## Verifiable Type-II Seesaw \& Dark Matter in a Gauged $U(1)_{B-L}$ Model arXiv no: 2002.07000

Satyabrata Mahapatra

Co-Authors: Nimmala Narendra, Narendra Sahu Department of Physics Indian Institute of Technology Hyderabad

$$
\text { June 28, } 2020
$$

## Overview

(1) Introduction
(2) Anomalies in a gauged $U(1)_{B-L}$ extension
(3) The Complete Model

- Neutrino Mass
- Scalar Masses and Mixing
(4) Dark Matter
- Right Handed Neutrinos \& Their Interactions
- Dark Matter
- Direct Detection
(5) Collider Signatures
(6) Coclusion


## Introduction

- Standard Model(SM) works remarkably well.


## But...

- Why Beyond Standard Model?
- Answer: Many Unsolved Mysteries
(1) Origin of Neutrino Mass
(2) Identity of Dark Matter
(3) Baryon-Antibaryon asymmetry
(4) ....


## Neutrino Mass

- Evidence $\rightarrow \nu$ - Oscillation Experiments
- Explained by dimension five operator $\mathcal{O}_{5}=\frac{\text { LLHH }}{\Lambda}$ Where L: Lepton H: Higgs


## and $\Lambda$ : Scale of new physics

- Neutrino Mass $M_{\nu}=\mathrm{O}\left(\frac{\langle H\rangle^{2}}{\Lambda}\right) \simeq 0.1 \mathrm{eV}$

$$
\text { for }\langle\hat{H}\rangle=10^{2} \mathrm{GeV} \text { and } \Lambda=10^{14} \mathrm{GeV} \text {. }
$$

- Seesaw Mechanisms
(1) Type-I (Gauge singlet RHN)
(2) Type-II ( $S U(2)$ triplet scalar)
(3) Type-III (SU(2) triplet fermion)


## Dark Matter

- Evidences: Galaxy Rotation curves, Gravitational Lensing, Large scale structure formation, Bullet clusters etc.
- Through Gravitaional Interactions
- Only microscopic property known is Relic Density

$$
\Omega_{D M} h^{2} \simeq 0.120 \pm 0.001
$$

## Anomalies in a gauged $U(1)_{B-L}$ extension

In chiral gauge theory, the Anomaly Coefficient;

$$
\mathcal{A}_{A b c}=\operatorname{Tr}\left(\gamma^{5} T_{A}\left[T_{b}, T_{c}\right]_{+}\right)=\operatorname{Tr}\left(T_{A}\left[T_{b}, T_{c}\right]_{+}\right)_{R}-\operatorname{Tr}\left(T_{A}\left[T_{b}, T_{c}\right]_{+}\right)_{L}
$$

- Gauging of $U(1)_{\mathrm{B}-\mathrm{L}}$ symmetry within the SM lead to the following triangle anomalies:

$$
\begin{gather*}
\mathcal{A}_{1}\left[U(1)_{B-L}^{3}\right]=3 \\
\mathcal{A}_{2}\left[(\text { Gravity })^{2} \times U(1)_{B-L}\right]=3 \tag{1}
\end{gather*}
$$

- For Anomaly cancellation $\rightarrow$ Add 3 RHNs ( $B-L$ charge -1 )(Most natural choice)
- Alternative ways of costructing anomaly free versions of $U(1)_{B-L}$ extension of the SM $\rightarrow$
3 RHNs with exotic $B-L$ charges $-4,-4,+5$

$$
\begin{align*}
\mathcal{A}_{1}\left[U(1)_{B-L}^{3}\right] & =\mathcal{A}_{1}^{S M}\left[U(1)_{B-L}^{3}\right]+\mathcal{A}_{1}^{\text {New }}\left[U(1)_{B-L}^{3}\right] \\
& =3+\left[(-4)^{3}+(-4)^{3}+(5)^{3}\right]=0 \\
\mathcal{A}_{2}\left[(\text { Gravity })^{2} \times U(1)_{B-L}\right] & =\mathcal{A}_{2}^{S M}\left[(\text { Gravity })^{2} \times U(1)_{B-L}\right] \\
& +\mathcal{A}_{2}^{\text {New }}\left[(\text { Gravity })^{2} \times U(1)_{B-L}\right] \\
& =3+[(-4)+(-4)+(5)]=0 \tag{2}
\end{align*}
$$

## The Proposed Model

## Motivation : A variant of type-II seesaw at TeV scale

- In a type-II seesaw framework, the SM is usually extended with a triplet scalar of hypercharge 2.
- In this case, gauging of $U(1)_{B-L}$ symmetry does not lead to any new anomalies.
- Therefore, we consider :

A type-II seesaw framework with gauged $U(1)_{\mathrm{B}-\mathrm{L}}$ symmetry, where the $B-L$ anomalies are cancelled by the introduction of 3 RHNs with exotic $B-L$ charges $-4,-4,+5$.

The model thus proposed explains the origin of neutrino mass and DM in a minimal set-up.

## Gauge Group : $S U(3)_{C} \otimes S U(2)_{L} \otimes U(1)_{Y} \otimes U(1)_{B-L}$

| Fields | $S U(3)_{c}$ | $S U(2)_{L}$ | $U(1)_{Y}$ | $U(1)_{B-L}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\nu_{R_{1,2}}$ | 1 | 1 | 0 | -4 |
| $\nu_{R_{3}}$ | 1 | 1 | 0 | 5 |
| $\Delta$ | 1 | 3 | 2 | 0 |
| $\xi$ | 1 | 3 | 2 | 2 |
| $\Phi_{B-L}$ | 1 | 1 | 0 | -1 |
| $\Phi_{12}$ | 1 | 1 | 0 | +8 |
| $\Phi_{3}$ | 1 | 1 | 0 | -10 |

Table: New particles and their quantum numbers under the imposed gauge symmetry.

## The Lagrangian involving the new fields

$$
\begin{align*}
L & \supset \overline{\nu_{R_{a}}} i \gamma^{\mu} D_{\mu} \nu_{R_{b}}+\overline{\nu_{R_{3}}} i \gamma^{\mu} D_{\mu} \nu_{R_{3}}+\left|D_{\mu} X\right|^{2} \\
& +Y_{i j}^{\xi} \overline{L_{i}^{c}} i \tau_{2} \xi L_{j}+Y_{a b} \Phi_{12} \overline{\left(\nu_{R_{a}}\right)^{c}} \nu_{R_{b}}+Y_{33} \Phi_{3} \overline{\left(\nu_{R_{3}}\right)^{c}} \nu_{R 3} \\
& +Y_{a 3} \overline{\left(\nu_{R_{a}}\right)^{c}} \Phi_{B-L} \nu_{R_{3}}+\text { h.c. }-\mathrm{V}\left(H, \Delta, \xi, \Phi_{B-L}, \Phi_{12}, \Phi_{3}\right) \tag{3}
\end{align*}
$$

where

$$
D_{\mu}=\left(\partial_{\mu}+i g_{B-L} Y_{B-L}\left(Z_{B-L}\right)_{\mu}\right)
$$

$g_{B-L}$ : gauge coupling associated with $U(1)_{B-L}$
$Z_{B-L}$ : corresponding gauge boson.
$X=\Phi_{B-L}, \Phi_{12}, \Phi_{3}, \Delta, \xi$
$i, j$ runs over $1,2,3 \& a, b$ runs over 1,2 .

The scalar potential of the model can be written as:

$$
\begin{aligned}
& \mathrm{V}\left(H, \Delta, \xi, \Phi_{B-L}, \Phi_{12}, \Phi_{3}\right)=-\mu_{H}^{2} H^{\dagger} H+\lambda_{H}\left(H^{\dagger} H\right)^{2}+M_{\Delta}^{2} \Delta^{\dagger} \Delta \\
& +\lambda_{\Delta}\left(\Delta^{\dagger} \Delta\right)^{2}+M_{\xi}^{2} \xi^{\dagger} \xi+\lambda_{\xi}\left(\xi^{\dagger} \xi\right)^{2}-\mu_{\Phi_{B-L}}^{2} \Phi_{B-L}^{\dagger} \Phi_{B-L} \\
& +\lambda_{\Phi_{B-L}}\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right)^{2}-\mu_{\Phi_{12}}^{2} \Phi_{12}^{\dagger} \Phi_{12}+\lambda_{\Phi_{12}}\left(\Phi_{12}^{\dagger} \Phi_{12}\right)^{2}-\mu_{\Phi_{3}}^{2} \Phi_{3}^{\dagger} \Phi_{3} \\
& +\lambda_{\Phi_{3}}\left(\Phi_{3}^{\dagger} \Phi_{3}\right)^{2}+\lambda_{H \Delta}\left(H^{\dagger} H\right)\left(\Delta^{\dagger} \Delta\right)+\lambda_{H \xi}\left(H^{\dagger} H\right)\left(\xi^{\dagger} \xi\right) \\
& +\lambda_{H \Phi_{B-L}}\left(H^{\dagger} H\right)\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right)+\lambda_{H \Phi_{12}}\left(H^{\dagger} H\right)\left(\Phi_{12}^{\dagger} \Phi_{12}\right)+\lambda_{H \Phi_{3}}\left(H^{\dagger} H\right)\left(\Phi_{3}^{\dagger} \Phi_{3}\right) \\
& +\lambda_{\Delta \xi}\left(\Delta^{\dagger} \Delta\right)\left(\xi^{\dagger} \xi\right)+\lambda_{\Delta \xi}^{\prime}\left(\Delta^{\dagger} \xi\right)\left(\xi^{\dagger} \Delta\right)+\lambda_{\Delta \Phi_{B-L}}\left(\Delta^{\dagger} \Delta\right)\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right) \\
& +\lambda_{\Delta \Phi_{12}}\left(\Delta^{\dagger} \Delta\right)\left(\Phi_{12}^{\dagger} \Phi_{12}\right)+\lambda_{\Delta \Phi_{3}}\left(\Delta^{\dagger} \Delta\right)\left(\Phi_{3}^{\dagger} \Phi_{3}\right)+\lambda_{\xi \Phi_{B-L}}\left(\xi^{\dagger} \xi\right)\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right) \\
& +\lambda_{\xi \Phi_{12}}\left(\xi^{\dagger} \xi\right)\left(\Phi_{12}^{\dagger} \Phi_{12}\right)+\lambda_{\xi \Phi_{3}}\left(\xi^{\dagger} \xi\right)\left(\Phi_{3}^{\dagger} \Phi_{3}\right) \\
& +\lambda_{\Phi_{B-L} \Phi_{12}}\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right)\left(\Phi_{12}^{\dagger} \Phi_{12}\right)+\lambda_{\Phi_{B-L}}^{\prime} \Phi_{12}\left(\Phi_{B-L}^{\dagger} \Phi_{12}\right)\left(\Phi_{12} \Phi_{B-L} \Phi_{B}\right) \\
& +\lambda_{\Phi_{B-L} \Phi_{3}}\left(\Phi_{B-L}^{\dagger} \Phi_{B-L}\right)\left(\Phi_{3}^{\dagger} \Phi_{3}\right)+\lambda_{\Phi_{B-L} \Phi_{3}}^{\prime}\left(\Phi_{B-L}^{\dagger} \Phi_{3}\right)\left(\Phi_{3}^{\dagger} \Phi_{B-L}\right) \\
& +\lambda_{\Phi_{12} \Phi_{3}}\left(\Phi_{12}^{\dagger} \Phi_{12}\right)\left(\Phi_{3}^{\dagger} \Phi_{3}\right)+\lambda_{\Phi_{12} \Phi_{3}}^{\prime}\left(\Phi_{12}^{\dagger} \Phi_{3}\right)\left(\Phi_{3}^{\dagger} \Phi_{12}\right)+\mu \Delta^{\dagger} H H \\
& +y \Phi_{B-L}^{2} \Delta^{\dagger} \xi+Y\left(\Phi_{B-L}^{\dagger}\right)^{2} \Phi_{3} \Phi_{12}+\text { h.c. }
\end{aligned}
$$

Two triplet (under $\left.S U(2)_{L}\right)$ scalars $\Delta$ and $\xi$ :

$$
\Delta=\left(\begin{array}{cc}
\frac{\delta^{+}}{\sqrt{2}} & \delta^{++}  \tag{5}\\
\delta^{0} & -\frac{\delta^{+}}{\sqrt{2}}
\end{array}\right) \quad \text { and } \quad \xi=\left(\begin{array}{cc}
\frac{\xi^{+}}{\sqrt{2}} & \xi^{++} \\
\xi^{0} & -\frac{\xi^{+}}{\sqrt{2}}
\end{array}\right)
$$

with $M_{\Delta} \sim 10^{14} \mathrm{GeV}$ and $M_{\xi} \sim \mathrm{TeV} \ll M_{\Delta}$.
The vevs are given as follows:

$$
\langle H\rangle=\frac{v}{\sqrt{2}}\binom{0}{1},\left\langle\Phi_{12}\right\rangle=\frac{v_{12}}{\sqrt{2}},\left\langle\Phi_{B-L}\right\rangle=\frac{v_{B-L}}{\sqrt{2}},\left\langle\Phi_{3}\right\rangle=\frac{v_{3}}{\sqrt{2}} .
$$

However, after electroweak phase transition, $\Delta$ and $\xi$ acquire induced vevs:

$$
\langle\Delta\rangle=\frac{u_{\Delta}}{\sqrt{2}}\left(\begin{array}{ll}
0 & 0 \\
1 & 0
\end{array}\right) \text { and }\langle\xi\rangle=\frac{u_{\xi}}{\sqrt{2}}\left(\begin{array}{ll}
0 & 0 \\
1 & 0
\end{array}\right)
$$

## Neutrino Mass

- In the conventional type-II seesaw, the lagrangian terms that violates Lepton number

$$
f_{\alpha \beta} \Delta L_{\alpha} L_{\beta}+\mu \Delta^{\dagger} H H
$$

where $\Delta$ does not acquire an explicit vacuum expectation value(vev).

- The electro-weak phase transition induces a small vev of $\Delta$ as:

$$
\langle\Delta\rangle=-\frac{\mu\langle H\rangle^{2}}{M_{\Delta}^{2}}
$$

- Thus for $\mu \sim M_{\Delta} \sim 10^{14} \mathrm{GeV}$

$$
M_{\nu}=f\langle\Delta\rangle \simeq f \frac{\langle H\rangle^{2}}{M_{\Delta}} \text { of order } \mathrm{O}(0.1) \mathrm{eV} \text { for } f \sim 1
$$

- Drawback: $M_{\Delta}$ is much larger than the energy attainable at the present generation colliders. Hence lack falsifiability.
- Alternatively, introduce two scalar triplets: $\Delta$ and $\xi$ with

$$
M_{\Delta} \sim 10^{14} \mathrm{GeV} \text { and } M_{\xi} \sim \mathrm{TeV} \ll M_{\Delta}
$$

- Relevant terms:

$$
\mu \Delta^{\dagger} H H+f \xi L L+y \Phi_{B-L}^{2} \Delta^{\dagger} \xi
$$

- At TeV scales $\Phi_{B-L}$ acquires a vev and break $B-L$ symmetry.
- $\left\langle\Phi_{B-L}\right\rangle$ generates a small mixing between $\Delta$ and $\xi$ of the order:

$$
\theta \sim \frac{\left\langle\Phi_{B-L}\right\rangle^{2}}{M_{\Delta}^{2}} \simeq 10^{-18}
$$

- This implies $\rightarrow \xi L L$ coupling can be large
\&
$\xi$ 's coupling with Higgs doublet is highly suppressed.
- $\Delta$ gets decoupled from the low energy effective theory
\&
Same sign dilepton signature of $\xi$ can be studied at colliders.


Figure: Generation of neutrino mass through the modified Type-II seesaw.

- $B-L$ quantum number of $\Delta$ is zero, So it can not generate Majorana masses of neutrinos.

$$
\langle\Delta\rangle=u_{\Delta}=-\frac{\mu v^{2}}{\sqrt{2}\left(2 M_{\Delta}^{2}+\lambda_{H \Delta} v^{2}+\lambda_{\Phi_{B-L} \Delta} v_{B-L}^{2}+\lambda_{\Phi_{12} \Delta} v_{12}^{2}+\lambda_{\Phi_{3} \Delta} v_{3}^{2}\right)}
$$

$$
\langle\xi\rangle=u_{\xi}=-\frac{y v_{B-L}^{2}}{2\left(2 M_{\xi}^{2}+\lambda_{\xi \Phi_{B-L}} v_{B-L}^{2}+\lambda_{\xi H} v^{2}+\lambda_{\xi \Phi_{12}} v_{12}^{2}+\lambda_{\xi \Phi_{3}} v_{3}^{2}\right)} u_{\Delta} .
$$

- If we assume

$$
y v_{B-L}^{2}=M_{\xi}^{2} \sim \lambda_{\xi \Phi_{B-L}} v_{B-L}^{2} \sim \lambda_{\xi H} v^{2} \sim \lambda_{\xi \Phi_{12}} v_{12}^{2} \sim \lambda_{\xi \Phi_{3}} v_{3}^{2}
$$ then we get $u_{\Delta} \simeq u_{\xi}$.

- Majorana mass matrix of the neutrinos

$$
\left(M_{\nu}\right)_{i j}=Y_{i j}^{\xi} u_{\xi}=Y_{i j}^{\xi} \frac{y v_{B-L}^{2}}{2\left(2 M_{\xi}^{2}+\lambda_{\xi \Phi_{B-L}} v_{B-L}^{2}+\lambda_{\xi H} v^{2}+\lambda_{\xi \Phi} v_{12}^{2}+\lambda_{\xi \Phi_{3}} v_{3}^{2}\right)}
$$

## Scalar Masses and Mixing

We parametrize the neutral scalars as:

$$
\begin{gather*}
H^{0}=\frac{v+h+i S}{\sqrt{2}}, \quad \Phi_{B-L}=\frac{v_{B-L}+\phi_{B-L}+i P}{\sqrt{2}} \\
\Phi_{12}=\frac{v_{12}+\phi_{12}+i Q}{\sqrt{2}}, \quad \Phi_{3}=\frac{v_{3}+\phi_{3}+i R}{\sqrt{2}} \\
\delta^{0}=\frac{u_{\Delta}+\delta+i \eta}{\sqrt{2}}, \quad \xi^{0}=\frac{u_{\xi}+\xi+i \rho}{\sqrt{2}} \tag{6}
\end{gather*}
$$

The physical mass square terms of the scalars are then given by

$$
\begin{aligned}
& M_{h}^{2} \simeq 2 \lambda_{H} v^{2}+\frac{\lambda_{H \Phi_{12}} v_{12}^{2}}{2}+\frac{\lambda_{H \Phi_{3}} v_{3}^{2}}{2}+\frac{\lambda_{H \Phi_{B-L}} v_{B-L}^{2}}{2}+\frac{\mu u_{\Delta}}{2} \\
& M_{\delta}^{2} \simeq-\frac{\mu v^{2}}{4 \sqrt{2} u_{\Delta}}-\frac{y v_{B-L}^{2} u_{\xi}}{8 u_{\Delta}} \\
& M_{\xi}^{2} \simeq-\frac{y v_{B-L}^{2} u_{\Delta}}{8 u_{\xi}}
\end{aligned}
$$

$$
\begin{align*}
M_{\phi_{12}}^{2} & \simeq 2 \lambda_{\Phi_{12}} v_{12}^{2}+\frac{\lambda_{H \Phi_{12}} v^{2}}{2}+\frac{\lambda_{\Phi_{12} \Phi_{3}} v_{3}^{2}}{2} \\
& +\frac{\lambda_{\Phi_{12} \Phi_{3}}^{\prime} v_{3}^{2}}{2}+\frac{\lambda_{\Phi_{12} \Phi_{B-L}} v_{B-L}^{2}}{2}+\frac{\lambda_{\Phi_{12} \Phi_{B-L}}^{\prime} v_{B-L}^{2}}{2}+\frac{Y v_{B-L}^{2} v_{3}}{8 v_{12}} \\
M_{\phi_{3}}^{2} & \simeq 2 \lambda_{\Phi_{3}} v_{3}^{2}+\frac{\lambda_{H \Phi_{3}} v^{2}}{2}+\frac{\lambda_{\Phi_{12} \Phi_{3} v_{12}^{2}}^{2}}{2} \\
& +\frac{\lambda_{\Phi \Phi_{3}}^{\prime} v_{12}^{2}}{2}+\frac{\lambda_{\Phi_{3} \Phi_{B-L}}^{2} v_{B-L}^{2}}{2}+\frac{\lambda_{\Phi_{3} \Phi_{B-L}}^{\prime} v_{B-L}^{2}}{2}+\frac{Y v_{B-L}^{2} v_{12}}{8 v_{3}} \\
M_{\phi_{B-L}}^{2} & \simeq 2 \lambda_{\Phi_{B-L}} v_{B-L}^{2}+\frac{\lambda_{H \Phi_{B-L} v^{2}}^{2}}{2}+\frac{\lambda_{\Phi_{12} \Phi_{B-L}} v_{12}^{2}}{2} \\
& +\frac{\lambda_{\Phi_{12} \Phi_{B-L}}^{\prime} v_{12}^{2}}{2}+\frac{\lambda_{\Phi_{3} \Phi_{B-L}} v_{3}^{2}}{2}+\frac{\lambda_{\Phi_{3} \Phi_{B-L}}^{\prime} v_{3}^{2}}{2}+\frac{Y v_{12}^{2} v_{3}}{2} \tag{8}
\end{align*}
$$

- The $Z_{B-L}$ boson acquires mass through the vevs of $\Phi_{B-L}, \Phi_{12}, \Phi_{3}$ which are charged under $U(1)_{B-L}$ and is given by:

$$
\begin{equation*}
M_{Z_{B-L}}^{2}=g_{B-L}^{2}\left(v_{B-L}^{2}+64 v_{12}^{2}+100 v_{3}^{2}\right) . \tag{9}
\end{equation*}
$$

- The assumption for mass heirarchy among scalars is:

$$
M_{\delta} \gg M_{\phi_{B-L}}, M_{\phi_{12}}>M_{h}, M_{\phi_{3}}, M_{\xi}
$$

- Mixing among the CP-even scalars $h, \phi_{3}$ and $\xi$ : $h-\xi$ mixing is highly suppressed.
$\xi-\Phi_{3}$ mixing is of the order $\mathrm{O}\left(\frac{u_{\xi}}{v_{3}}\right)$ and hence negligibly small.
Phenomenologically relevant mixing is between $H$ and $\Phi_{3}$.
- Minimising the scalar potential with respect to $\langle H\rangle=v$ and $\left\langle\Phi_{3}\right\rangle=v_{3}$, we obtain:

$$
\begin{equation*}
v_{3}=\sqrt{\frac{2 \lambda_{H \Phi_{3}} M_{H}^{2}-4 \lambda_{H} M_{\Phi_{3}}^{2}}{4 \lambda_{H} \lambda_{\Phi_{3}}-\lambda_{H \Phi_{3}}^{2}}} \quad \text { and } \quad v=\sqrt{\frac{2 \lambda_{H \Phi_{3}} M_{\Phi_{3}}^{2}-4 \lambda_{\Phi_{3}} M_{H}^{2}}{4 \lambda_{H} \lambda_{\Phi_{3}}-\lambda_{H \Phi_{3}}^{2}}} . \tag{10}
\end{equation*}
$$

The mass matrix of $h$ and $\phi_{3}$ can be written as:

$$
\mathcal{M}^{2}\left(h, \phi_{3}\right)=\left(\begin{array}{cc}
\lambda_{H} v^{2} & \lambda_{H \Phi_{3}} v v_{3}  \tag{11}\\
\lambda_{H \Phi_{3}} v v_{3} & \lambda_{\Phi_{3}} v_{3}^{2}
\end{array}\right) .
$$

The masses of the physical Higgses can:

$$
\begin{aligned}
M_{h_{1}}^{2} & =\frac{1}{2}\left[\left(\lambda_{H} v^{2}+\lambda_{\Phi_{3}} v_{3}^{2}\right)-\sqrt{\left(\lambda_{\Phi_{3}} v_{3}^{2}-\lambda_{H} v^{2}\right)^{2}+4\left(\lambda_{H \Phi_{3}} v v_{3}\right)^{2}}\right] \\
M_{h_{2}}^{2} & =\frac{1}{2}\left[\left(\lambda_{H} v^{2}+\lambda_{\Phi_{3}} v_{3}^{2}\right)+\sqrt{\left(\lambda_{\Phi_{3}} v_{3}^{2}-\lambda_{H} v^{2}\right)^{2}+4\left(\lambda_{H \Phi_{3}} v v_{3}\right)^{2}}\right](12)
\end{aligned}
$$

The mass eigen states of these scalars can be given as:

$$
\begin{align*}
& h_{1}=\cos \beta h+\sin \beta \phi_{3} \\
& h_{2}=-\sin \beta h+\cos \beta \phi_{3} \tag{13}
\end{align*}
$$

where

$$
\begin{equation*}
\tan 2 \beta=\left(\frac{2 \lambda_{H \Phi_{3}} v v_{3}}{\lambda_{\Phi_{3}} v_{3}^{2}-\lambda_{H} v^{2}}\right) . \tag{14}
\end{equation*}
$$

## Right Handed Neutrinos \& Their Interactions

The mass matrix of right handed neutrinos:

$$
-L_{\nu_{R}}^{\text {mass }}=\frac{1}{2}\left(\begin{array}{lll}
\left(\nu_{R 1}\right)^{c} & \overline{\left(\nu_{R 2}\right)^{c}} & \overline{\left(\nu_{R 3}\right)^{c}}
\end{array}\right) \mathcal{M}\left(\begin{array}{l}
\nu_{R 1}  \tag{15}\\
\nu_{R 2} \\
\nu_{R 3}
\end{array}\right)
$$

where

$$
\mathcal{M}=\left(\begin{array}{ccc}
Y_{11} v_{12} & Y_{12} v_{12} & Y_{13} v_{B-L}  \tag{16}\\
Y_{12} v_{12} & Y_{22} v_{12} & Y_{23} v_{B-L} \\
Y_{13} v_{B-L} & Y_{23} v_{B-L} & Y_{33} v_{3}
\end{array}\right)=\left(\begin{array}{cc}
{\left[M_{12}\right]} & {\left[M^{\prime}\right]} \\
{\left[M^{\prime}\right]^{T}} & M_{3}
\end{array}\right)
$$

Here $M_{12}, M^{\prime}, M_{3}$ are:

$$
M_{12}=\left(\begin{array}{ll}
Y_{11} v_{12} & Y_{12} v_{12} \\
Y_{12} v_{12} & Y_{22} v_{12}
\end{array}\right), M^{\prime}=\binom{Y_{13} v_{B-L}}{Y_{23} v_{B-L}}, M_{3}=\left(Y_{33} v_{3}\right)
$$

The right handed neutrino Majorana mass matrix $\mathcal{M}$ can be block-diagonalised using a rotation matrix of the form :

$$
\mathcal{R}_{\theta}=\left(\begin{array}{cc}
\cos \theta\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right] & \sin \theta\left[\begin{array}{l}
1 \\
1
\end{array}\right]  \tag{17}\\
-\sin \theta\left[\begin{array}{ll}
1 & 1
\end{array}\right] & \cos \theta
\end{array}\right)=\left(\begin{array}{ccc}
\cos \theta & 0 & \sin \theta \\
0 & \cos \theta & \sin \theta \\
-\sin \theta & -\sin \theta & \cos \theta
\end{array}\right)
$$

This transformation is orthogonal in the linear over $\theta$ approximation as,

$$
\left(\mathcal{R}_{\theta}\right)^{T} \mathcal{R}_{\theta}=\left(\begin{array}{ccc}
1 & \sin ^{2} \theta & 0  \tag{18}\\
\sin ^{2} \theta & 1 & 0 \\
0 & 0 & 1+\sin ^{2} \theta
\end{array}\right) \simeq I_{3} .
$$

Here $\theta$ is essentially the mixing between $M_{12}$ and $M_{3}$.

$$
\begin{equation*}
\sin 2 \theta \simeq \frac{Y_{13} v_{B-L}}{Y_{11} v_{12}+Y_{12} v_{12}-Y_{33} v_{3}} \simeq \frac{Y_{23} v_{B-L}}{Y_{11} v_{12}+Y_{12} v_{12}-Y_{33} v_{3}} . \tag{19}
\end{equation*}
$$

Then we can completely diagonalise the block-diagonalised matrix $\mathcal{M}$ further by performing another orthogonal transformation using the transformation matrix

$$
\mathcal{R}_{\alpha}=\left(\begin{array}{ccc}
\cos \alpha & \sin \alpha & 0  \tag{20}\\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{array}\right)
$$

where the mixing angle $\alpha$ is given by

$$
\begin{equation*}
\tan 2 \alpha \simeq \frac{Y_{12} v_{12}}{Y_{11} v_{12}-Y_{22} v_{12}} \tag{21}
\end{equation*}
$$

Thus the mass eigen states of the right handed neutrinos can be written as:

$$
\left(\begin{array}{l}
N_{1 R}  \tag{22}\\
N_{2 R} \\
N_{3 R}
\end{array}\right)=\left(\begin{array}{ccc}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
\cos \theta & 0 & \sin \theta \\
0 & \cos \theta & \sin \theta \\
-\sin \theta & -\sin \theta & \cos \theta
\end{array}\right)\left(\begin{array}{l}
\nu_{R 1} \\
\nu_{R 2} \\
\nu_{R 3}
\end{array}\right)
$$

Here it is worth noting that the matrix $\mathcal{R}=\mathcal{R}_{\alpha} \mathcal{R}_{\theta}$ is also orthogonal (i.e., $\mathcal{R}^{T} \mathcal{R}=I$ ) in the linear over $\theta$ approximation.

- For simplicity we assume a strong hierarchy between $\nu_{R 3}$ and $\nu_{R 1}$, $\nu_{R 2}$. This implies that $\sin \theta \rightarrow 0$.
- In this limit, $\nu_{R 3}$ completely decouples from $\nu_{R 1}$ and $\nu_{R 2}$.
- As a result the diagonalisation of above mass matrix gives the mass eigen values corresponding to the states $N_{1 R}, N_{2 R}$ and $N_{3 R}$ as

$$
\begin{align*}
M_{1,2} & =\frac{1}{2}\left[\left(Y_{11} v_{12}+Y_{22} v_{12}\right) \pm \sqrt{\left.\left(Y_{11} v_{12}-Y_{22} v_{12}\right)^{2}+4\left(Y_{12} v_{12}\right)^{2}\right]}\right. \\
M_{3} & =Y_{33} v_{3} . \tag{23}
\end{align*}
$$

## Interactions

The interaction terms of the right handed neutrinos with $Z_{B-L}$ in the mass eigen basis can be written as :

$$
\begin{align*}
\mathcal{L}_{Z_{B-L}} & =g_{B-L}\left[\left\{\left(4 \cos ^{2} \theta-5 \sin ^{2} \theta\right)(1+\sin 2 \alpha)\right\} \overline{N_{1 R}} \gamma^{\mu} N_{1 R}\right. \\
& +\left\{16 \sin ^{2} \theta-5 \cos ^{2} \theta\right\} \overline{N_{3 R}} \gamma^{\mu} N_{3 R} \\
& +\left\{\left(4 \cos ^{2} \theta-5 \sin ^{2} \theta\right)(1-\sin 2 \alpha)\right\} \overline{N_{2 R}} \gamma^{\mu} N_{2 R} \\
& +\cos 2 \alpha\left\{4 \cos ^{2} \theta-5 \sin ^{2} \theta\right\}\left(\overline{N_{1 R}} \gamma^{\mu} N_{2 R}+\overline{N_{2 R}} \gamma^{\mu} N_{1 R}\right) \\
& -\frac{13}{2} \sin 2 \theta\{\cos \alpha+\sin \alpha\}\left(\overline{N_{1 R}} \gamma^{\mu} N_{3 R}+\overline{N_{3 R}} \gamma^{\mu} N_{1 R}\right) \\
& \left.-\frac{13}{2} \sin 2 \theta\{\cos \alpha-\sin \alpha\}\left(\overline{N_{2 R}} \gamma^{\mu} N_{3 R}+\overline{N_{3 R}} \gamma^{\mu} N_{2 R}\right)\right]\left(Z_{B-L}\right)_{\mu} \tag{24}
\end{align*}
$$

$Y_{33} \overline{\left(\nu_{R 3}\right)}{ }^{c} \Phi_{3} \nu_{R 3}$
$=Y_{33}\left[\sin ^{2} \theta(1+\sin 2 \alpha) \overline{\left(N_{1 R}\right)^{c}} \phi_{3} N_{1 R}+\sin ^{2} \theta(1-\sin 2 \alpha) \overline{\left(N_{2 R}\right)^{c}} \phi_{3} N_{2 R}\right.$
$+\cos ^{2} \theta \overline{\left(N_{3 R}\right)^{c}} \phi_{3} N_{3 R}+\sin ^{2} \theta \cos 2 \alpha\left(\overline{\left(N_{1 R}\right)^{c}} \phi_{3} N_{2 R}+\overline{\left(N_{2 R}\right)^{c}} \phi_{3} N_{1 R}\right)$
$+\frac{1}{2} \sin 2 \theta(\cos \alpha+\sin \alpha)\left(\overline{\left(N_{1 R}\right)^{c}} \phi_{3} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi_{3} N_{1 R}\right)$
$\left.+\frac{1}{2} \sin 2 \theta(\cos \alpha-\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} \phi_{3} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi_{3} N_{2 R}\right)\right]$.

$$
\begin{align*}
& Y_{33} \overline{\left(\nu_{R 3}\right)^{c}} \Phi_{3} \nu_{R 3} \\
& =Y_{33} \sin \beta\left[\sin ^{2} \theta(1+\sin 2 \alpha) \overline{\left(N_{1 R}\right)^{c}} h_{1} N_{1 R}+\sin ^{2} \theta(1-\sin 2 \alpha) \overline{\left(N_{2 R}\right)^{c}} h_{1} N_{2 R}\right. \\
& +\cos ^{2} \theta \overline{\left(N_{3 R}\right)^{c}} h_{1} N_{3 R}+\sin ^{2} \theta \cos 2 \alpha\left(\overline{\left(N_{1 R}\right)^{c}} h_{1} N_{2 R}+\overline{\left(N_{2 R}\right)^{c}} h_{1} N_{1 R}\right) \\
& +\frac{1}{2} \sin 2 \theta(\cos \alpha+\sin \alpha)\left(\overline{\left(N_{1 R}\right)^{c}} h_{1} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} h_{1} N_{1 R}\right) \\
& \left.+\frac{1}{2} \sin 2 \theta(\cos \alpha-\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} h_{1} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} h_{1} N_{2 R}\right)\right] \\
& +Y^{33} \cos \beta\left[\sin ^{2} \theta(1+\sin 2 \alpha) \overline{\left(N_{1 R}\right)^{c}} h_{2} N_{1 R}+\sin ^{2} \theta(1-\sin 2 \alpha) \overline{\left(N_{2 R}\right)^{c}} h_{2} N_{2}\right. \\
& +\cos ^{2} \theta \overline{\left(N_{3 R}\right)^{c}} h_{2} N_{3 R}+\sin ^{2} \theta \cos 2 \alpha\left(\overline{\left(N_{1 R}\right)^{c}} h_{2} N_{2 R}+\overline{\left(N_{2 R}\right)^{c}} h_{2} N_{1 R}\right) \\
& +\frac{1}{2} \sin 2 \theta(\cos \alpha+\sin \alpha)\left(\overline{\left(N_{1 R}\right)^{c}} h_{2} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} h_{2} N_{1 R}\right) \\
& \left.+\frac{1}{2} \sin 2 \theta(\cos \alpha-\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} h_{2} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} h_{2} N_{2 R}\right)\right] \tag{26}
\end{align*}
$$

## Dark Matter

- At a TeV scale, the $U(1)_{\mathrm{B}-\mathrm{L}} \rightarrow Z_{2}$.
- RHNs: odd under the $Z_{2}$ symmetry, (while all other particles even) $\rightarrow$ lightest RHN is the viable candidate of DM.
- $N_{3}=\left(N_{3 R}+N_{3 R}^{c}\right) / \sqrt{2}$ is the candidate of DM.


Figure: DM annihilation channels.

## Relic Density of DM

- The relic density of DM is calculated using micrOMEGAs.


Figure: Relic density of DM as a function of its mass. Contribution to the relic density of DM is only through its annihilation channels. The Green horizontal line shows the observed relic density of DM.

## Relic Density of DM



## Direct Detection



Figure: The spin-independent scattering cross-section of DM-nucleon via Higgs portal.

- The spin-independent scattering of DM is possible through $\phi_{3}-h$ mixing.
- The spin-independent elastic scattering cross-section of DM per nucleon can be expressed as:

$$
\begin{equation*}
\sigma_{S I}^{h_{1} h_{2}}=\frac{\mu_{r}^{2}}{\pi A^{2}}\left[Z f_{p}+(A-Z) f_{n}\right]^{2} \tag{27}
\end{equation*}
$$

where $\mu_{r}=M_{N_{3}} m_{n} /\left(M_{N_{3}}+m_{n}\right)$ :reduced mass
$f_{p} \& f_{n}$ : The interaction strengths of proton and neutron with DM

$$
\begin{equation*}
f_{p, n}=\sum_{q=u, d, s} f_{T_{q}}^{p, n} \alpha_{q} \frac{m_{p, n}}{m_{q}}+\frac{2}{27} f_{T G}^{p, n} \sum_{q=c, t, b} \alpha_{q} \frac{m_{p, n}}{m_{q}}, \tag{28}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{q}=\frac{Y_{33}^{e f f} \sin 2 \beta}{2 \sqrt{2}}\left(\frac{m_{q}}{v}\right)\left[\frac{1}{M_{h_{2}}^{2}}-\frac{1}{M_{h_{1}}^{2}}\right] \tag{29}
\end{equation*}
$$

- The spin-independent cross-section can be re-expressed as:

$$
\begin{align*}
\sigma_{\mathrm{SI}}^{h_{1} h_{2}} & =\frac{\mu_{r}^{2}}{\pi A^{2}}\left(\frac{Y_{33}^{e f f} \sin 2 \beta}{2 \sqrt{2}}\right)^{2}\left[\frac{1}{M_{h_{2}^{2}}}-\frac{1}{M_{h_{1}^{2}}}\right]^{2} \\
& \times\left[Z\left(\frac{m_{p}}{v}\right)\left(f_{T u}^{p}+f_{T d}^{p}+f_{T s}^{p}+\frac{2}{9} f_{T G}^{p}\right)\right. \\
& +(A-Z)\left(\frac{m_{n}}{v}\right)\left(f_{T u}^{n}+f_{T d}^{n}+f_{T s}^{n}+\frac{2}{9} f_{T G}^{n}\right)^{2} \tag{30}
\end{align*}
$$

- The most stringent bound on direct detection come from the XENON1T where it rules out spin-independent cross-section down to $\sigma_{\mathrm{SI}} \approx 10^{-47} \mathrm{~cm}^{2}$.




## Collider Signatures

- If $M_{Z_{B-L}}<M_{\xi}, M_{\Phi_{B-L}}, M_{\Phi_{12}}, M_{\Phi_{3}}$, dominant decay channel $Z_{B-L} \rightarrow N_{3 R} \overline{N_{3 R}}$ or $Z_{B-L} \rightarrow N_{i R} \overline{N_{i R}}$ with $\mathrm{i}=1,2$
- Also decay of $Z_{B-L}$ to a pair of SM leptons will be dominant as compared to it's decay to quarks.
- But alternatively if $M_{Z_{B-L}}>M_{\xi}, M_{\Phi_{B-L}}, M_{\Phi_{12}}, M_{\Phi_{3}}$ then the total decay width of $Z_{B-L}$ significantly increases as it additionally decays to $\phi_{3} \phi_{3}^{*}, \phi_{12} \phi_{12}^{*}, \xi^{ \pm \pm} \xi^{\mp \mp}, \xi^{ \pm} \xi^{\mp}, \xi^{0} \xi^{0 *}$ and $\phi_{B-L} \phi_{B-L}^{*}$.
- Depending on the relative magnitudes of $M_{Z_{B-L}}$ and $M_{\xi^{ \pm \pm}}$, the production cross-section of $\xi^{ \pm \pm}$and $Z_{B-L}$ will vary.
- If $M_{Z_{B-L}}>2 M_{\xi^{ \pm \pm}}$, then at LHC $\xi^{ \pm \pm}$particles can be pair produced via $Z_{B-L}$ decay with a significant branching fraction. But if $M_{Z_{B-L}}<2 M_{\xi^{ \pm \pm}}$, then at LHC $\xi^{ \pm \pm}$particles can be produced via Drell-Yan process $\left(q \bar{q} \rightarrow \xi^{ \pm \pm} \xi^{\mp \mp}\right)$
- The $\xi^{ \pm \pm}$particle can decay to two like-sign charged leptons $\left(I_{\alpha}^{+} I_{\beta}^{+}, \alpha, \beta=e, \mu, \tau\right)$ or to $W^{+} W^{+}$. Though it can also decay to $W^{+} \xi^{+}$, but this decay rate $\left[\Gamma\left(\xi^{++} \rightarrow W^{+} \xi^{+}\right)\right.$] is phase space suppressed since mass of $\xi^{+}$is of the same order of mass of $\xi^{++}$.
- It is worth mentioning that for small $u_{\xi}, \Gamma\left(\xi^{++} \rightarrow I_{\alpha}^{+} I_{\beta}^{+}\right)$dominates over $\Gamma\left(\xi^{++} \rightarrow W^{+} W^{+}\right)$:

$$
\begin{equation*}
\Gamma\left(\xi^{++} \rightarrow I_{\alpha}^{+} I_{\beta}^{+}\right)=\frac{M_{\xi^{++}}}{4 \pi u_{\xi}^{2}\left(1+\delta_{\alpha \beta}\right)}\left|\left(M_{\nu}\right)_{\alpha \beta}\right|^{2} \tag{31}
\end{equation*}
$$

and

$$
\begin{align*}
\Gamma\left(\xi^{++} \rightarrow W^{+} W^{+}\right)= & g^{4} u_{\xi}^{2} M_{\xi^{++}}^{3}\left[1-4\left(\frac{M_{W}}{M_{\xi^{++}}}\right)^{2}\right]^{\frac{1}{2}} \\
& {\left[1-4\left(\frac{M_{W}}{M_{\xi^{++}}}\right)^{2}+\left(\frac{M_{W}}{M_{\xi^{++}}}\right)^{4}\right] } \tag{32}
\end{align*}
$$

- This can be well analysed by plotting contours of the ratio

$$
\begin{equation*}
R=\frac{\Gamma\left(\xi^{++} \rightarrow I_{\alpha}^{+} I_{\beta}^{+}\right)}{\Gamma\left(\xi^{++} \rightarrow W^{+} W^{+}\right)} \tag{33}
\end{equation*}
$$



Figure: The contour for R in the plane of $\log _{10}\left(u_{\xi}\right)$ and $M_{\xi}$.

- This like-sign dilepton channel of $\xi^{++}$is almost background free and can be seen at LHC. The mass of $\xi^{++}$is approximately about the invariant mass of the two like sign leptons.


## Conclusion

- In this paper we studied a gauged $U(1)_{B-L}$ extension of a TeV scale type-II seesaw. We implemented it by introducing two scalar triplets

$$
\Delta\left(M_{\Delta} \simeq 10^{14} \mathrm{GeV}\right) \text { and } \xi\left(M_{\xi} \leq 1 \mathrm{TeV}\right) \text { with } M_{\xi} \ll M_{\Delta}
$$

- Even though there is orders of magnitude difference between the masses of these two scalars but their contribution to the neutrino mass is identical because of the small mixing between them which arises at TeV scale when $U(1)_{B-L}$ is broken by the vev of the singlet scalar $\Phi_{B-L}$.
- $\Delta$ being super heavy is decoupled from the low energy effective theory however the decay of leptophillic $\xi^{++}$is almost background free and can be studied at LHC.
- To make the theory free from anomalies we introduced three right handed neutrino fields $\nu_{R_{i}}(i=1,2,3)$ which have charges under $U(1)_{B-L}$ as $-4,-4,+5$ respectively.
- The $U(1)_{B-L}$ charges of right handed neutrinos precludes any coupling with the SM fermions. The lightest one among these right handed neutrinos is the DM candidate of the model whose stability is guaranteed by a remnant $Z_{2}$ symmetry after $U(1)_{B-L}$ breaking.
- We showed a combined parameter space which allows both the relic and direct detection bounds.


## Thank You

## Symmetries and Anomalies

Whatever be the Gauge symmetry chosen for costructing a model, the model should be Anomaly-free.

## Anomalies:

- Nöether's Theorem : corresponding to a continuous symmetry there exists a four vector, called the Nöether's current whose divergence is zero ( Originally proved in CFT)
- But when quantum corrections are included, there are cases where the theorem no longer holds.

$$
\partial_{\mu} J^{\mu} \neq 0
$$

- Such case where a symmetry of the classical lagrangian is broken by quantum corrections is called Anomaly.


## Gauge Anomalies

- In Gauge theories the conservation of the axial vector current is incompatible with gauge invariance and Radiative corrections supply a nonzero operator that appears on the RHS inplace of Zero. (Adler-Bell-Jackiw Anomaly)
- In GWS Theory the Gauge bosons couple to fermions in a chiral fashion.
- But these chiral couplings pesent a potential problem at One-loop correction.
- In a gauge theory in which gauge bosons couple to chiral currents, there appears triangle diagrams with the Axial current and two gauge currents at it's vertices, and then the Axial current acquires a non-zero divergence.


## Gauge Anomalies



- In general, if the lines connected to the triangle diagrams correspond to the currents $\tilde{J}_{\mu}^{A}, J_{\nu}^{b}, J_{\lambda}^{c}$ where b,c are gauge indices; then

$$
\partial_{\mu} \tilde{J}_{\mu}^{A}=\frac{g_{b} g_{c}}{16 \pi^{2}} \operatorname{Tr}\left(T_{A}\left[T_{b}, T_{c}\right]_{+}\right) F_{b}^{\alpha \beta} \tilde{F}_{\alpha \beta}^{c}
$$

- In chiral gauge theory, The anomalous term of triangle diagram is proportional to the group theoretic invariant $\mathcal{A}_{A b c}=\operatorname{Tr}\left(\gamma^{5} T_{A}\left[T_{b}, T_{c}\right]_{+}\right)=\operatorname{Tr}\left(T_{A}\left[T_{b}, T_{c}\right]_{+}\right)_{R}-\operatorname{Tr}\left(T_{A}\left[T_{b}, T_{c}\right]_{+}\right)_{L}$


## Gauge Anomalies



- All of these anomalies must vanish for the theory to be consistent. i.e. $\mathcal{A}_{A b c}=0$

We know $\quad \operatorname{Tr}\left(\tau_{a}\right)=0 \quad \& \operatorname{Tr}\left(\lambda_{a}\right)=0$

$$
\operatorname{Tr}\left(\tau_{a}, \tau_{b}\right)=2 \delta_{a b} \quad \& \quad \operatorname{Tr}\left(\lambda_{a}, \lambda_{b}\right)=2 \delta_{a b}
$$

## Gauge Anomalies

## Anomaly Cancellation for Gauge currents

- $[S U(3)]^{2} \times[S U(2)] \propto \operatorname{Tr}\left(\tau_{a}\left[\lambda_{b}, \lambda_{c}\right]_{+}\right)=0$
- $[S U(3)] \times[S U(2)]^{2} \propto \operatorname{Tr}\left(\lambda_{a}\left[\tau_{b}, \tau_{c}\right]_{+}\right)=0$
- $[S U(2)]^{3} \propto \operatorname{Tr}\left(\tau_{a}\left[\tau_{b}, \tau_{c}\right]_{+}\right)=0$
- $[S U(3)] \times[S U(2)] \times[U(1)] \propto \operatorname{Tr}\left(\lambda_{a}\left[\tau_{b}, Y\right]_{+}\right)=0$
- $[S U(3)] \times[U(1)]^{2} \propto \operatorname{Tr}\left(Y^{2} \lambda_{a}\right)=0$
- $[S U(2)] \times[U(1)]^{2} \propto \operatorname{Tr}\left(Y^{2} \tau_{a}\right)=0$
- $[S U(3)]^{2} \times[U(1)] \propto \operatorname{Tr}\left(Y\left[\lambda_{b}, \lambda_{c}\right]_{+}\right)=2 Y_{Q}-Y_{u}-Y_{d}$
- $[S U(2)]^{2} \times[U(1)] \propto \operatorname{Tr}\left(Y\left[\tau_{b}, \tau_{c}\right]_{+}\right)=3 Y_{Q}+Y_{L}$
- $[U(1)]^{3} \propto \operatorname{Tr}\left(Y^{3}\right)=2 Y_{L}^{3}-Y_{e}^{3}+6 Y_{Q}^{3}-3 Y_{u}^{3}-3 Y_{d}^{3}$
- $[\text { Grav. }]^{2} \times[U(1)] \propto \operatorname{Tr}(Y)=2 Y_{L}-Y_{e}+6 Y_{Q}-3 Y_{u}-3 Y_{d}$

The Yukawa interactions in the mass basis can be written as:

$$
\begin{align*}
Y_{a b} \overline{\left(\nu_{R a}\right)^{c}} \Phi \nu_{R b} & =Y\left[\cos ^{2} \theta(1+\sin 2 \alpha) \overline{\left(N_{1 R}\right)^{c}} \phi N_{1 R}\right. \\
& +\cos ^{2} \theta(1-\sin 2 \alpha) \overline{\left(N_{2 R}\right)^{c}} \phi N_{2 R} \\
& +4 \sin ^{2} \theta \overline{\left(N_{3 R}\right)^{c} \phi N_{3 R}} \\
& +\cos ^{2} \theta \cos 2 \alpha\left(\overline{\left(N_{1 R}\right)^{c}} \phi N_{2 R}+\overline{\left(N_{2 R}\right)^{c}} \phi N_{1 R}\right) \\
& -\sin 2 \theta(\cos \alpha+\sin \alpha)\left(\overline{\left(N_{1 R}\right)^{c}} \phi N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi N_{1 R}\right) \\
& \left.-\sin 2 \theta(\cos \alpha-\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} \phi N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi N_{2 R}\right)\right] \tag{34}
\end{align*}
$$

Similarly
$Y_{a 3} \overline{\left(\nu_{R a}\right)^{c}}{ }^{c} \Phi_{B-L} \nu_{R 3}$
$=Y\left[\sin 2 \theta(1+\sin 2 \alpha) \overline{\left(N_{1 R}\right)^{c}} \phi_{B-L} N_{1 R}+\sin 2 \theta(1-\sin 2 \alpha) \overline{\left(N_{2 R}\right)^{c}} \phi_{B-L} N_{2 R}\right.$
$-2 \sin 2 \theta \overline{\left(N_{3 R}\right)^{c}} \phi_{B-L} N_{3 R}$
$+\frac{1}{2} \sin 2 \theta(\cos 2 \alpha+\sin 2 \alpha)\left(\overline{\left(N_{1 R}\right)^{c}} \phi_{B-L} N_{2 R}+\overline{\left(N_{2 R}\right)^{c}} \phi_{B-L} N_{1 R}\right)$
$+\left(\cos ^{2} \theta-2 \sin ^{2} \theta\right)(\cos \alpha-\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} \phi_{B-L} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi_{B-L} N_{2 R}\right)$
$\left.+\left(\cos ^{2} \theta-2 \sin ^{2} \theta\right)(\cos \alpha+\sin \alpha)\left(\overline{\left(N_{2 R}\right)^{c}} \phi_{B-L} N_{3 R}+\overline{\left(N_{3 R}\right)^{c}} \phi_{B-L} N_{2 R}\right)\right]$.
(35)

