The electroweak phase transition with a complex singlet and real Triplet

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Outline

1. Electroweak phase transition in Standard Model
2. Effect of cubic term in BSM scenarios
3. Finite temperature effective potential
4. Gravitational waves production

If the minimum of $V_{eff}^{T=0}(\phi)$ ocuurs at $<\phi>=\sigma\neq 0$, for sufficiently high temperatures, th minimum of $V_{eff}^{\beta}(\phi)$ occurs at $<\phi(T)>=0$.

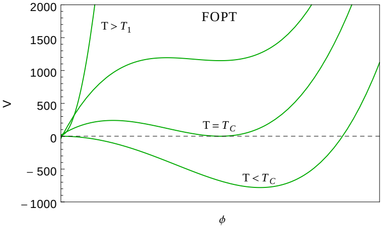
This phenomena is known as symmetry restoration at high temperature, and gives rise to the phase transition from $\phi(T)=0$ to $\phi=\sigma$.

The phase transition may be first or second order.

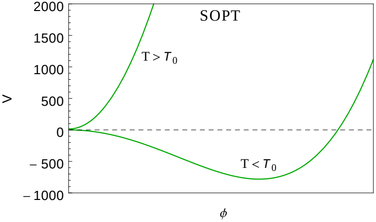
- First-order phase transitions have out of equilibrium symmetric states when the temperature decreases and are used for baryogenesis process.
- Second-order phase transitions are used in the so-called new inflationary models.

$$V(\Phi, T) = D(T^2 - T_0^2)\Phi^2 + \lambda(T)\Phi^4 - ET(\Phi^3)$$

$$V(\Phi, T) = D(T^2 - T_0^2)\Phi^2 + \lambda(T)\Phi^4$$



(a)



(b)

- Cubic term in effective potential is essential to generate a potential barrier between the symmetric and broken phases.
- It can provide the phase transition to be of the first order.

Electroweak phase transition in Standard Model

- In Standard Model, this cubic term, E, is contributed only by the electroweak gauge bosons.
- The parameter E is the cubic term of the effective potential of order of

$$E \sim \frac{2M_W^3 + M_Z^3}{4\pi v^3} \sim 0.01$$

- The Higgs self-coupling parameter has very small value

$$\lambda \sim 2E \sim 0.02 \rightarrow m_h = 49.2 GeV.$$

If the Higgs self-coupling is weak then we have first-order phase transition.

- This is incompatible with observed Higgs boson mass

$$M_h = \sqrt{2\lambda} v \sim 125.5 GeV \rightarrow \lambda \sim 0.13$$

In Standard Model the electroweak phase transition is second order.

M. Gogberashvili,

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In BSM scenarios, additional contribution from bosons to the cubic term in effective potential can trigger the first order phase transition.

Why first order?

The first-order electroweak phase transition may solve some cosmological problems, like the generation of baryon asymmetry of the universe.

- The effective potential for high field values is written as

$$V_{\rm eff}(h,\mu) \;\simeq\; \lambda_{\rm eff}(h,\mu)\,\frac{h^4}{4}\,, \quad \text{with } h \gg v\,,$$

- Where λ_{eff} is given by

$$\lambda_{\rm eff}(h,\mu) \simeq \underbrace{\lambda_h(\mu)}_{\text{tree-level}} + \underbrace{\frac{1}{16\pi^2}\sum_{\substack{i=W^\pm,Z,t,\\ h,\chi}} n_i\kappa_i^2\left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from SM}} + \underbrace{\frac{1}{16\pi^2}\sum_{i=T^0,T^\pm} n_i\kappa_i^2\left[\log\frac{\kappa_i h^2}{\mu^2} - c_i\right]}_{\text{Contribution from inert triplet}}.$$

where

$$C_W = C_Z = \frac{5}{6}$$

$$C_h = C_\chi = C_t = \frac{3}{2}$$

and n_i are the degrees of freedom

$$n_W = 6,\; n_Z = 3,\; n_h = 1,\; n_\chi = 3,\; n_t = -12$$

The Φ- dependent part of the effective potential can be written in the high-temperature expansion as

$$V_{eff}(\Phi, T) = V_{tree} + \Delta V_B + \Delta V_F$$

where

$$\Delta V_B = \sum_{i=h,\chi,W_L,Z_L,\gamma_L,W_T,Z_T,\gamma_T,T} g_i \Delta V_i$$

$$\Delta V_i = \frac{m_i^2(\Phi)T^2}{24} - \frac{\mathcal{M}_i^3(\Phi)T}{12\pi} - \frac{m_i^4(\Phi)}{64\pi^2}\left[\log\frac{m_i^2(v)}{c_B T^2} - 2\frac{m_i^2(v)}{m_i^2(\Phi)} + \delta_{i\chi}\log\frac{m_h^2(v)}{m_i^2(v)}\right],$$

and

$$\Delta V_F = g_t\left[\frac{m_{top}^2(\Phi)T^2}{48} + \frac{m_{top}^4(\Phi)}{64\pi^2}\left[\log\frac{m_{top}^2(v)}{c_F T^2} - 2\frac{m_{top}^2(v)}{m_{top}^2(\Phi)}\right]\right]$$

J. R. Espinosa and M. Quiros,

Phys. Lett. B 305 (1993), 98-105

The number of degrees of freedom g_i are given by

$$g_h = 1,\ g_\chi = 3,\ g_T = 3,\ g_t = 12$$

$$g_{W_L} = g_{Z_L} = g_{\gamma_L} = 1,$$

$$g_{W_T} = g_{Z_T} = g_{\gamma_T} = 2$$

while the coefficients c_B and c_F are defined by: $\log c_B$=3.9076, $\log c_F$=1.1350.

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Phys. Lett. B 305 (1993), 98-105

The Debye masses $\mathcal{M}_i^2(\Phi)$ for $i = h, \chi, T, W_L, W_T, Z_T, \gamma_T$ are

$$\mathcal{M}_i^2 = m_i^2(\Phi) + \Pi_i(\Phi, T)$$

where the self-energies $\Pi_i(\Phi, T)$ are given by

$$\begin{aligned}
\Pi_h(\Phi, T) &= \left(\frac{3g^2 + 3g'^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^4}{4} + \frac{\lambda_{ht}}{12}\right) T^2 \\
\Pi_\chi(\Phi, T) &= \left(\frac{3g^2 + 3g'^2}{16} + \frac{\lambda_1}{2} + \frac{y_t^4}{4} + \frac{\lambda_{ht}}{12}\right) T^2 \\
\Pi_T(\Phi, T) &= \frac{2\lambda_t + \lambda_{ht}}{6} T^2 \\
\Pi_{W_L}(\Phi, T) &= \frac{11}{6} g^2 T^2 \\
\Pi_{W_T}(\Phi, T) &= \Pi_{Z_T}(\Phi, T) = \Pi_{\gamma T} = 0
\end{aligned}$$

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Phys. Lett. B 305 (1993), 98-105

$$\mathcal{M}^2_{ZL} = \frac{1}{2}\left[m_Z^2(\Phi) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W}T^2 + \Delta(\Phi,T)\right]$$

$$\mathcal{M}^2_{\gamma L} = \frac{1}{2}\left[m_Z^2(\Phi) + \frac{11}{6}\frac{g^2}{\cos^2\theta_W}T^2 - \Delta(\Phi,T)\right]$$

with

$$\Delta^2(\Phi,T) = m_Z^4(\Phi) + \frac{11}{3}\frac{g^2\cos^2 2\theta_W}{\cos^2\theta_W}\left[m_Z^2(\Phi) + \frac{11}{12}\frac{g^2}{\cos^2\theta_W}T^2\right]T^2$$

Sef energy contribution to the transverse components of W, Z, γ is zero.

Photon is also contributing to the effective potential through non-zero self energy contribution in longitudinal component.

How do we conserve this extra degree of freedom (photon mass contribution)?

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Phys. Lett. B 305 (1993), 98-105

Dominant contribution from BSM-boson

The Φ dependent part of the effective potential can be written as

$$V(\Phi) = A(T)\Phi^2 + B(T)\Phi^4 + C(T)(\Phi^2 + K^2(T))^{\frac{3}{2}}$$

where

$$\begin{aligned} A(T) &= -\frac{1}{2}\mu_T^2 + \frac{1}{4}\left(\frac{\lambda_{ht}}{6} + \frac{y_t^2}{2}\right) \\ B(T) &= \frac{1}{4}\lambda_T \\ C(T) &= -\left(\frac{\lambda_{ht}}{2}\right)^{\frac{3}{2}}\frac{T}{6\pi} \\ K^2(T) &= \frac{(\lambda_{ht} + 2\lambda_t)T^2 + 6m_t^2}{3\lambda_{ht}} \end{aligned}$$

where

$$\mu_T^2 = \mu^2 - \frac{\lambda_{ht}}{16\pi^2}\left(\sum_{i=T_0,T^\pm} m_i^2(v) + m_T^2 \sum_{i=T_0,T^\pm} log\frac{c_B T^2}{m_i^2(v)}\right) + \frac{3}{8\pi^2} y_t^2 m_{top}^2(v) log\frac{m_{top}^2(v)}{c_F T^2}$$

$$\lambda_T = \lambda_1 + \frac{\lambda_{ht}^2}{32\pi^2} log\frac{c_B T^2}{m_T^2(v)} + \frac{3}{16\pi^2} y_t^4 log\frac{m_{top}^2(v)}{c_F T^2}$$

Critical Temperature

At some temperature the origin is a minimum and there is a maximum at $\Phi_-(T)$ and a minimum at $\Phi_+(T)$ given by the condition $V'(0) = 0$

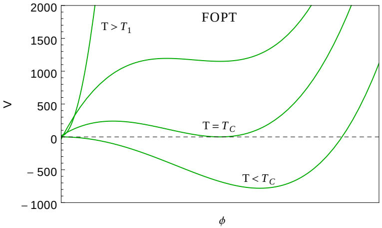
$$\Phi_\pm(T) = \frac{1}{32B^2}\left(9C^2 - 16AB \pm |C|\sqrt{9C^2 + 32(2B^2K^2 - AB)}\right)$$

The critical temperature T_C for the first-order transition is determined by the condition

$$V(\Phi = 0; T_C) = V(\Phi_+(T_C); T_C)$$

The degree to which the phase transition is first-order is characterized by

$$\frac{\Phi_+(T_C)}{T_C}$$



(a)

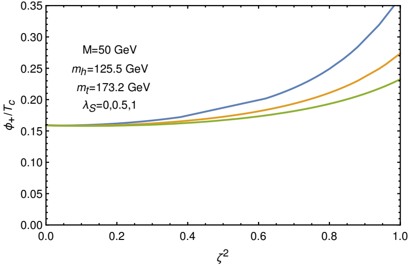
Variation with quartic coupling

The condition for a strongly first-order phase transition has typically taken to be

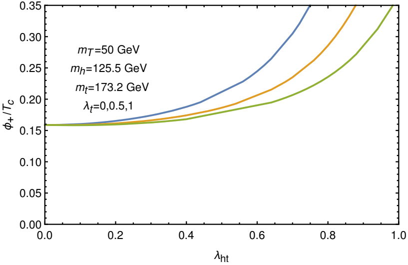
$$\frac{\Phi_+(T_C)}{T_C} \geq 1.$$

D. E. Morrissey, M. J. Ramsey-Musolf

New J. Phys. 14 (2012), 125003



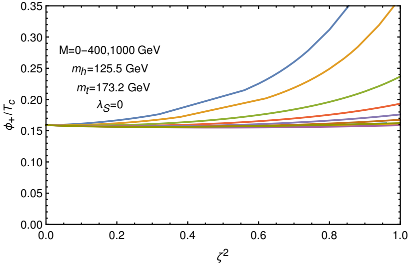
(a) Singlet with m_h=125.5 GeV, m_t=173.2 GeV.



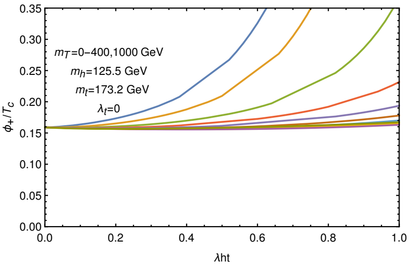
(b) Triplet with m_h=125.5 GeV, m_t=173.2 GeV.

$\frac{\Phi_+(T_C)}{T_C}$ is decreasing with increase in quartic coupling for particular value of soft mass parameter.

Blue color corresponds to $\lambda_s/\lambda_t=0$.



(a) Singlet with m_h=125.5 GeV, m_t=173.2 GeV.



(b) Triplet with m_h=125.5 GeV, m_t=173.2 GeV.

$\frac{\Phi_+(T_C)}{T_C}$ is decreasing with increase in soft mass parameter for particular value of quartic coupling.

Blue color corresponds to $\mathrm{M}/\mathrm{m_T}=0$.

The modified one-loop β-function for the SM-like quartic coupling λ_h and other quartic couplings λ_t, λ_{ht} by the addition of inert triplet are as follows

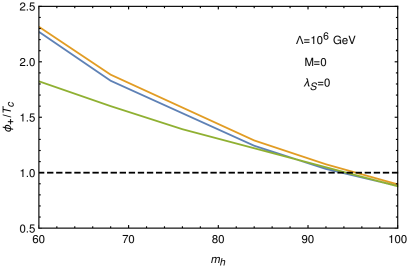
$$\begin{aligned}
\Delta\beta_{\lambda} &= 8\lambda_{ht}^2 \\
\beta_{\lambda_t} &= \frac{1}{16\pi^2}\left[-24g_2^2\lambda_t + 88\lambda_t^2 + 8\lambda_{ht}^2 + \frac{3}{2}g_2^4\right] \\
\beta_{\lambda_{ht}} &= \frac{1}{16\pi^2}\left[\frac{3}{4}g_2^4 - \frac{9}{10}g_1^2\lambda_{ht} - \frac{33}{2}g_2^2\lambda_{ht} + 12\lambda\lambda_{ht} + 16\lambda_{ht}^2 + 24\lambda_{ht}\lambda_t + 6y_t^2\lambda_{ht}\right]
\end{aligned}$$

For $\lambda_t = 0$, we consider maximum allowed value of λ_{ht} at the electroweak scale for which the theory remains perturbative at a particular scale.

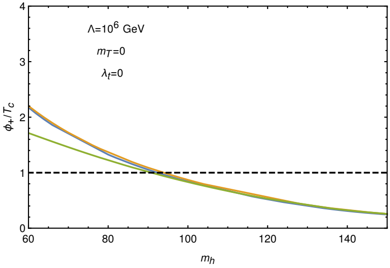
Λ (GeV)	$m_h = 65.0$ (GeV)			$m_h = 125.5$ (GeV)		
	m_t (GeV)			m_t (GeV)		
	90	120	173.2	90	120	173.2
10^4	1.4558	1.4358	1.3950	1.4390	1.4190	1.3710
10^6	0.7825	0.7672	0.7290	0.7601	0.7435	0.7067
10^8	0.5568	0.5420	0.5120	0.5330	0.5171	0.4873
10^{11}	0.4051	0.3920	0.3700	0.3785	0.3645	0.3437
10^{16}	0.2948	0.2839	0.2712	0.2633	0.2516	0.2439
10^{19}	0.2587	0.2489	0.2403	0.2239	0.2135	0.2129

Λ is the perturbative scale where any of the coupling diverges.

Orange, blue and green color corresponds to different top quark mass $m_t = 90, 120, 173.2$ GeV respectively for $m_h = 125.5$ GeV.



(a) Singlet

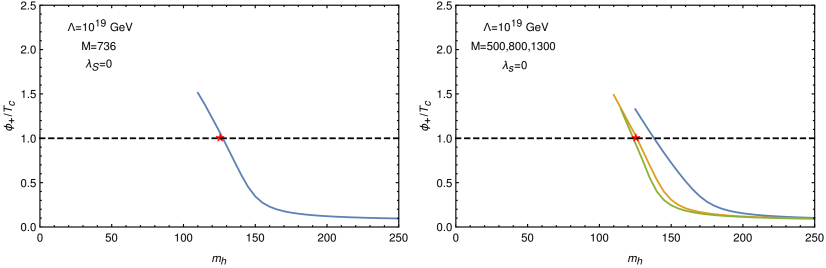


(b) Triplet

- λ_{ht}^{max} values are lower in case of triplet because of more number of scalars.
- Lower values of λ_{ht}^{max} corresponds to lower values of $\frac{\Phi_+(T_C)}{T_C}$.
- In singlet case, Higgs mass bound comes out to be 95 GeV and in triplet it is 93 GeV.
- For Planck scale perturbativtiy first-order phase transition does not satisfy the current Higgs mass bound.

Λ (GeV)	λ_{ht}^{max} (GeV)	$\zeta_{max}^2/\lambda_{hs}$ (GeV)
10^{19}	1.95	4.0

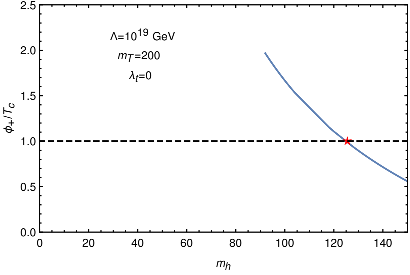
- λ_{ht} is the quartic coupling for itm and $\zeta^2 = \lambda_{hs}$ is the quartic coupling corresponding to singlet.
- Two loop analysis allows higher values of λ_{ht}^{max}.
- Higher values of λ_{ht}^{max} will give higher $\frac{\Phi_+(T_C)}{T_C}$ and will satisfy the current Higgs mass bound.



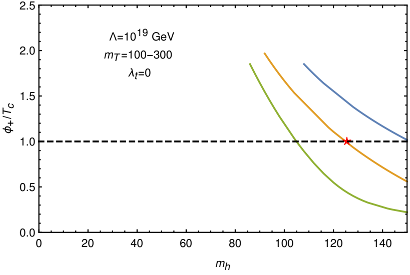
(a) Singlet (b) Singlet

- For Planck scale perturbativity, the upper bound on the soft mass parameter is 736 GeV satisfying the current Higgs mass and strongly first-order phase transition.
- Blue, orange and green color corresponds to $M = 500, 800, 1300$ GeV respectively.

Bounds on soft mass pareameter: Triplet



(a) Triplet



(b) Triplet

- For Planck scale perturbativity, the upper bound on the soft mass parameter is 200 GeV satisfying the current Higgs mass and strongly first-order phase transition.
- Blue, orange and green color corresponds to $m_T = 100, 200, 300$ GeV respectively.

Gravitaional waves production

In first-order phase transition, true vacuum bubble starts to nucleate at some temperature and then they start expanding because of pressure difference between the false and true vacuua.

Uncollided bubbles do not radiate Gravitational waves beacuse of the spherical symmetry of each bubble, the collision process breaks the symmetry and Gravitational waves are produced.

The GWs are produced from the strong first-order electroweak phase transition maily by three mechanisms;

- Bubble collisions
- Sound waves in hot plasma
- Magnetohydrodynamic turbulence of bubbles in the early universe

Total GW intensity $\Omega_{GW}\mathrm{h}^2$ as a function of frequency is expressed as sum of

$$\Omega_{GW}h^2 = \Omega_{coll}h^2 + \Omega_{SW}h^2 + \Omega_{turb}h^2$$

C. Caprini, M. Hindmarsh, S. Huber,et al.
JCAP 04 (2016), 001

- Photon also contributes to the effective potential through non-zero self energy contribution in longitudinal component at finite temperature.
- Current Higgs mass bound is not satisfied for one-loop beta function analysis.
- For singlet, the upper bound on soft mass parameter is 760 GeV satisfying the strongly first-order phase transition and current Higgs mass bound.
- For triplet, the upper bound on soft mass parameter is 200 GeV satisfying the strongly first-order phase transition and current Higgs mass bound.

- M. Gogberashvili, Adv. High Energy Phys. **2018** (2018), 4653202 doi:10.1155/2018/4653202 [arXiv:1702.08445 [gr-qc]].
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- D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. **14** (2012), 125003 doi:10.1088/1367-2630/14/12/125003 [arXiv:1206.2942 [hep-ph]].
- C. Caprini, M. Hindmarsh, S. Huber, T. Konstandin, J. Kozaczuk, G. Nardini, J. M. No, A. Petiteau, P. Schwaller and G. Servant, *et al.* JCAP **04** (2016), 001 doi:10.1088/1475-7516/2016/04/001 [arXiv:1512.06239 [astro-ph.CO]].

Thank You