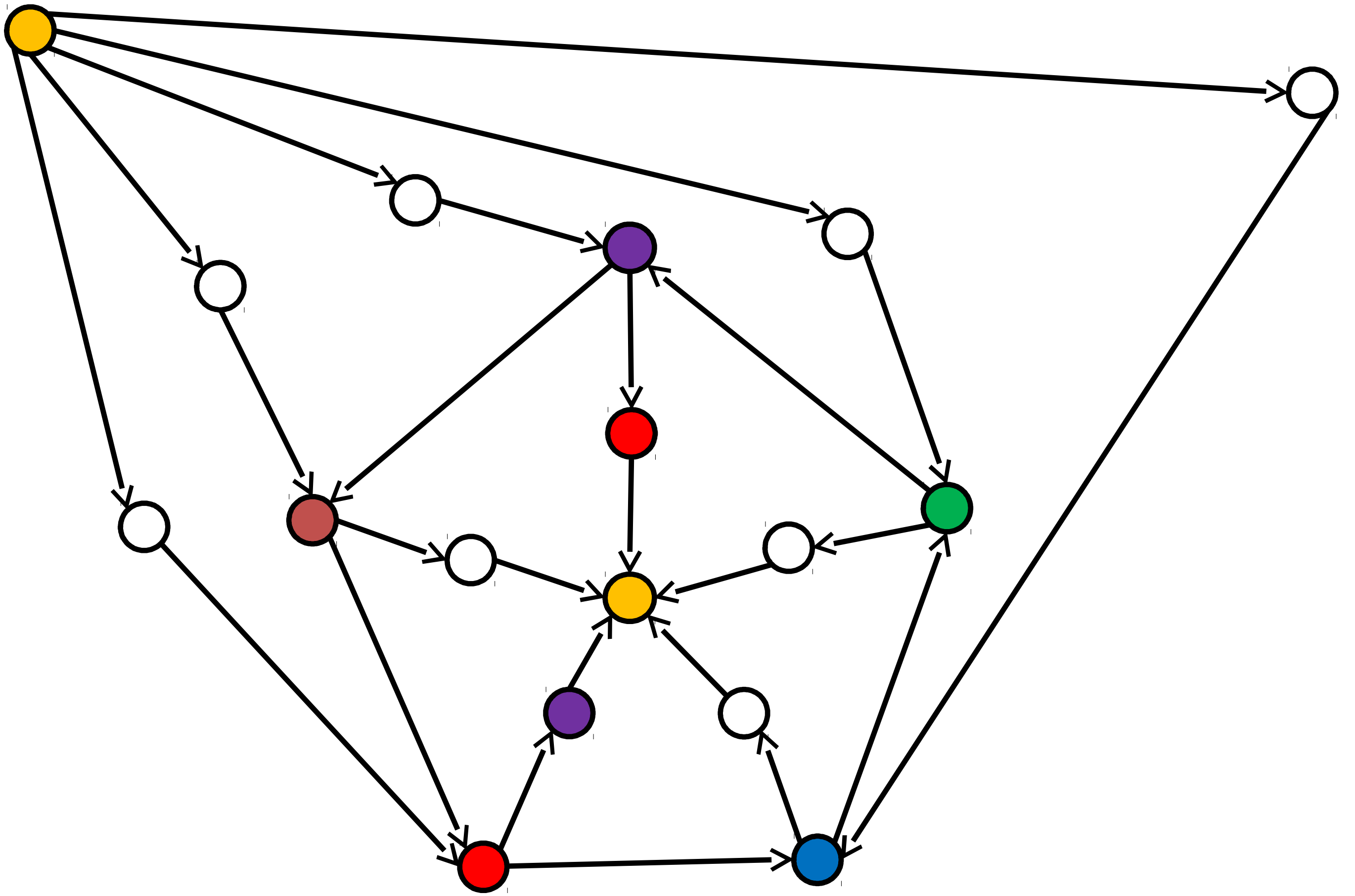

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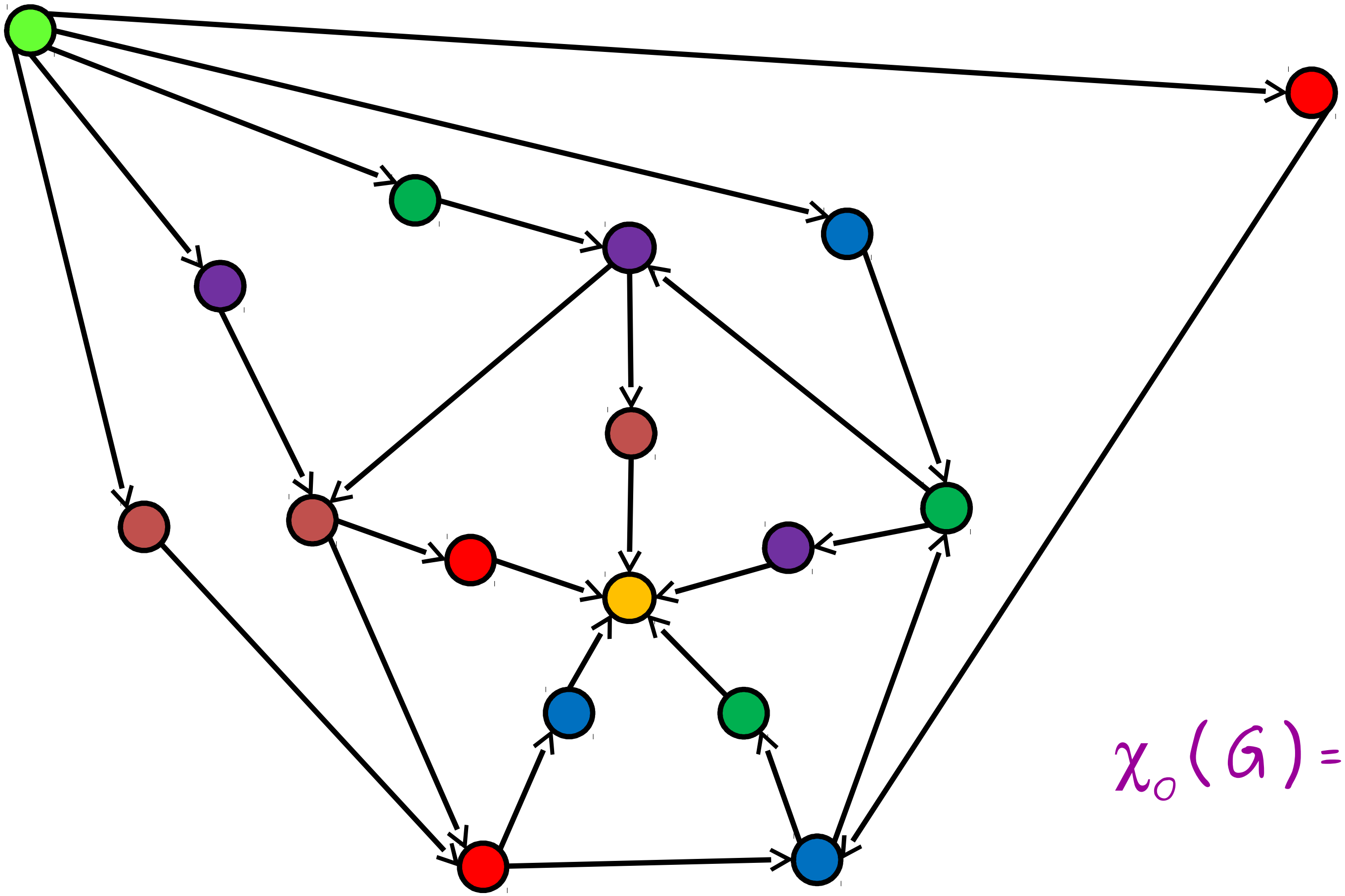


$\chi_o(G) = 7$

Oriented coloring

Studied for:

i. complexity
ii. acyclic chromatic number
iii. planar graphs with girth at least g,
iv. graphs with bounded max avg degree
v. subcubic graphs,
vi. graphs with maximum degree Δ,
vii. notion of cliques extended and studied

viii. ....

Homomorphisms of signed graphs

switch

Pushable chromatic number

Pushable chromatic number

oriented coloring + switch

Pushable chromatic number

push

oriented coloring + ~~switch~~

Pushable chromatic number

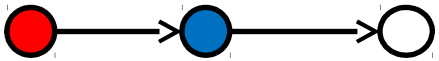
push + partition into independent sets such that all arcs between two independent sets are from one to another.

$\chi_p(G) = \min\{\#\text{independent sets}\}$

Examples

Examples

Examples



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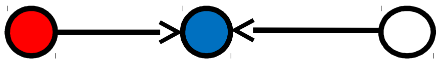

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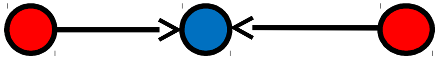
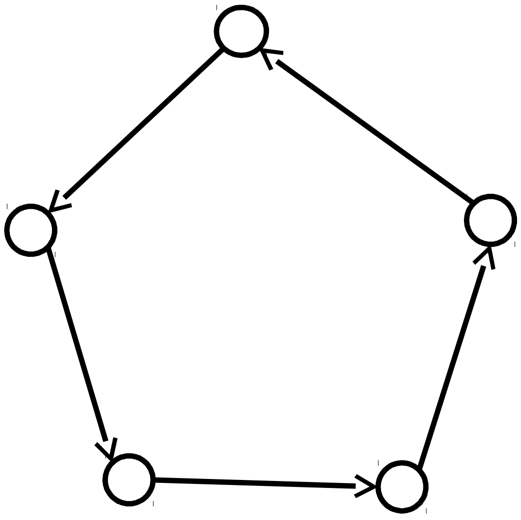
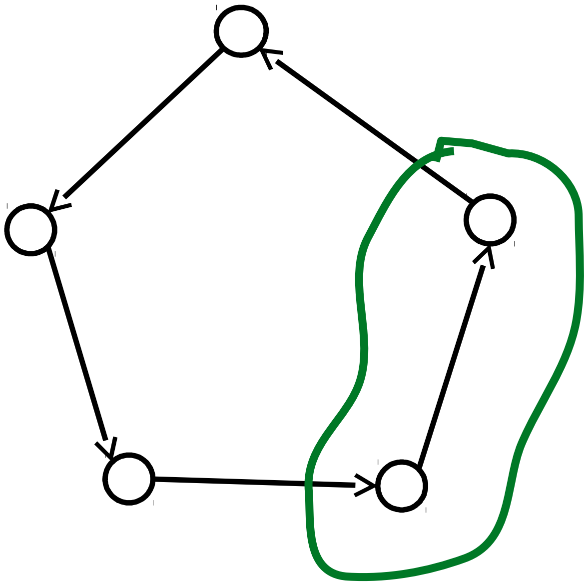


$\chi_p(G) = 2$

Examples



Examples



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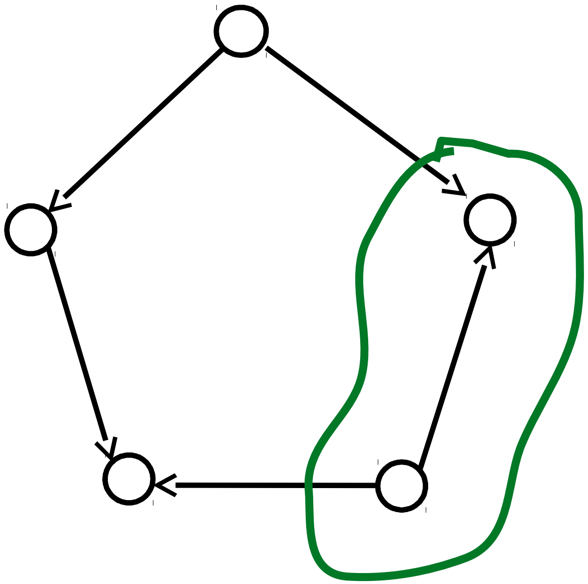
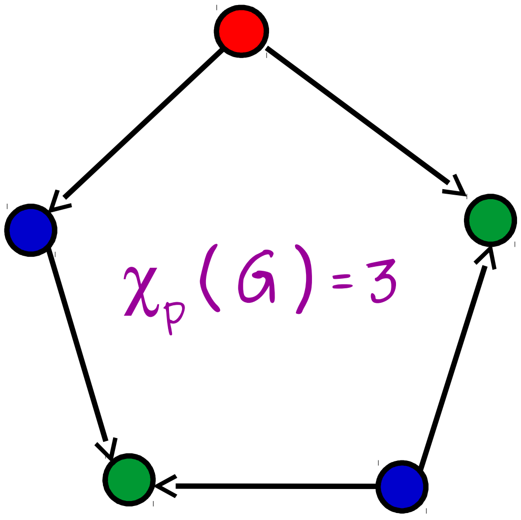


Examples



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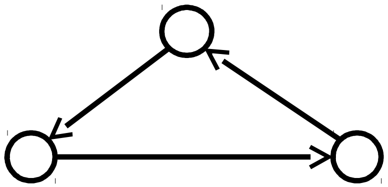

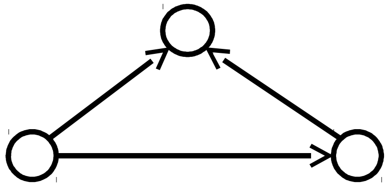

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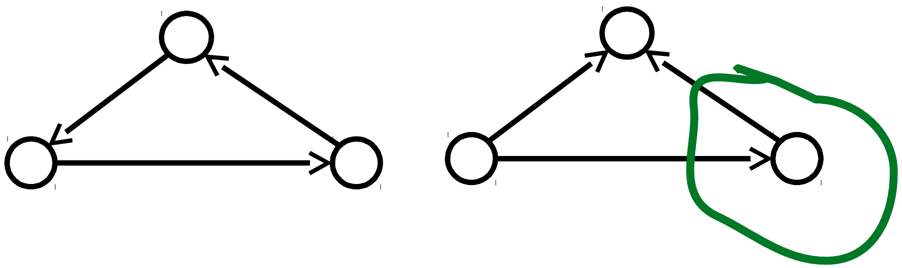

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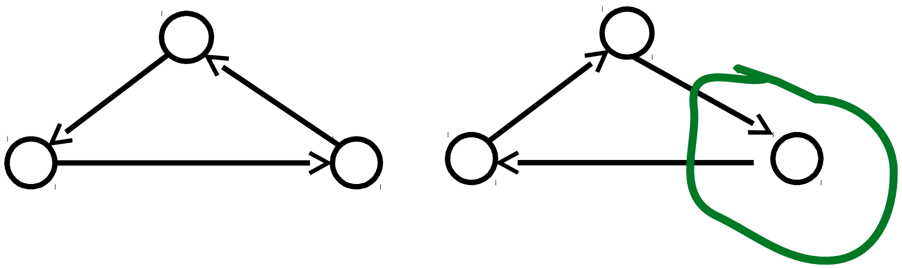

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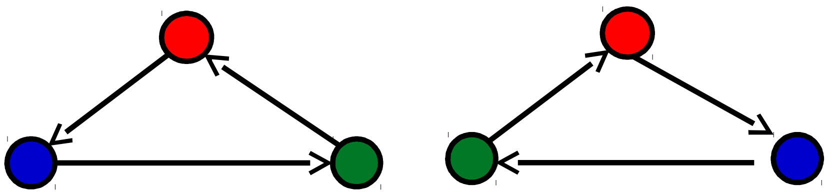
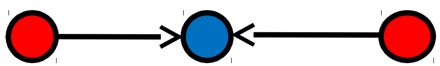
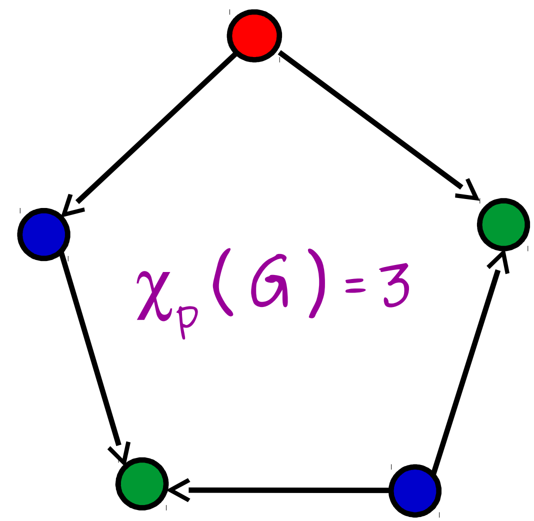


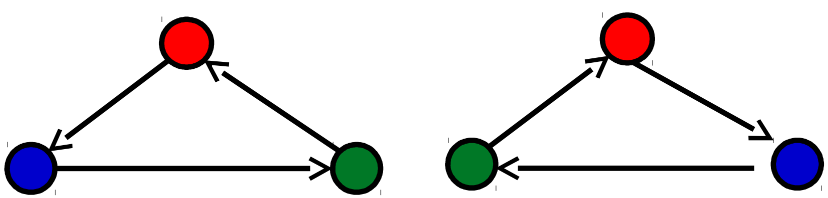
$\chi_p(G) = 3$

Examples



$\chi_p(G) = 2$





$\chi_p(G) = 3$

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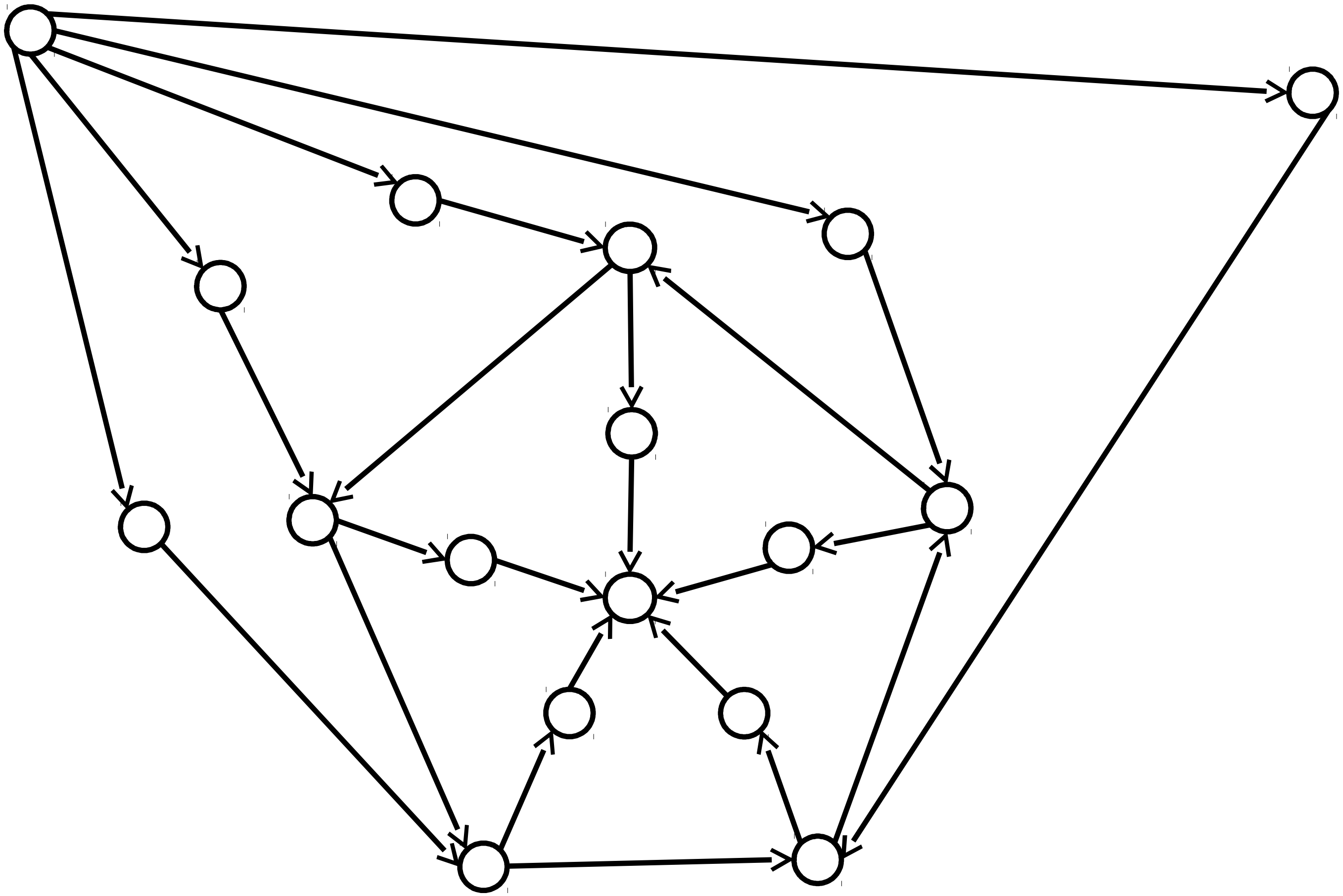

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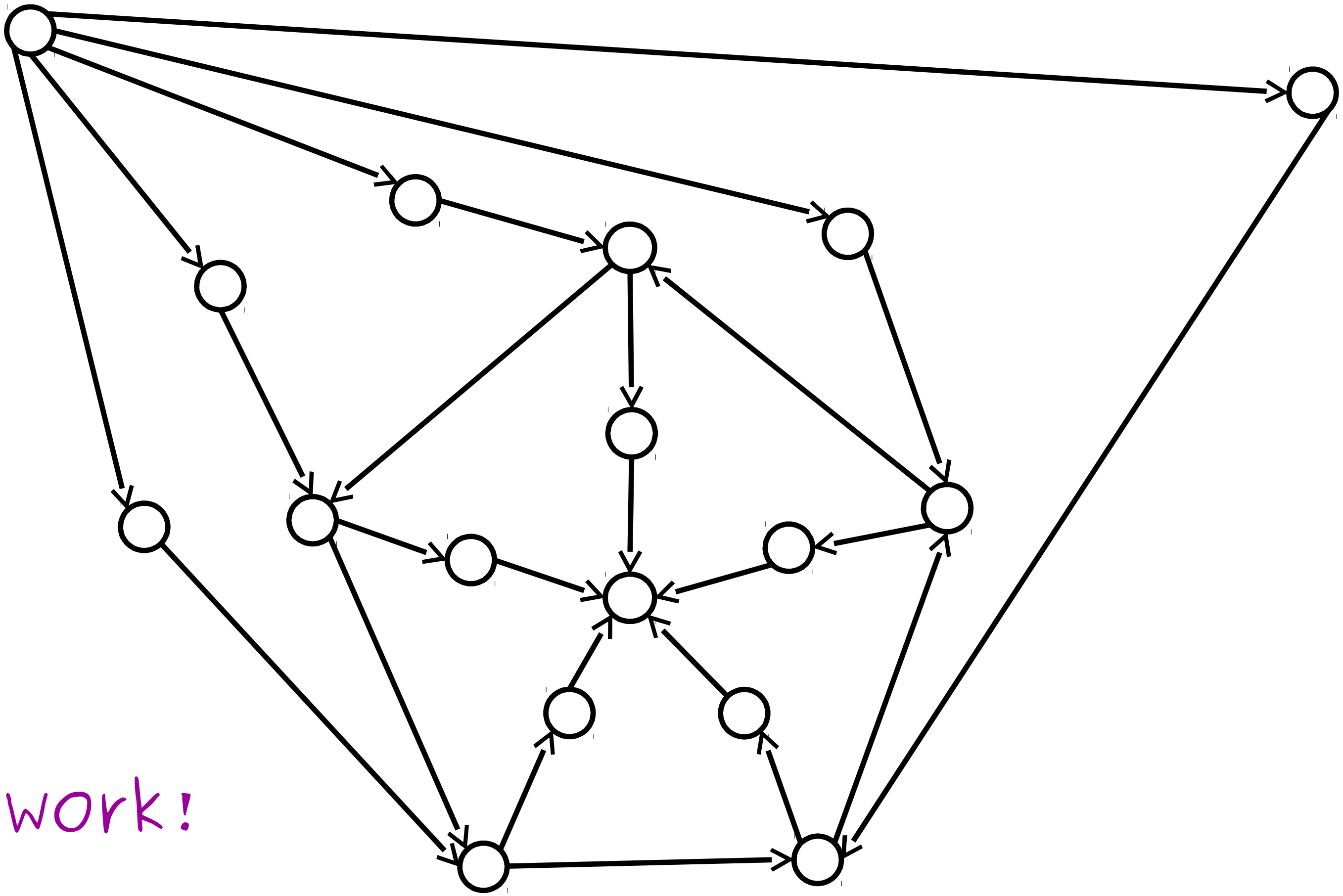

homework!

Pushable homomorphism

Pushable homomorphism

push + vertex mapping

preserving arcs.

Pushable homomorphism

push + vertex mapping

preserving arcs.

$$\chi_p(G) = \min\{|H| : G \to_p H\}$$

Results

i.

ii.

iii.

iv.

v.

Results

i. $H \to_p C_4$ polynomial, NP-complete otherwise.
[Klostermeyer and MacGillivray 2004]

ii.

iii.

iv.

v.

Results

i. $H \rightarrow_p C_4$ polynomial, NP-complete otherwise.

[Klostermeyer and MacGillivray 2004]

ii. $\chi_p(G) \leq \chi_o(G) \leq 2.\chi_p(G)$

[Klostermeyer and MacGillivray 2004]

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Results

i. $H \rightarrow_p C_4$ polynomial, NP-complete otherwise.

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ii. $\chi_p(G) \leq \chi_o(G) \leq 2.\chi_p(G)$

[Klostermeyer and MacGillivray 2004]

iii. Conjecture: $G \cong_p H \iff AT(G) \cong_o AT(H)$

[Klostermeyer and MacGillivray 2004]

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v.

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[Sen 2017]

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[Sen 2017]

iv. $\chi_p(\mathcal{O}) = 4$

[Klostermeyer and MacGillivray 2004]

v.

Results

i. $H \rightarrow_p C_4$ polynomial, NP-complete otherwise.

[Klostermeyer and MacGillivray 2004]

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[Klostermeyer and MacGillivray 2004]

iii. ~~Conjecture:~~ $G \cong_p H$ <=> $AT(G) \cong_o AT(H)$

[Sen 2017]

iv. $\chi_p(\mathcal{O}) = 4$

[Klostermeyer and MacGillivray 2004]

v. Problem: Which outerplanar graphs are 3-col ?

[Klostermeyer and MacGillivray 2004]

Results

i. $H \rightarrow_p C_4$ polynomial, NP-complete otherwise.

[Klostermeyer and MacGillivray 2004]

ii. $\chi_p(G) \leq \chi_o(G) \leq 2.\chi_p(G)$

[Klostermeyer and MacGillivray 2004]

iii. ~~Conjecture:~~ $G \cong_p H \;\; <=> \;\; AT(G) \cong_o AT(H)$

[Sen 2017]

iv. $\chi_p(\mathcal{O}) = 4$

[Klostermeyer and MacGillivray 2004]

v. ~~Problem:~~ $\chi_p(\mathcal{O}_5) = 3$

[Sen 2017]

Results

Results

i. $\chi_p(AC_k) = k.2^{k-2}$ [Sen 2017]

ii.

iii.

iv.

v.

vi.

vii.

viii.

Results

i. $\chi_p(AC_k) = k.2^{k-2}$ [Sen 2017]

ii. $10 \leq \chi_p(P_3) \leq 40$ [Sen 2017]

iii.

iv.

v.

vi.

vii.

viii.

Results

i. $\chi_p(AC_k) = k.2^{k-2}$ [Sen 2017]

ii. $10 \leq \chi_p(P_3) \leq 40$ [Sen 2017]

iii. $\chi_p(Mad < 8/3) = 4$ [Sen 2017]

iv.

v.

vi.

vii.

viii.

Results

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iv. $5 \leq \chi_p(Mad < 3) \leq 7$ [Bensmail, Das, Nandi, Paul, Pirron, Sen, Sopena (submitted) 2020]

v.

vi.

vii.

viii.

Results

i. $\chi_p(AC_k) = k.2^{k-2}$ [Sen 2017]

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v. $\chi_p(P_{16}) = 3$ [Guégan and Ochem 2015]

vi.

vii.

viii.

Results

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v. $\chi_p(P_{16}) = 3$ [Guégan and Ochem 2015]

vi. $\chi_p(P_8) = 4$ [Sen 2017]

vii. $2^{\Delta/2} \leq \chi_p(\text{con. max deg} = \Delta) \leq (\Delta-3)(\Delta-2)2^{\Delta}+2$ [Bensmail, Das, Nandi, Paul, Pirron, Sen, Sopena (submitted) 2020]

viii.

Results

i. $\chi_p(AC_k) = k.2^{k-2}$ [Sen 2017]

ii. $10 \leq \chi_p(P_3) \leq 40$ [Sen 2017]

iii. $\chi_p(Mad < 8/3) = 4$ [Sen 2017]

iv. $5 \leq \chi_p(Mad < 3) \leq 7$ [Bensmail, Das, Nandi, Paul, Pirron, Sen, Sopena (submitted) 2020]

v. $\chi_p(P_{16}) = 3$ [Guégan and Ochem 2015]

vi. $\chi_p(P_8) = 4$ [Sen 2017]

vii. $2^{\Delta/2} \leq \chi_p(\text{con. max deg} = \Delta) \leq (\Delta-3)(\Delta-2)2^{\Delta+2}$ [Bensmail, Das, Nandi, Paul, Pirron, Sen, Sopena (submitted) 2020]

viii. $6 \leq \chi_p(\text{Subcubic}) \leq 7$ [Bensmail, Das, Nandi, Paul, Pirron, Sen, Sopena (submitted) 2020]

Results

Results

i. push clique: $\chi_p(C) = |V(C)|$

[Bensmail, Nandi, Sen 2017]

ii.

iii.

iv.

Results

i. push clique: $\chi_p(C) = |V(C)|$

[Bensmail, Nandi, Sen 2017]

ii. recognition of PUSH CLIQUE is NP-complete

[Bensmail, Nandi, Sen 2017]

iii.

iv.

Results

i. push clique: $\chi_p(C) = |V(C)|$

[Bensmail, Nandi, Sen 2017]

ii. recognition of PUSH CLIQUE is NP-complete

[Bensmail, Nandi, Sen 2017]

iii. If G is a planar push clique, then $|V(G)| \leq 8$

[Bensmail, Nandi, Sen 2017]

iv.

Results

i. push clique: $\chi_p(C) = |V(C)|$

[Bensmail, Nandi, Sen 2017]

ii. recognition of PUSH CLIQUE is NP-complete

[Bensmail, Nandi, Sen 2017]

iii. If G is a planar push clique, then $|V(G)| \leq 8$

[Bensmail, Nandi, Sen 2017]

iv. If G is a planar push clique, then G contains one of the 16 graphs from a list (omitted) as a underlying spanning subgraph

[Bensmail, Nandi, Sen 2017]

Keep pushing..

Thank you