"la Caixa" Foundation

Institut de Física d'Altes Energies

Phase Transitions in the Early Universe

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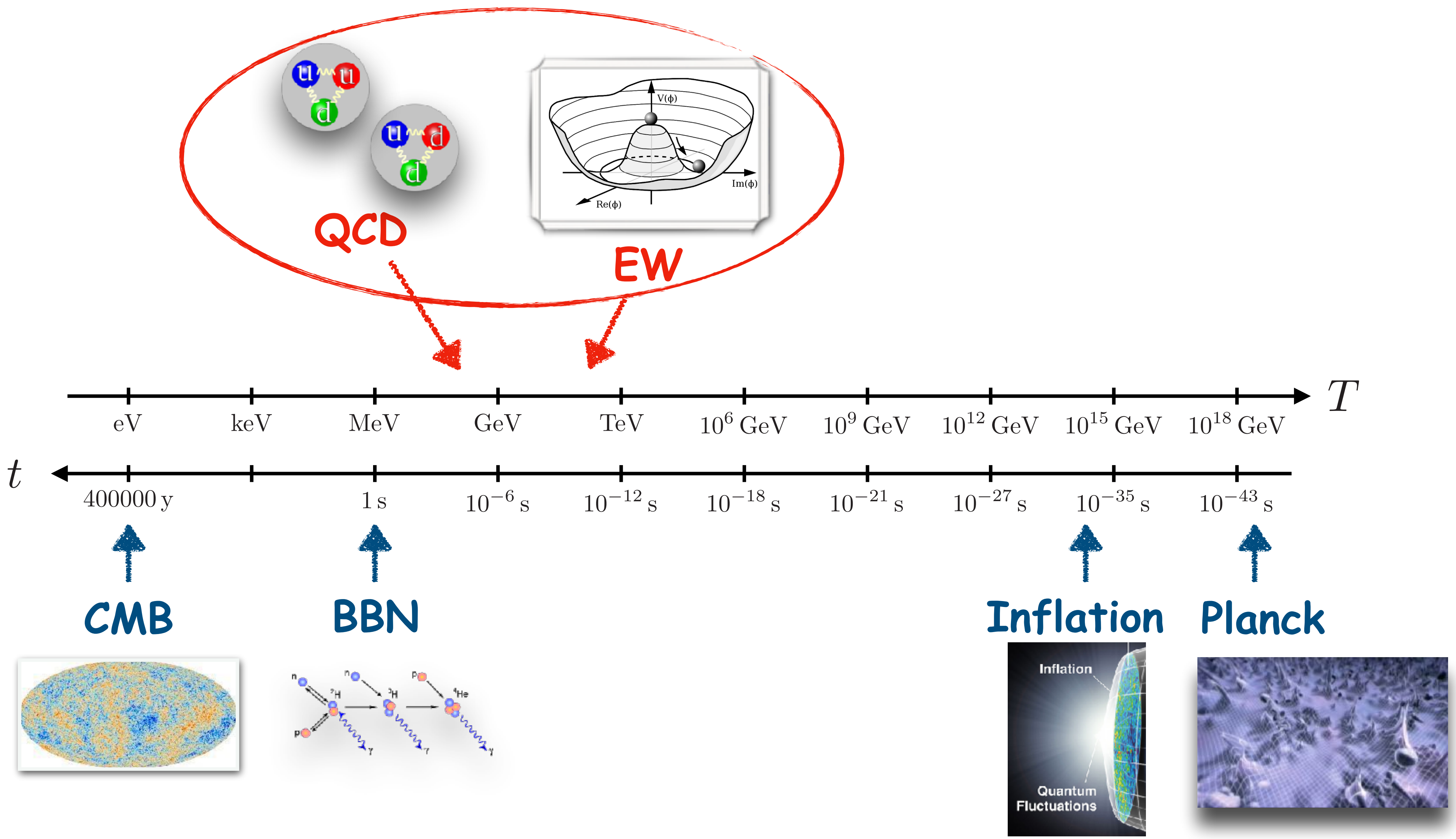
26/02/2021

based on JHEP 12 (2019) 149 *arXiv:1909.07894,*
JHEP 04 (2020) 025, *arXiv:1912.06139*

Thermal History of the Universe

Phase transitions are important events in the evolution of the Universe

- the SM predicts two of them (QCD confinement and EW symmetry breaking)



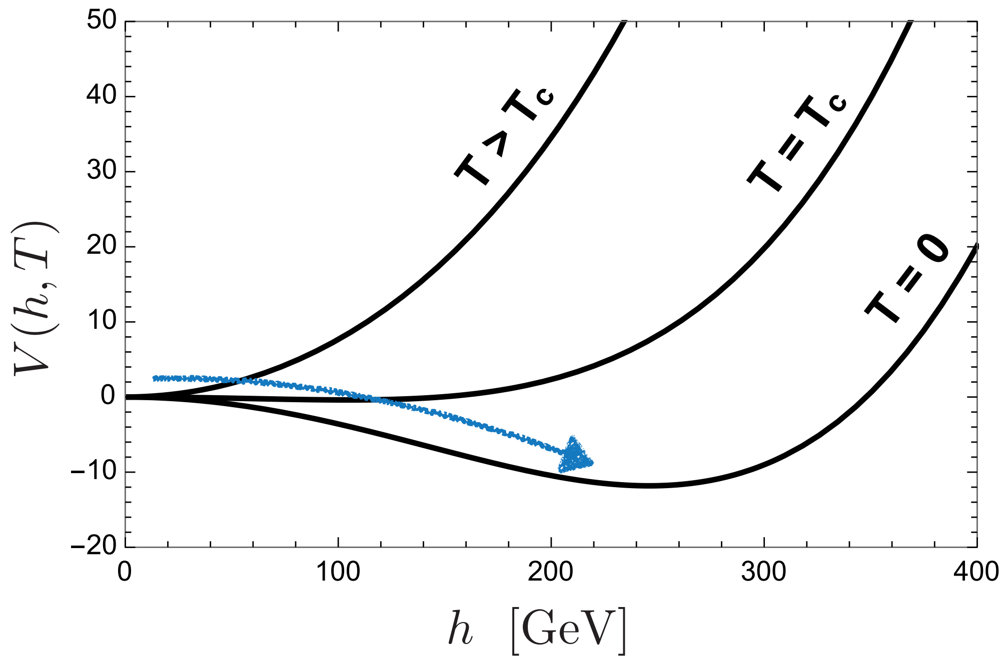
Phase transitions in the SM

In the SM the QCD and EW PhT are extremely weak

→ the two phases are smoothly connected (cross over)

- no barrier is present in the effective potential

- the field gently "rolls down" towards the global minimum when $T < T_c$



- no strong breaking of thermal equilibrium

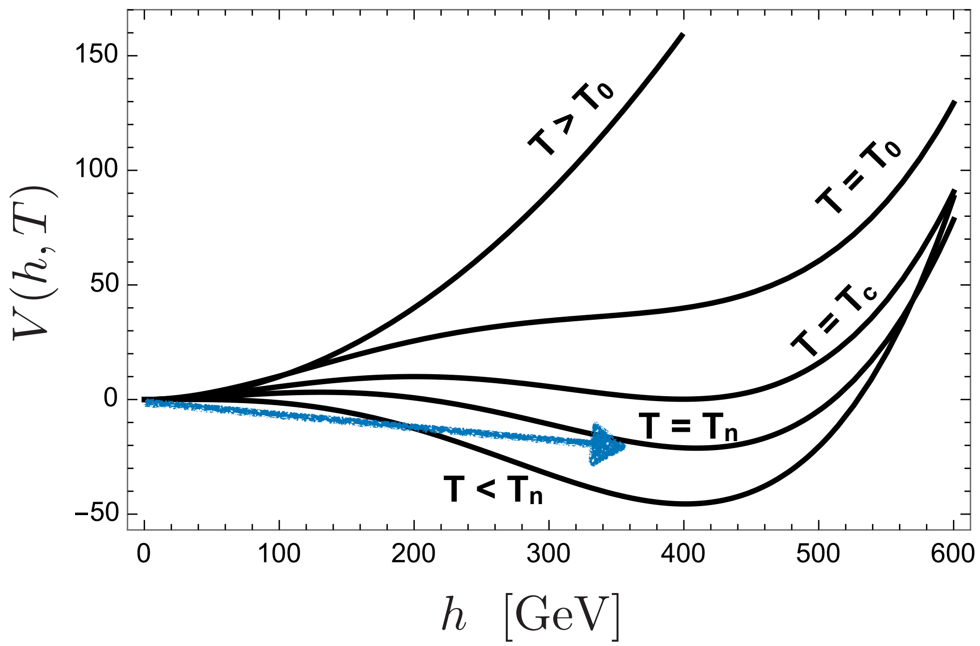
- no distinctive experimental signatures

Phase transitions beyond the SM

New physics may provide **first order** phase transitions

- a barrier in the potential may be generated from tree-level deformations, thermal or quantum effects

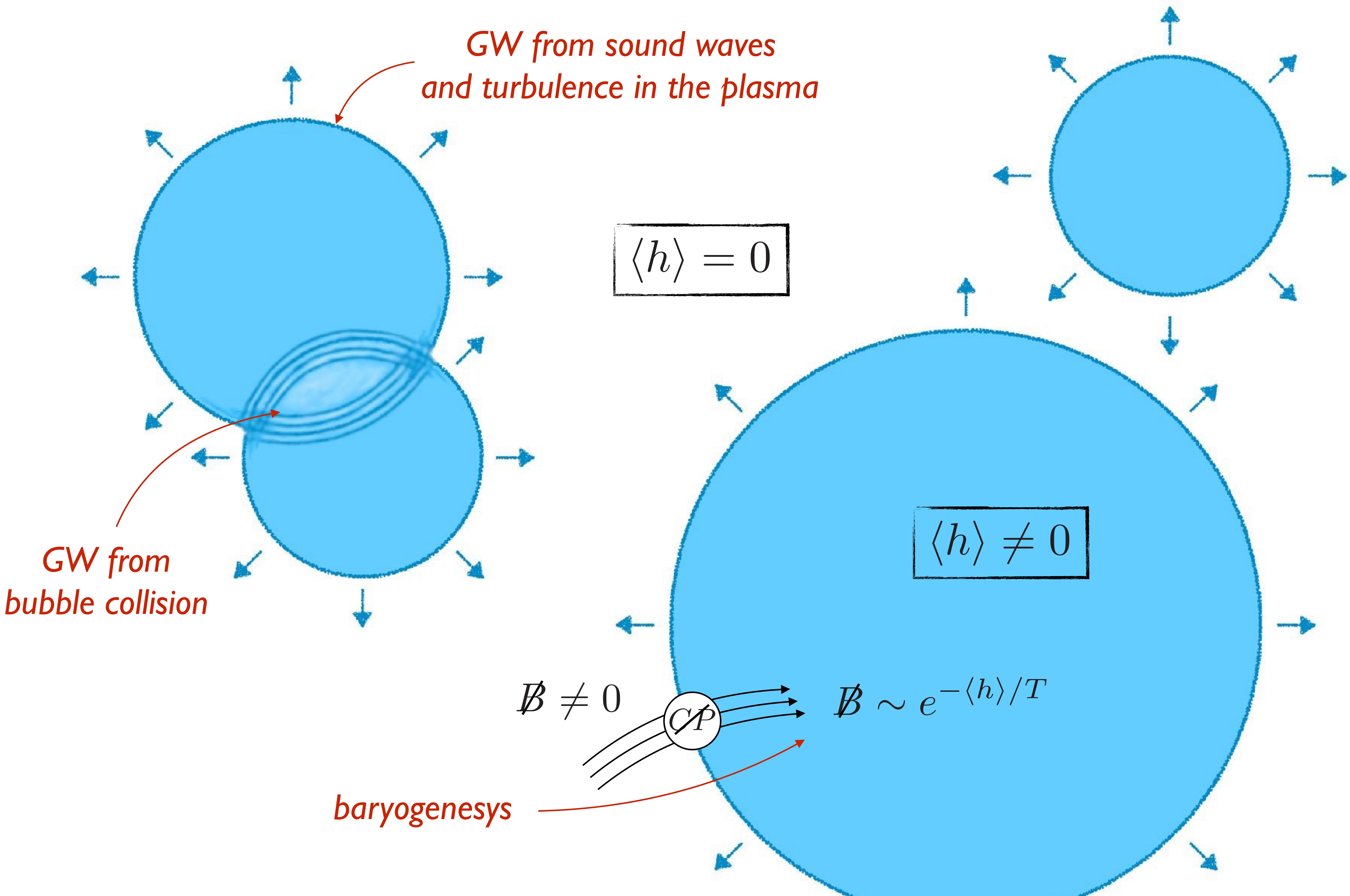
- the field tunnels from false to true minimum at $T = T_n < T_c$



- the transition proceeds through bubble nucleation
- significant breaking of thermal equilibrium
- interesting experimental signatures (eg. gravitational waves)

Bubble nucleation

Bubble dynamics can produce **gravitational waves** and **baryogenesys**

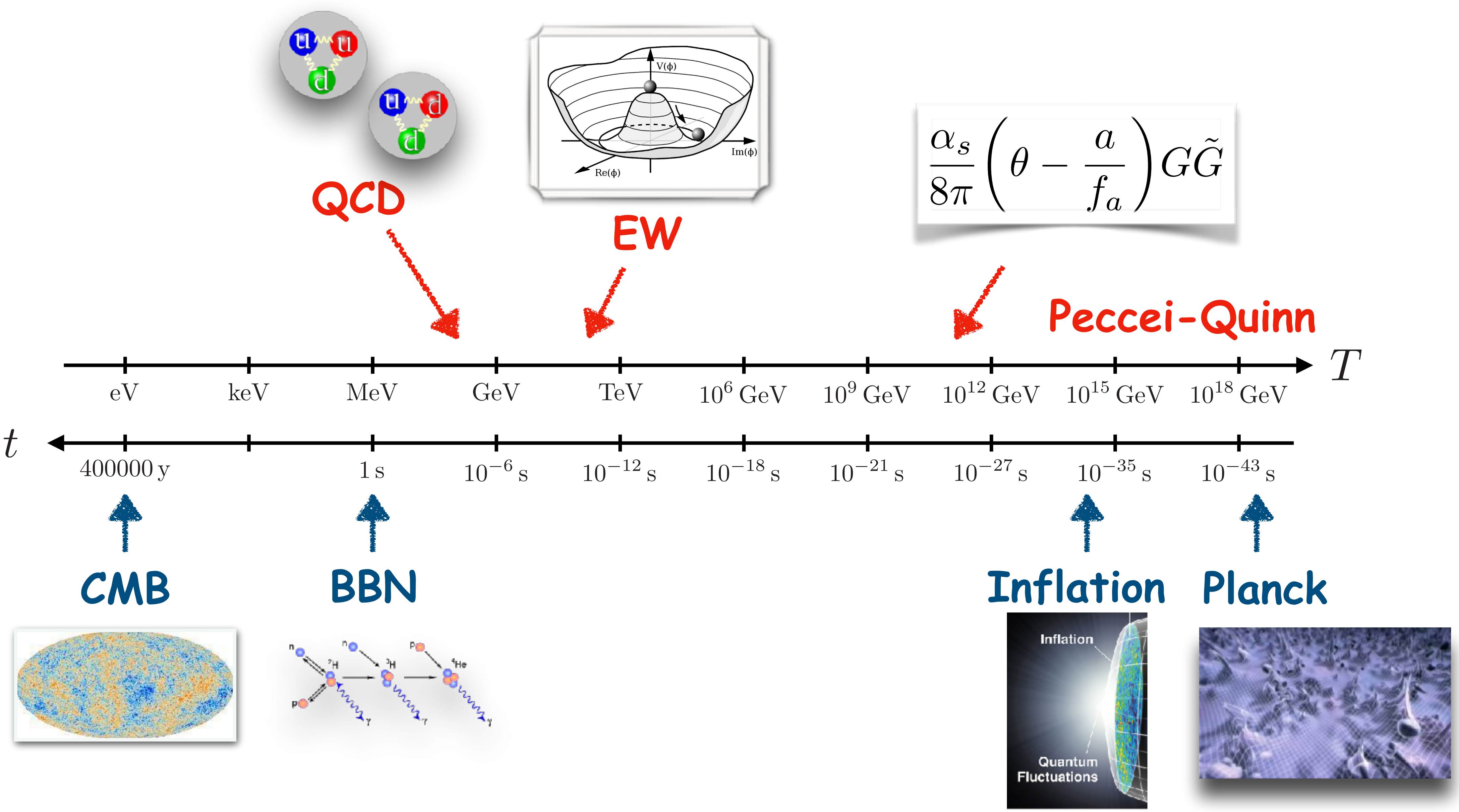


Thermal History of the Universe

Additional phase transitions could be present due to **new-physics**

well motivated example:

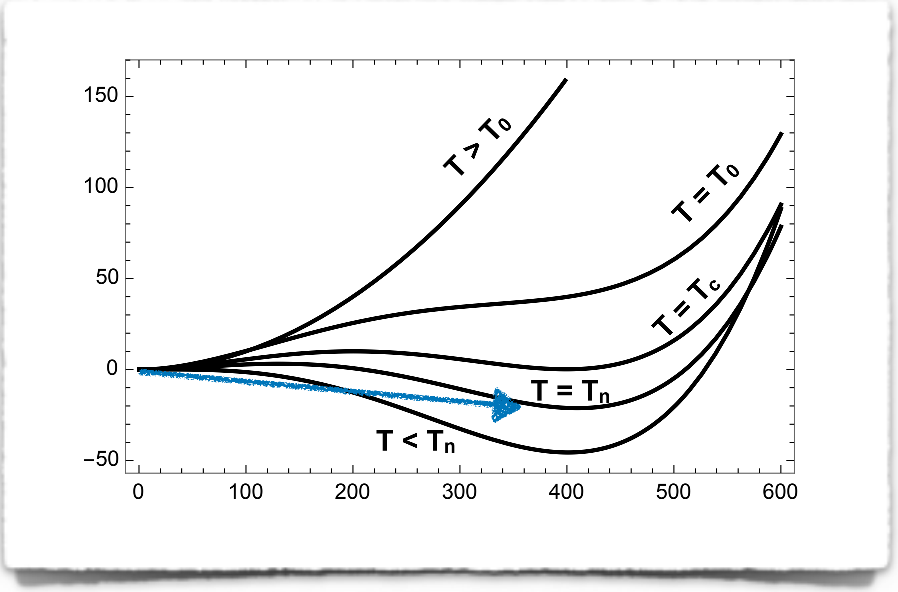
- Peccei-Quinn symmetry breaking connected to QCD axion



How to get a first-order PhT

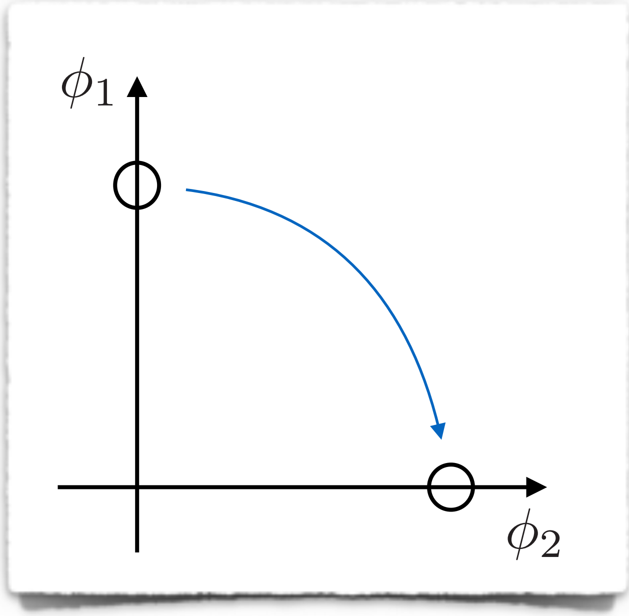
I. **"Single field"** transitions

- barrier coming from:
  - quantum corrections due to additional fields
  - thermal effects

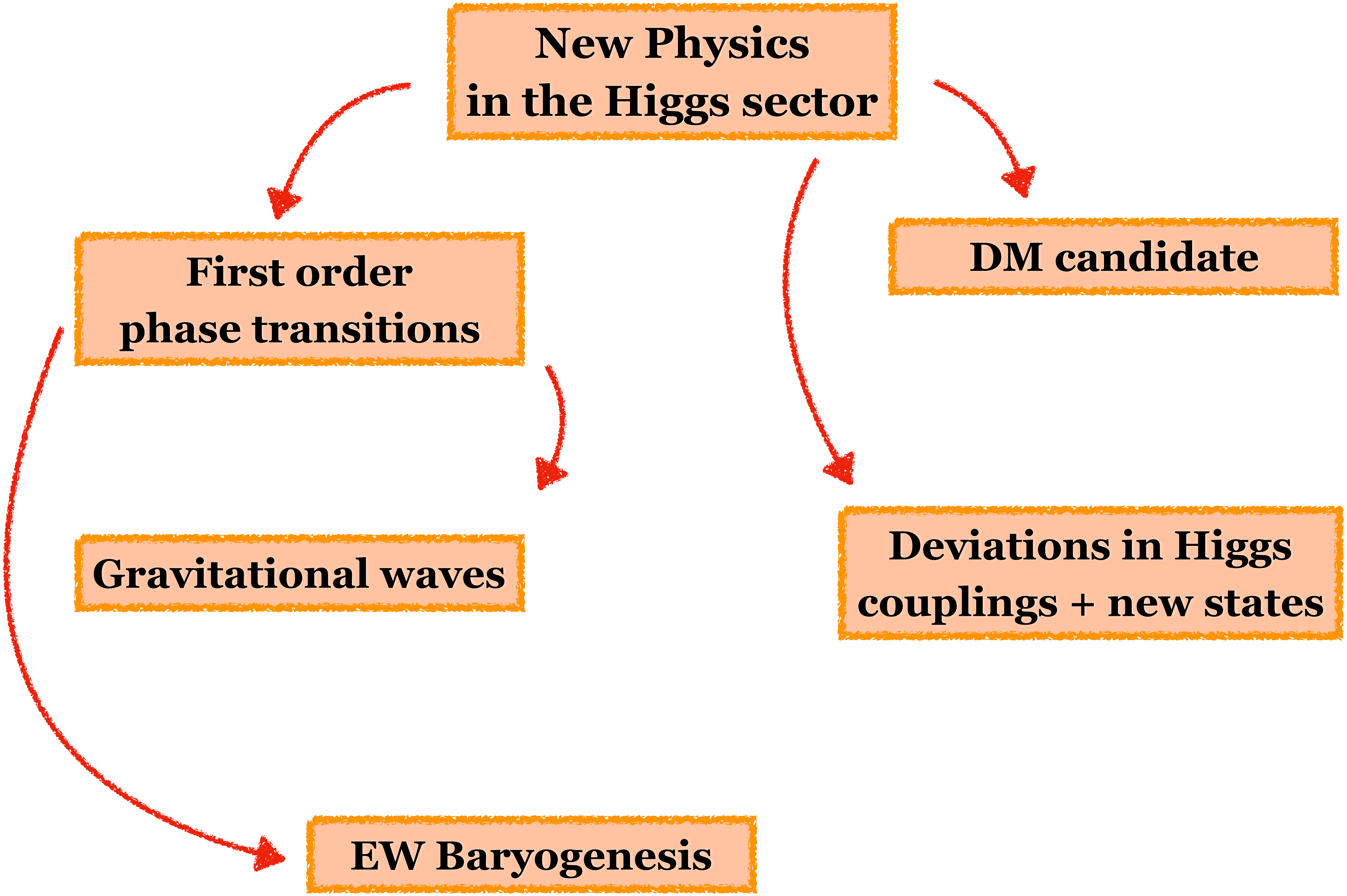


II. **"Multiple field"** transitions

- barrier can be present already at tree-level and T=0
- minima in different directions in field space



Extended Higgs sectors


New Physics
in the Higgs sector
First order
phase transitions
DM candidate
Gravitational waves
Deviations in Higgs
couplings + new states
EW Baryogenesis

New Physics
in the Higgs sector

First order
phase transitions

DM candidate

Collider - cosmology synergy

Gravitational waves

testable at
future interferometers

Deviations in Higgs
couplings + new states

testable at
future colliders

EW Baryogenesis

SM + singlet scalar

Higgs + singlet scalar potential (Z_2 symmetric)
in the high-temperature limit

$$V(h,\eta,T) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2 + \left(c_h\frac{h^2}{2} + c_\eta\frac{\eta^2}{2}\right)T^2$$

with thermal masses

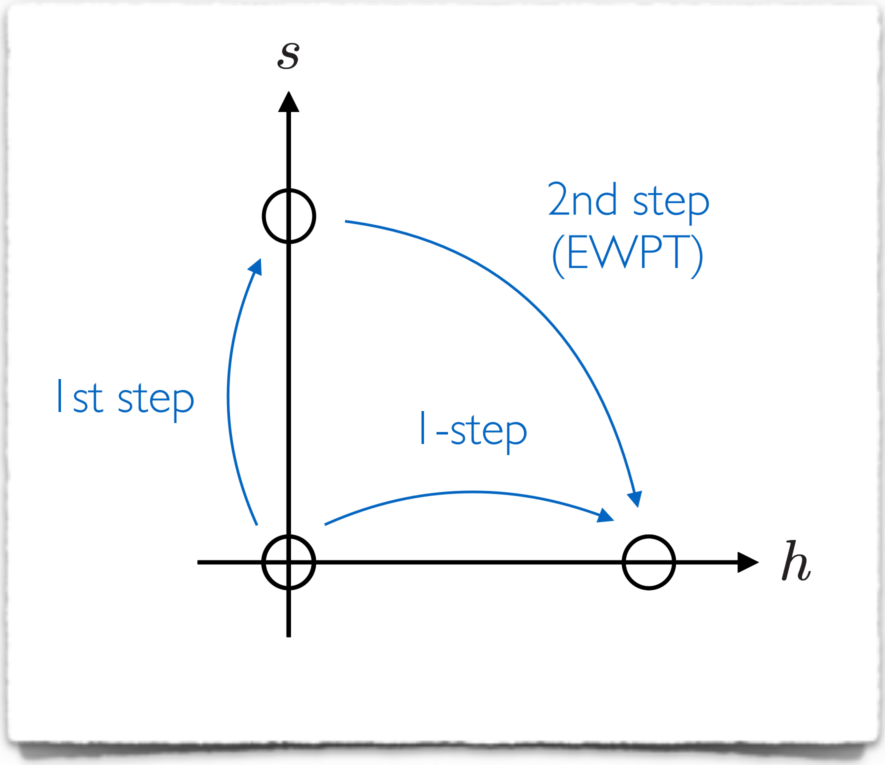
important to create
a barrier in the potential

$$c_h = \frac{1}{48}\left(9g^2 + 3g'^2 + 12y_t^2 + 24\lambda_h + 2\lambda_{h\eta}\right) \qquad c_\eta = \frac{1}{12}\left(4\lambda_{h\eta} + \lambda_\eta\right)$$

- EW symmetry is restored at very high T

$$\langle h,\eta\rangle = (0,0)$$

- Two interesting patterns of symmetry breaking (as the Universe cools down)
  i. 1-step PhT $(0,0)\to(v,0)$
  ii. 2-step PhT $(0,0)\to(0,w)\to(v,0)$
    - 2-step naturally realized since singlet is destabilized before the Higgs ($c_\eta < c_h$)



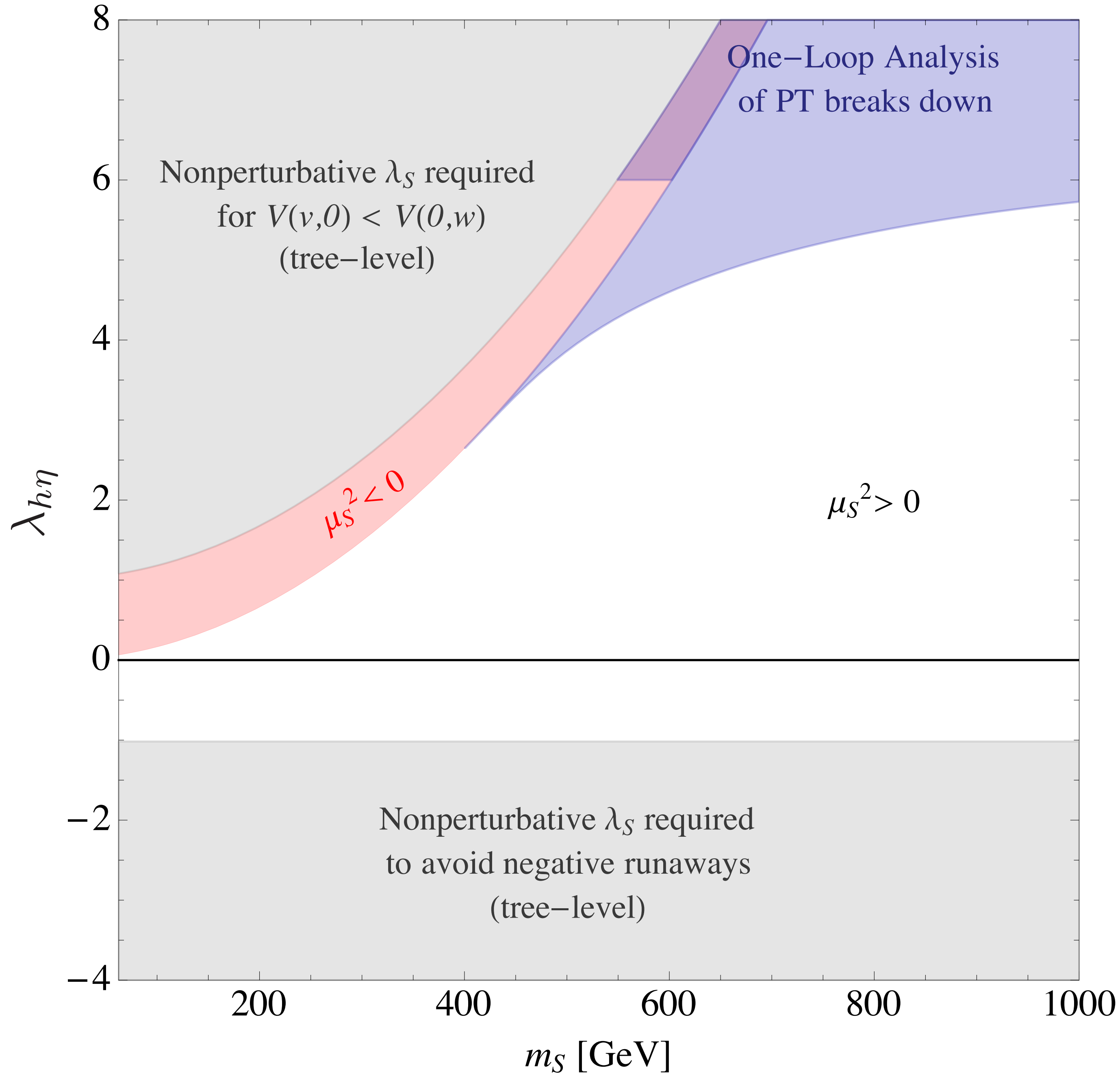
Phenomenology

Very weak constraints

- $m_\eta < m_h/2$ excluded by invisible Higgs decays
- direct searches very challenging: only possible at FCC 100 TeV
  (interesting channel: $pp \to \eta\eta jj$ (VBF))
- indirect searches:
  - modification of Higgs self couplings ($\lambda_3 = \frac{m_h^2}{2v} + \frac{\lambda_{h\eta}^3}{24\pi^2}\frac{v^3}{m_\eta^2} + \cdots$)
  - corrections to Zh cross section at lepton colliders
- dark matter direct detection
  - the singlet can contribute to DM abundance (but can not provide all DM)
  - constraints are very model dependent
    (cosmological history depends on hidden sector details)

The parameter space

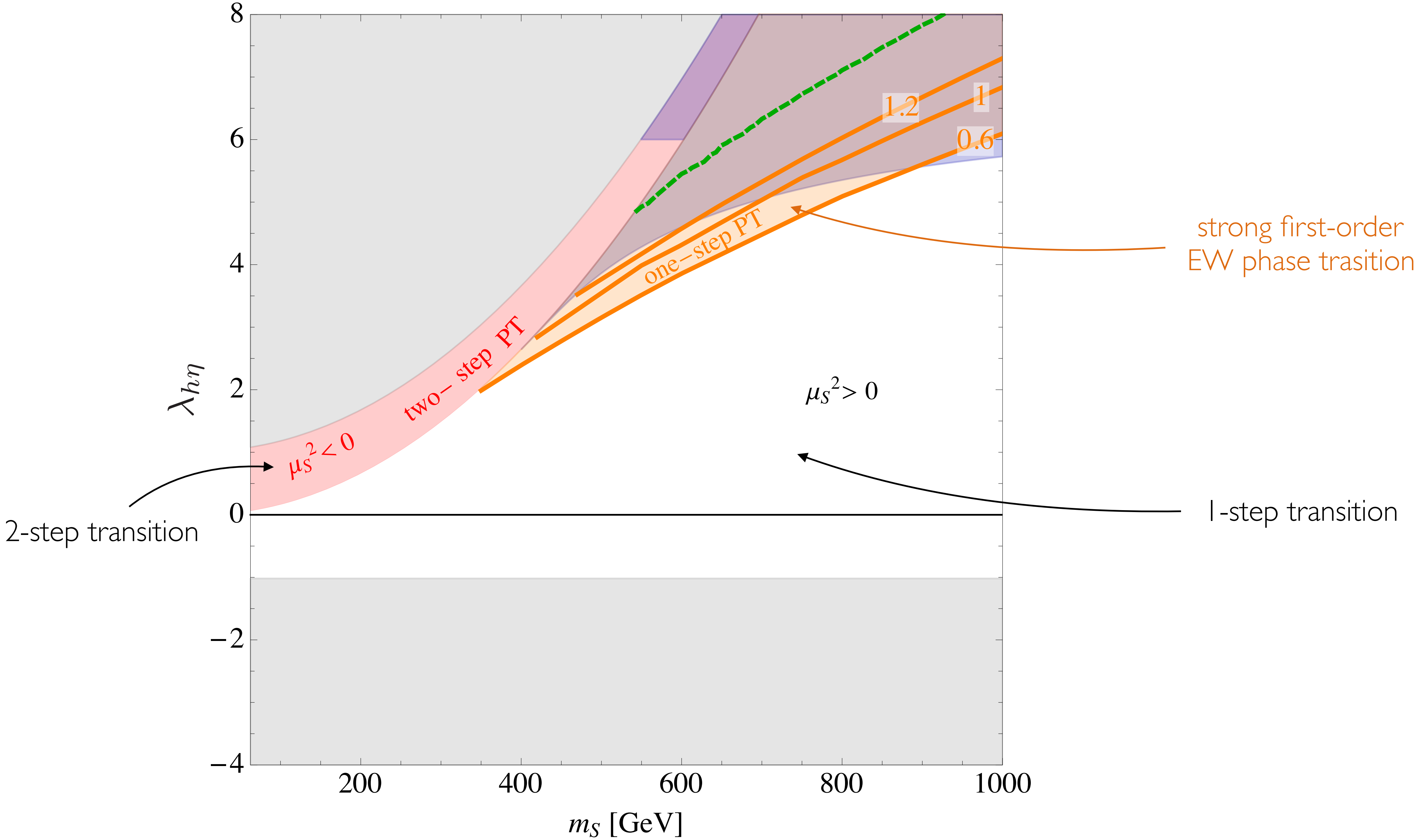
[Curtin, Meade, Yu '14]



Note: PhT parameter space shrinks if nucleation probability is taken into account

The parameter space

[Curtin, Meade, Yu '14]



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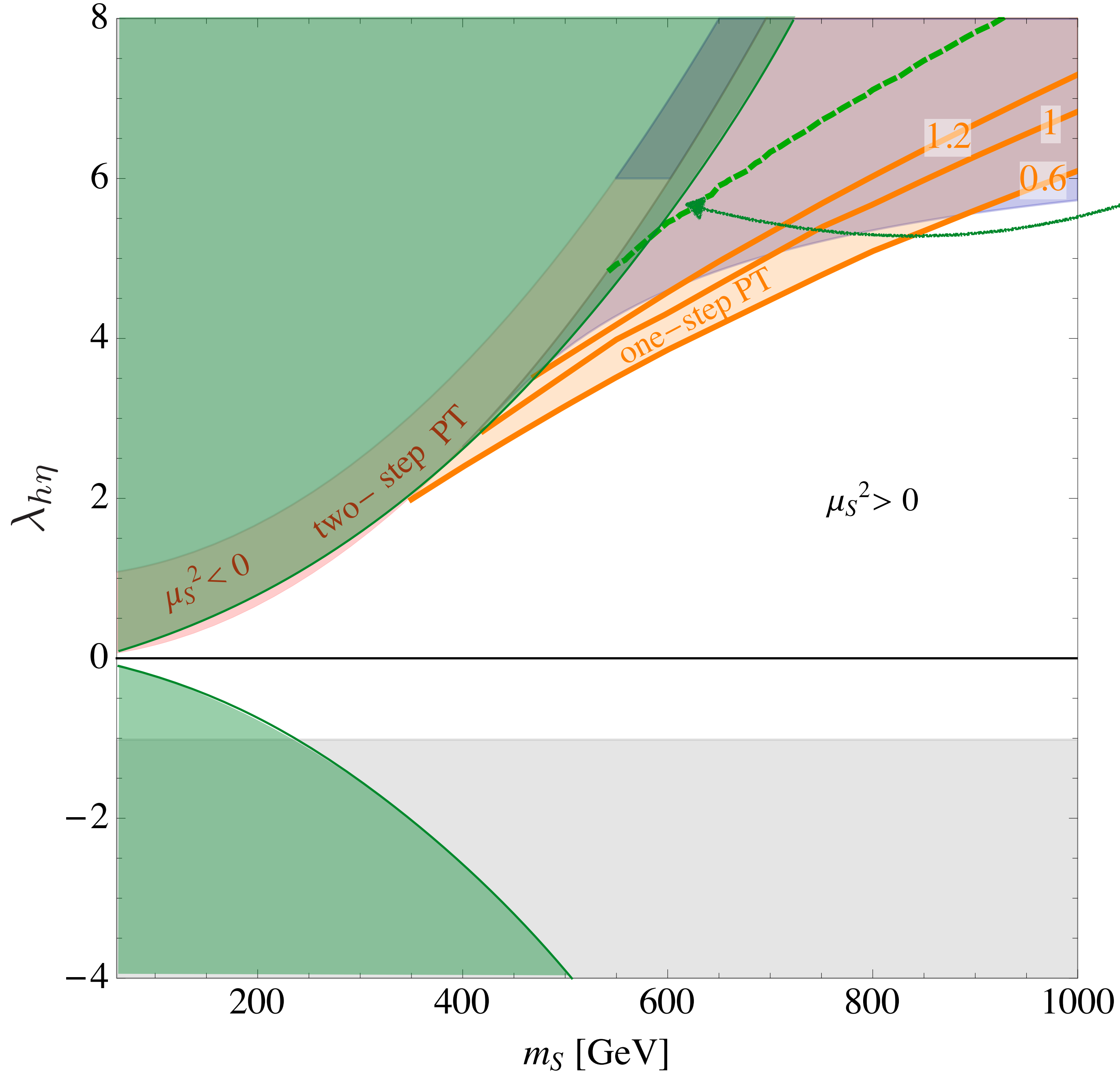
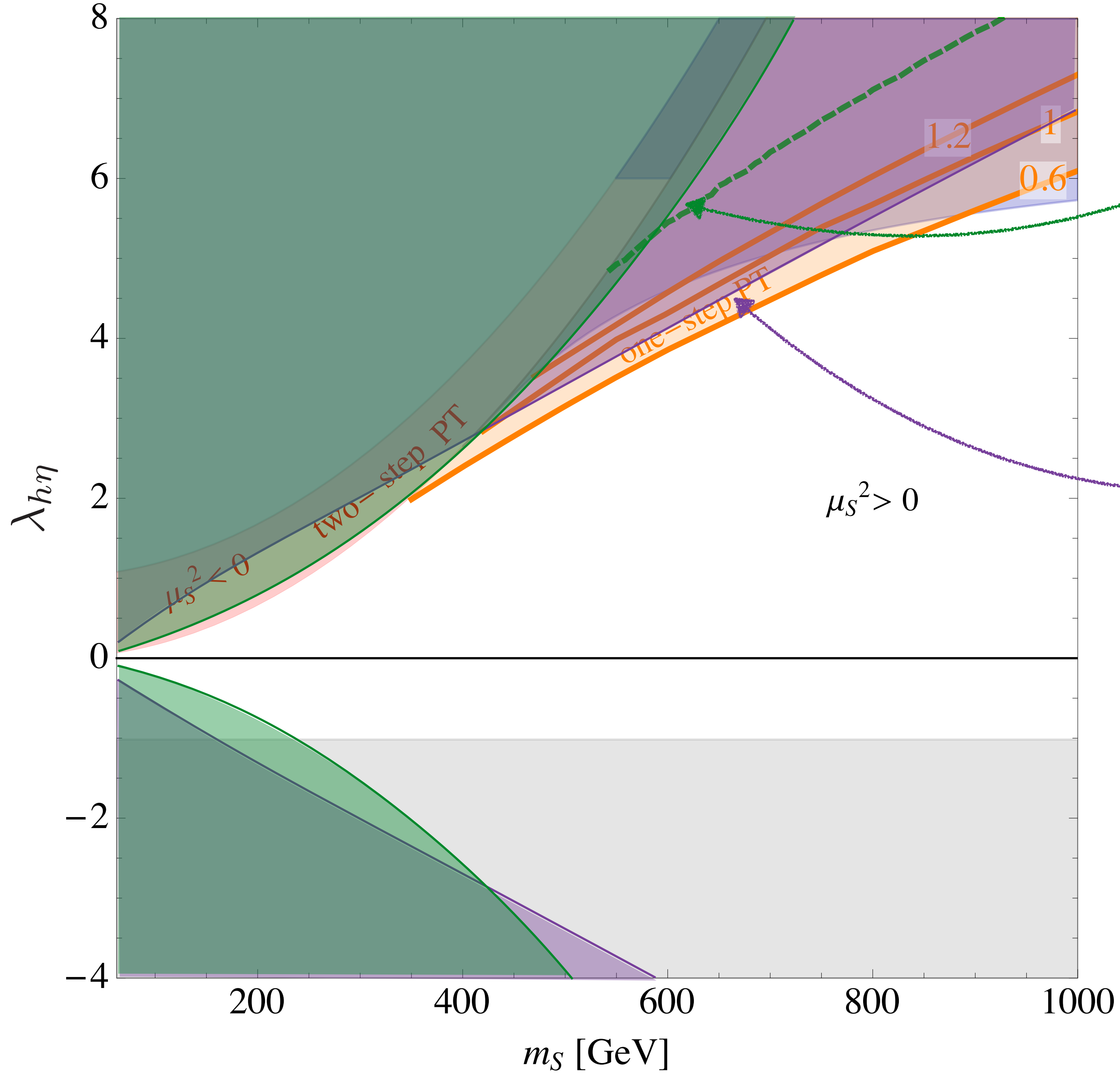


$S/\sqrt{B} > 2$ in VBF
accessible at FCC-hh with 30/ab

Note: PhT parameter space shrinks if nucleation probability is taken into account

The parameter space

[Curtin, Meade, Yu '14]



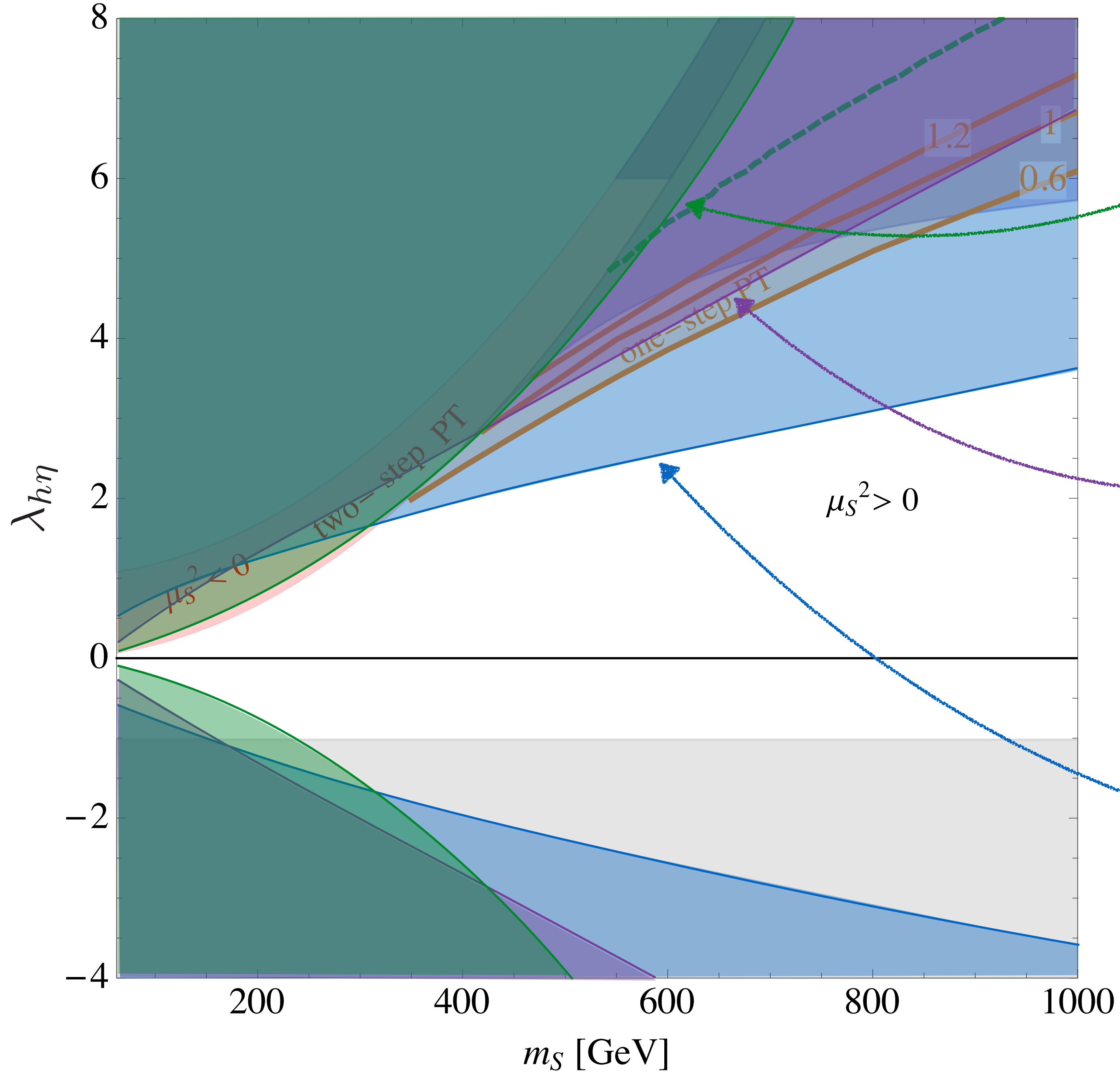
$S/\sqrt{B} > 2$ in VBF
accessible at FCC-hh with 30/ab

σ_{Zh} modified by more than 0.6%
accessible at FCC-ee

Note: PhT parameter space shrinks if nucleation probability is taken into account

The parameter space

[Curtin, Meade, Yu '14]



$S/\sqrt{B} > 2$ in VBF
accessible at FCC-hh with 30/ab

σ_{Zh} modified by more than 0.6%
accessible at FCC-ee

λ_3 modified by more than 10%
accessible at FCC-hh with 30/ab

Note: PhT parameter space shrinks if nucleation probability is taken into account

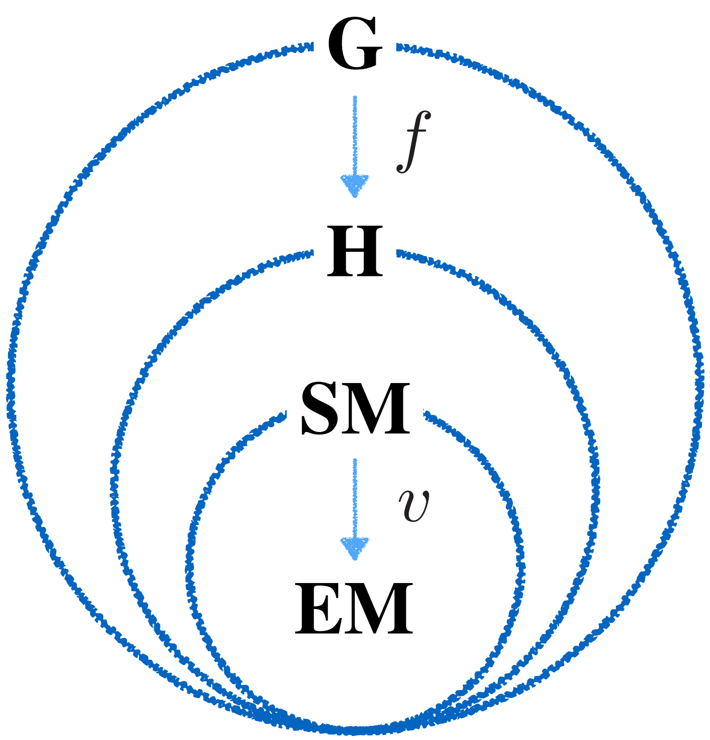
A strongly coupled realisation

with S. De Curtis and G. Panico
JHEP 12 (2019) 149, *arXiv:1909.07894*

Phase transitions in Composite Higgs

Higgs as a **Goldstone** from spontaneously broken global symmetry in a strongly-coupled sector

Multiple phase transitions expected:



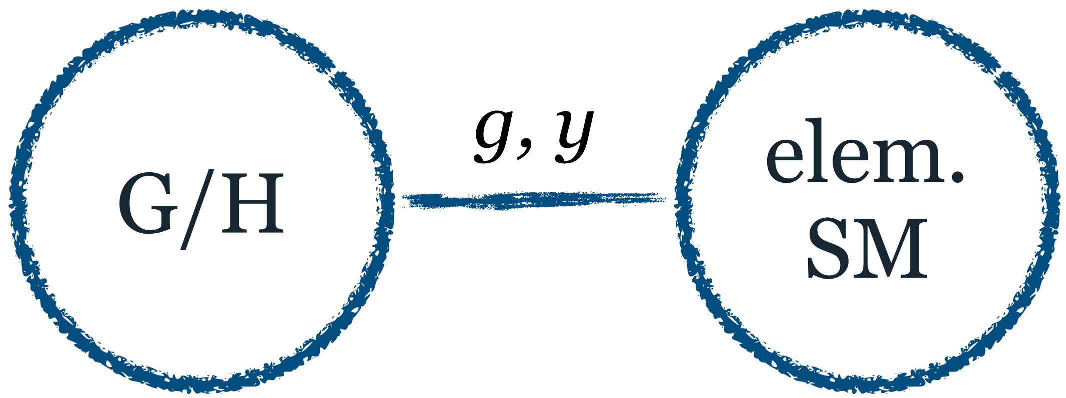
- breaking of the global symmetry in the strong sector

  $G \to H$ at $T \sim \mathrm{TeV}$

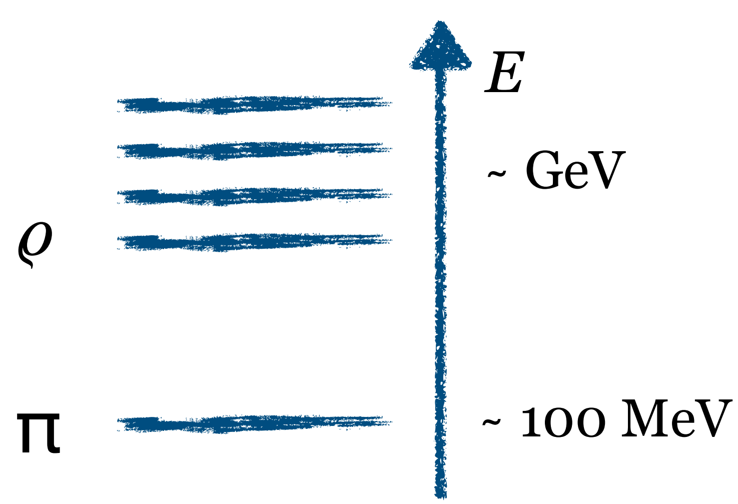
- EW symmetry breaking

  $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{EM}}$ at $T \sim 100\,\mathrm{GeV}$

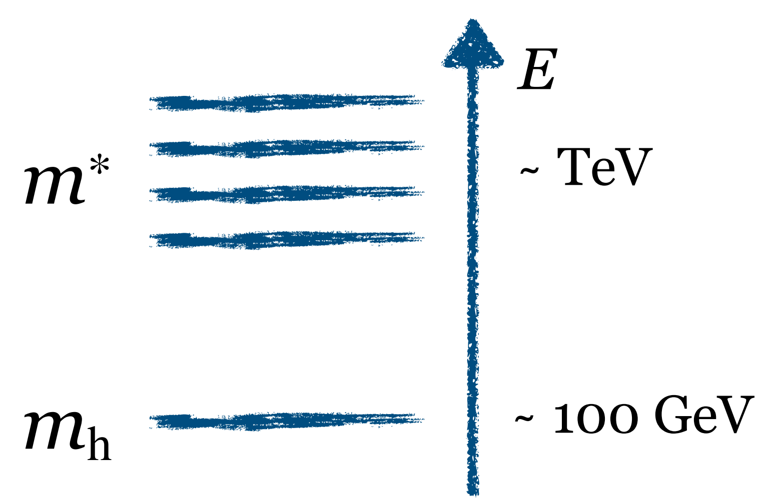
Mass spectra



we borrow the idea from QCD
where we observe that the
(pseudo) scalars are the lightest states



the Higgs could be a kind of pion
arising from a new strong sector



Symmetry structure of the strong sector

G	H	N_G	NGBs rep.$[H]$ = rep.$[\mathrm{SU}(2)\times\mathrm{SU}(2)]$
SO(5)	SO(4)	4	$\mathbf{4} = (\mathbf{2},\mathbf{2})$
SO(6)	SO(5)	5	$\mathbf{5} = (\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(6)	SO(4) × SO(2)	8	$\mathbf{4}_{+\mathbf{2}} + \bar{\mathbf{4}}_{-\mathbf{2}} = 2\times(\mathbf{2},\mathbf{2})$
SO(7)	SO(6)	6	$\mathbf{6} = 2\times(\mathbf{1},\mathbf{1}) + (\mathbf{2},\mathbf{2})$
SO(7)	G_2	7	$\mathbf{7} = (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	SO(5) × SO(2)	10	$\mathbf{10}_{\mathbf{0}} = (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3}) + (\mathbf{2},\mathbf{2})$
SO(7)	$[\mathrm{SO}(3)]^3$	12	$(\mathbf{2},\mathbf{2},\mathbf{3}) = 3\times(\mathbf{2},\mathbf{2})$
Sp(6)	Sp(4) × SU(2)	8	$(\mathbf{4},\mathbf{2}) = 2\times(\mathbf{2},\mathbf{2}), (\mathbf{2},\mathbf{2}) + 2\times(\mathbf{2},\mathbf{1})$
SU(5)	SU(4) × U(1)	8	$\mathbf{4}_{-5} + \bar{\mathbf{4}}_{+\mathbf{5}} = 2\times(\mathbf{2},\mathbf{2})$
SU(5)	SO(5)	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$

Mrazek et al., 2011

Symmetry structure of the strong sector

Minimal scenario: SO(5)/SO(4) **one Higgs doublet**

G	H	N_G	NGBs rep.$[H]$ = rep.$[SU(2) \times SU(2)]$
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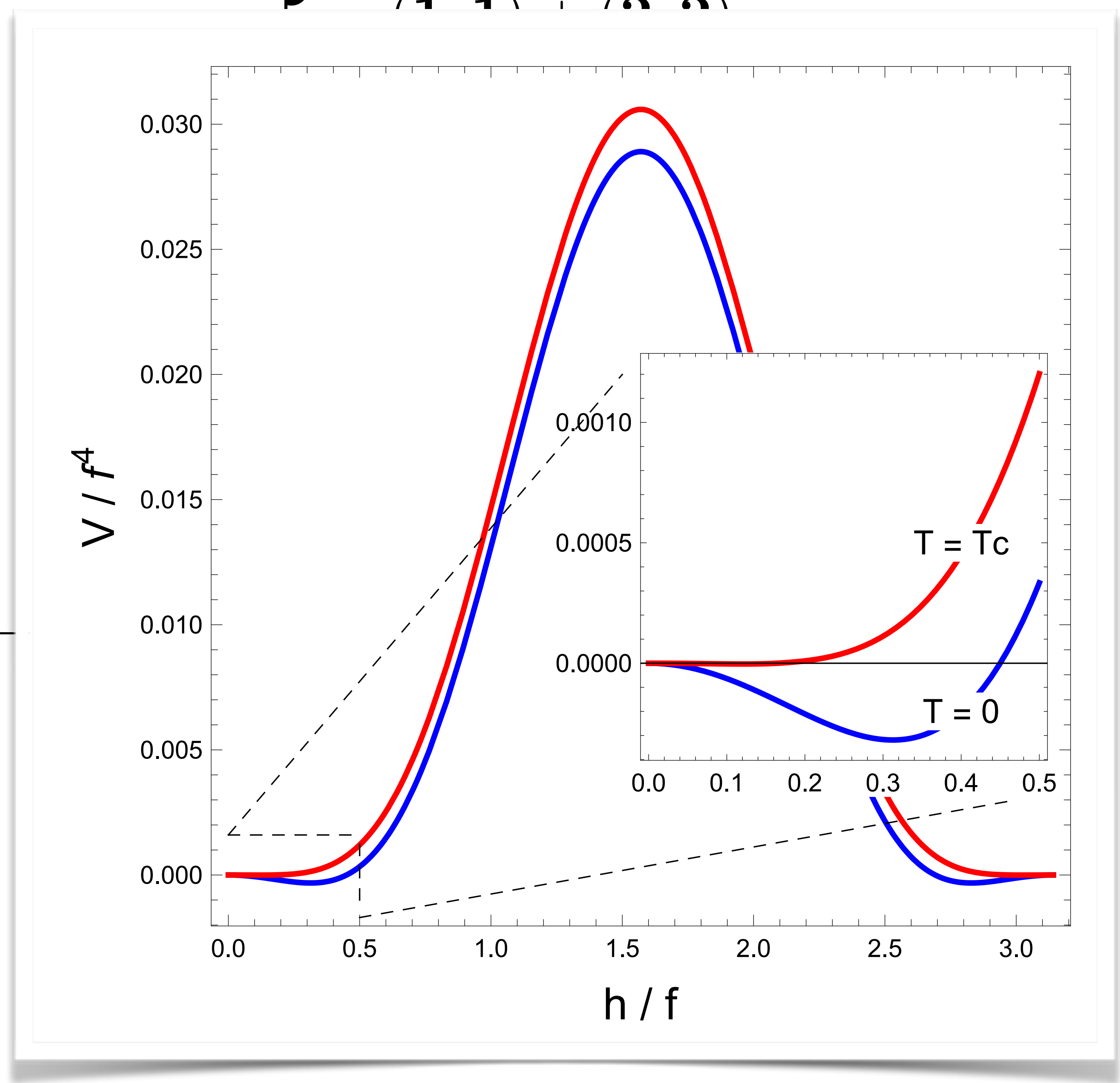

PhT similar to the SM
due to the pheno constraint

$\xi = v^2/f^2 \lesssim 0.1$ no Ist order PhT

Symmetry structure of the strong sector

Next to minimal scenario: SO(6)/SO(5)

one Higgs doublet + a scalar singlet

G	H	N_G	NGBs rep.$[H]$ = rep.$[\mathrm{SU}(2)\times\mathrm{SU}(2)]$
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SU(5)	SO(5)	14	$\mathbf{14} = (\mathbf{3},\mathbf{3}) + (\mathbf{2},\mathbf{2}) + (\mathbf{1},\mathbf{1})$

the scalar potential

$$V(h,\eta) = \frac{\mu_h^2}{2}h^2 + \frac{\lambda_h}{4}h^4 + \frac{\mu_\eta^2}{2}\eta^2 + \frac{\lambda_\eta}{4}\eta^4 + \frac{\lambda_{h\eta}}{2}h^2\eta^2$$

Top partners

The quantum numbers of the fermionic top partners under SO(6) control the Higgs potential

- ☑ 4 — not suitable for the top quark: large Zb_Lb_L coupling
- ☑ 10 — no potential for the scalar singlet η
- ☑ 6, 15, 20' — viable representations for the top quark

☑ $(q_L, t_R) \sim (\mathbf{6}, \mathbf{6})$

typically predicts $\lambda_\eta \simeq 0, \quad \lambda_{h\eta} \simeq \lambda_h/2$

it requires large tuning in bottom quark sector

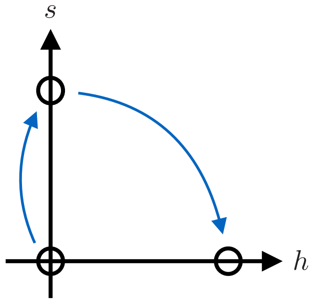
☑ $(q_L, t_R) \sim (\mathbf{15}, \mathbf{6})$

less-tuned scenario: no need to rely on bottom partners
but λ_η still small

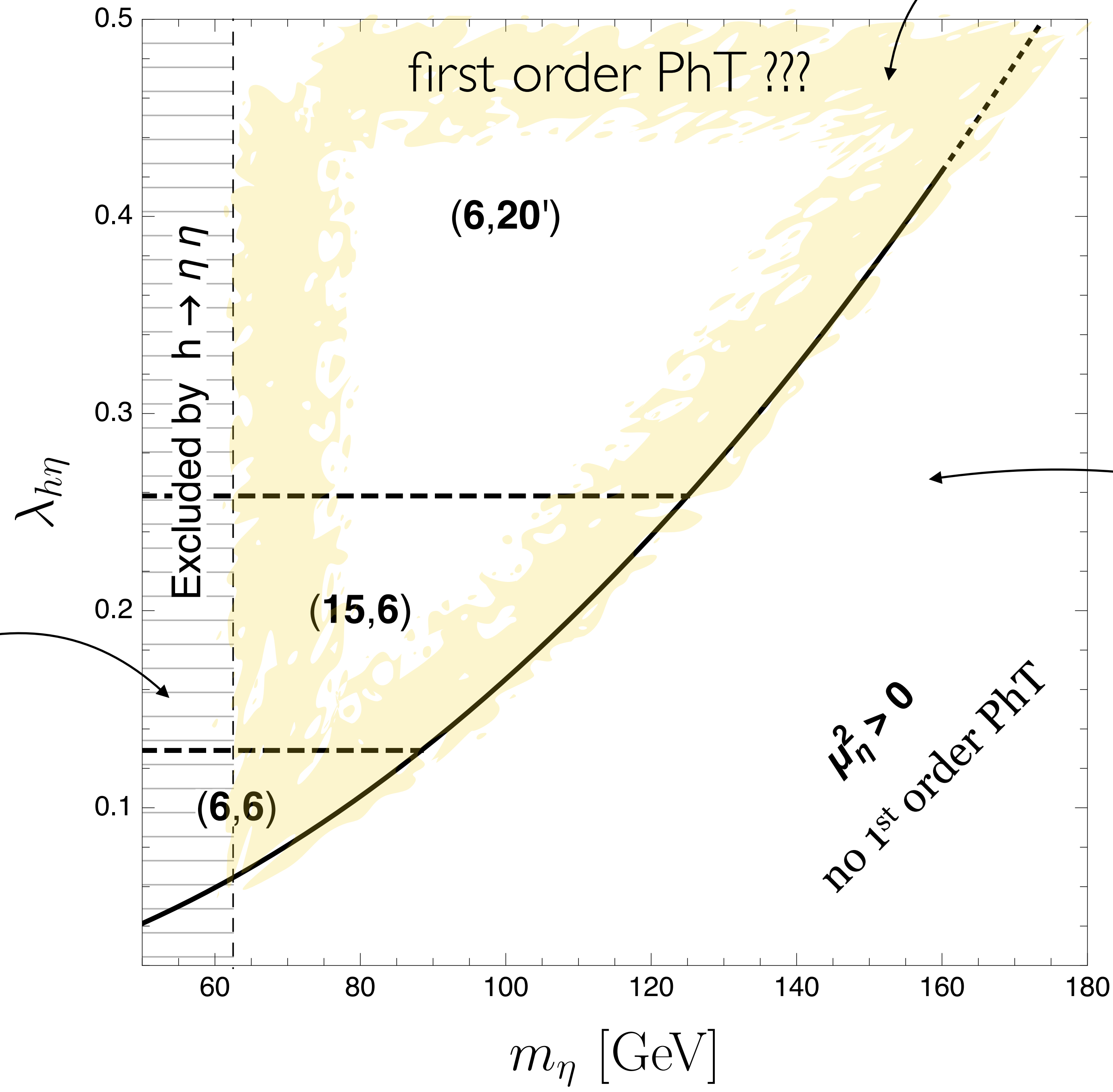
☑ $(q_L, t_R) \sim (\mathbf{6}, \mathbf{20'})$

large parameter space available without large tuning

Parameter space



singlet can get a VEV
(possible 2-step transition)



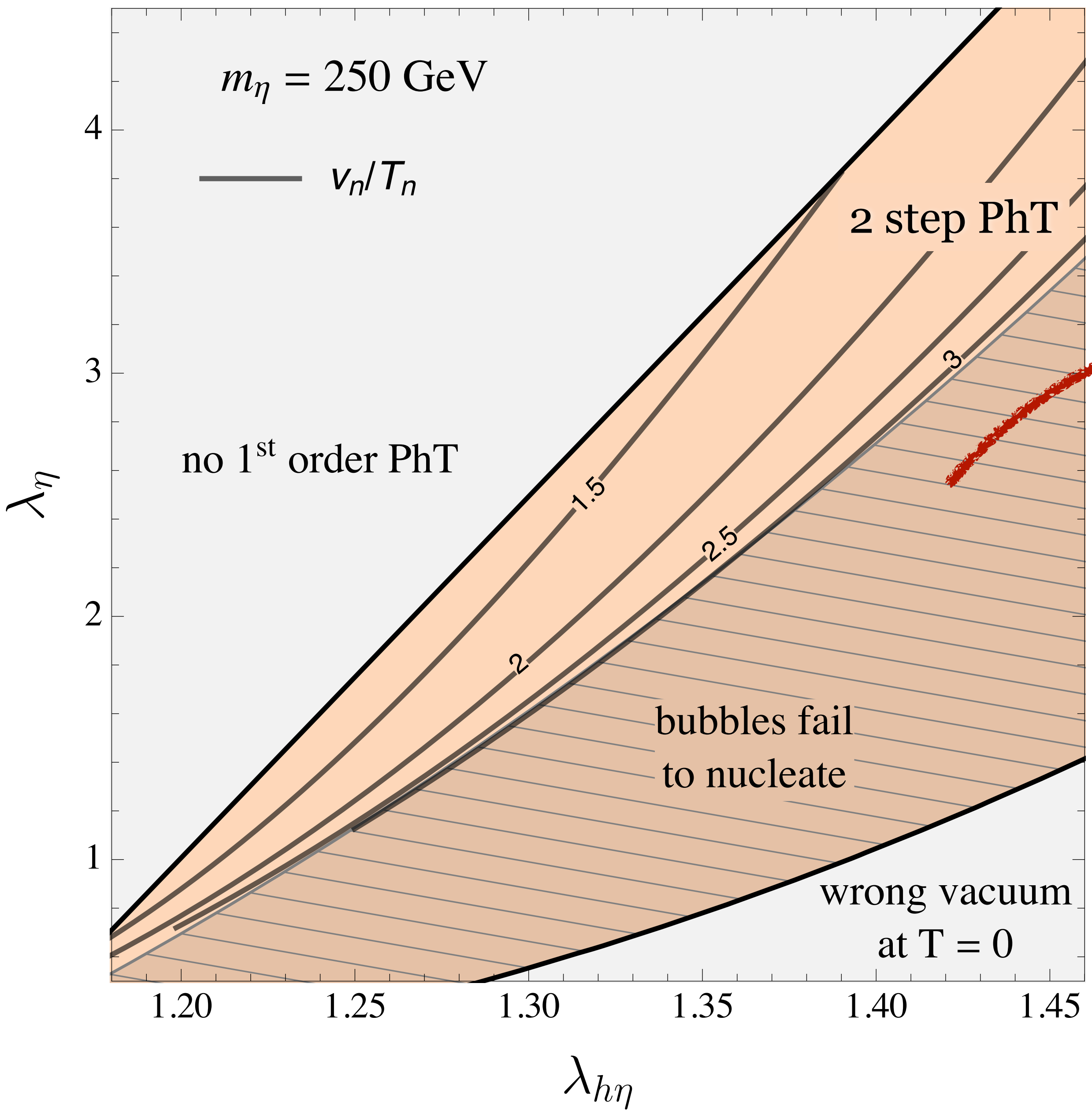
singlet too light

singlet does not get a VEV
(no 2-step transition)

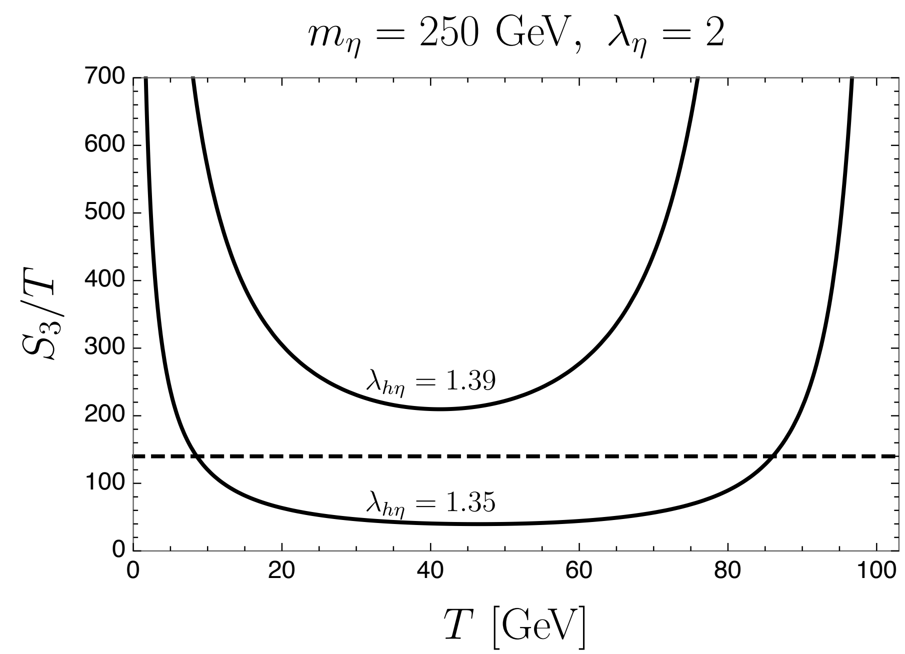
portal too small to induce
barrier in Higgs potential

Properties of the EWPhT

$(q_L, t_R) \sim (6, 20')$



bubbles fail to nucleate:
the system is trapped in the false metastable vacuum
(it may decay to the true EW vacuum at zero temperature)



the bounce action is bounded from below

v_n/T_n: strength of the PhT

Gravitational waves

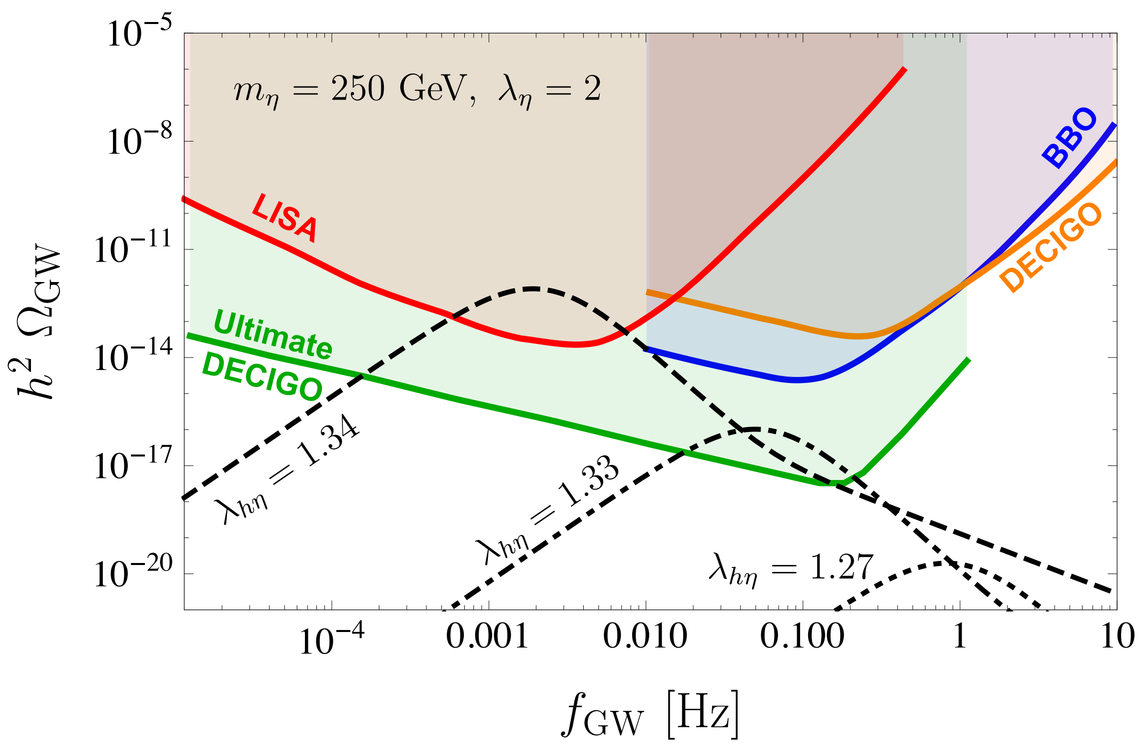
1st order phase transitions are sources of a stochastic background of GW:

- bubble collision
- sound waves in the plasma
- turbulence in the plasma

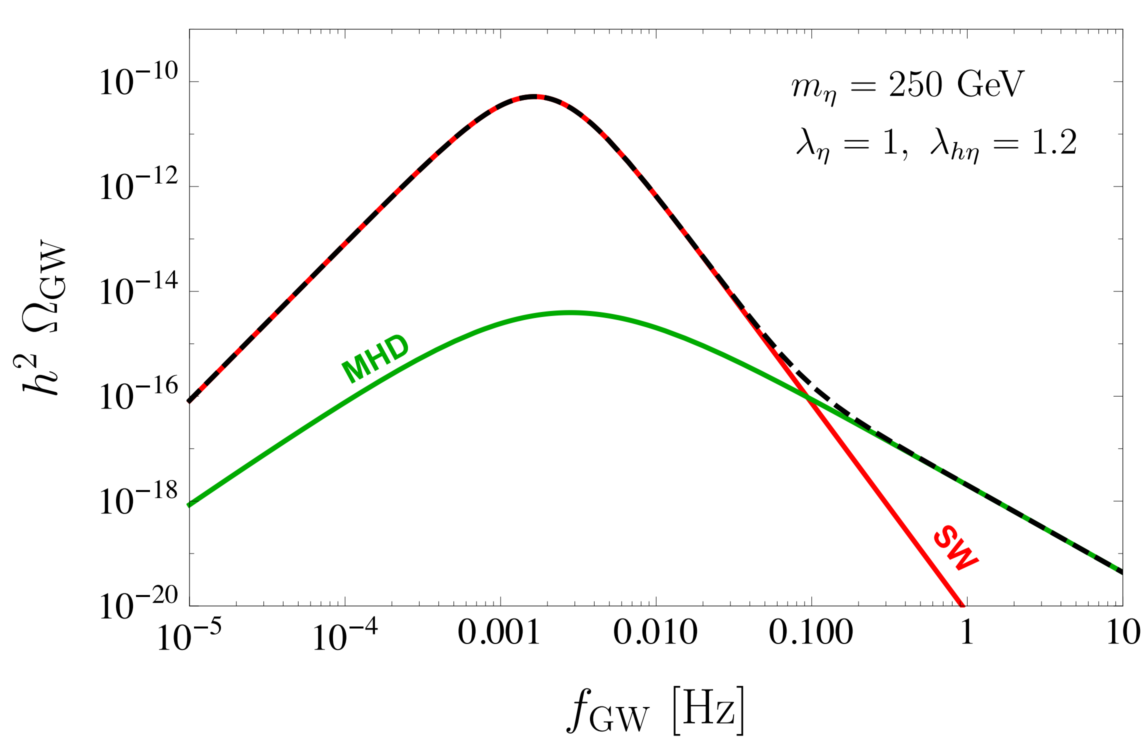
$$f_{\rm peak} = f_* \frac{a_*}{a_0} \sim 10^{-3}\,\mathrm{mHz} \left(\frac{f_*}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100\,\mathrm{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

$$f_*/\beta \equiv (f_*/\beta)(v_w)$$

$$\beta/H_* \simeq \mathcal{O}(10^2) - \mathcal{O}(10^3)$$



peak frequencies within the sensitivity reach of future experiments for a significant part of the parameter space



GW spectra with non trivial structure

EW Baryogenesis

Sakharov's conditions

		SM		SO(6)/SO(5)
✱ B violation	✓	*EW sphaleron processes violate B+L*	✓	*as in the SM*
✱ Out of equilibrium dynamics	✗	*EWPhT not first order*	✓	*EWPhT can be 1st order and sufficiently strong*
✱ C and CP violation	✗	CP violation too small	✓	*CP violation in the $\eta \bar{t} t$ coupling*

EW Baryogenesis: CP violation

an additional source of CP violation is naturally present due to the non-linear dynamics of the Goldstones

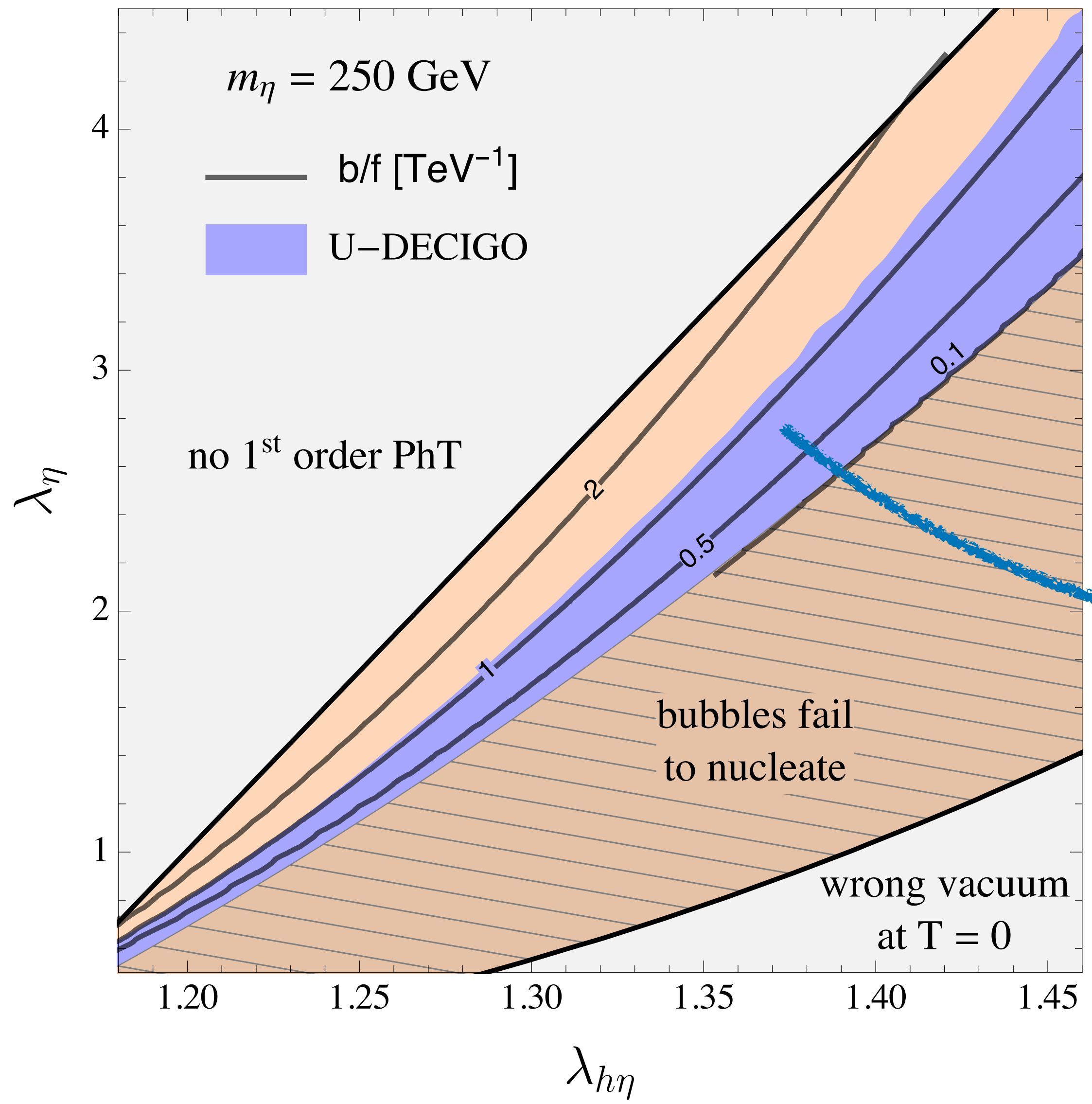
$$\mathcal{O}_t = y_t \left(1 + i\frac{b}{f}\eta\right) \frac{h}{\sqrt{2}}\, \bar{t}_L t_R + \text{h.c.}$$

A phase in the quark mass is generated. The phase becomes physical during the EW phase transition at $T \neq 0$, when η changes its vev

this is realised in the two-step phase transition

(0,0) -> (0,w) -> (v, 0)

EW Baryogenesis

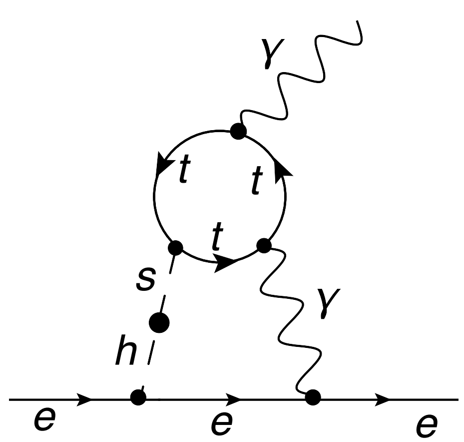


b/f ~ phase in the top mass needed to guarantee the amount of CPV for EWBG

b/f ≲ TeV^{-1} is enough to reproduce the observed baryon asymmetry

there is a region where EWBG and an observable GW spectrum can be achieved simultaneously

note: if Z_2 is broken ($w \neq 0$) at T = 0 constrains from EDM can challenge EWBG



The Peccei-Quinn phase transition

with G. Panico, M. Redi and A. Tesi
JHEP 04 (2020) 025, *arXiv:1912.06139*

The axion

The **axion** offers an elegant solution to the strong CP problem

$$\mathscr{L} \supset -\frac{\alpha_s}{8\pi}\left(\frac{a}{f_a} - \theta\right) G^{A}_{\mu\nu}\tilde{G}^{A\mu\nu}$$

[Peccei-Quinn; Weinberg-Wilczek]

Small size of θ angle explained dynamically

- Goldstone boson of a spontaneously broken U(1) anomalous under QCD
- symmetry breaking at very high scale $f_a \gtrsim 10^9\,\mathrm{GeV}$

- **Is the phase transition of the PQ symmetry first order?**
- **Is there any signal of gravity waves?**

The minimal PQ model

Single scalar field (the **axion**) coupled to coloured fermions

$$\mathcal{L} = \lambda_X(|X|^2 - f^2/2)^2 + (yXQQ^c + \text{h.c.})$$

It displays a **second order** phase transition for several reasons:

I. No massless bosonic states coupled to X where PQ is restored

II. Fermion contribution to 1-loop Coleman-Weinberg has "wrong" sign

III. Potential is always well approximated by $m^2(T)|X|^2 + \lambda(T)|X|^4$

Peccei-Quinn breaking must be **non-minimal**
to have first-order phase transition

The Higgs portal

Coupling with the Higgs boson is typically present

$$V = -\mu^2|H|^2 + \lambda|H|^4 + \underline{\lambda_{XH}|X|^2|H|^2} + \lambda_X(|X|^2 - f^2/2)^2$$

[Dev, Ferrer, Zhang, Zhang '19]

Lagrangian similar to the Higgs + singlet case, but with crucial differences:

I. huge **hierarchy** of scales $v \lll f$

- tuning of parameters: $\mu^2 = \lambda_{XH}/2f^2 + O((100\text{GeV})^2)$
- matching to the Higgs mass: $\frac{M_h^2}{2v^2} = \lambda - \frac{\lambda_{XH}^2}{4\lambda_X}$

II. both fields must have VEV at T=0

- two step transition not possible (due to minimum structure of tree-level potential)

Radiative PQ breaking at weak coupling

Radiative PQ breaking

Collection of scalar fields (some of which charged under PQ)

[Gildener, Weinberg '76]

$$V = \frac{\lambda_{ijkl}}{4} \phi_i \phi_j \phi_k \phi_l$$

Flat direction in the potential at scale Λ (generic feature due to RG running)

$$\lambda_{\text{eff}}(\mu) = \lambda_{ijkl}(\mu) n_i n_j n_k n_l \,, \qquad \lambda_{\text{eff}}(\Lambda) = 0 \,, \qquad \phi_i = n_i \sigma$$

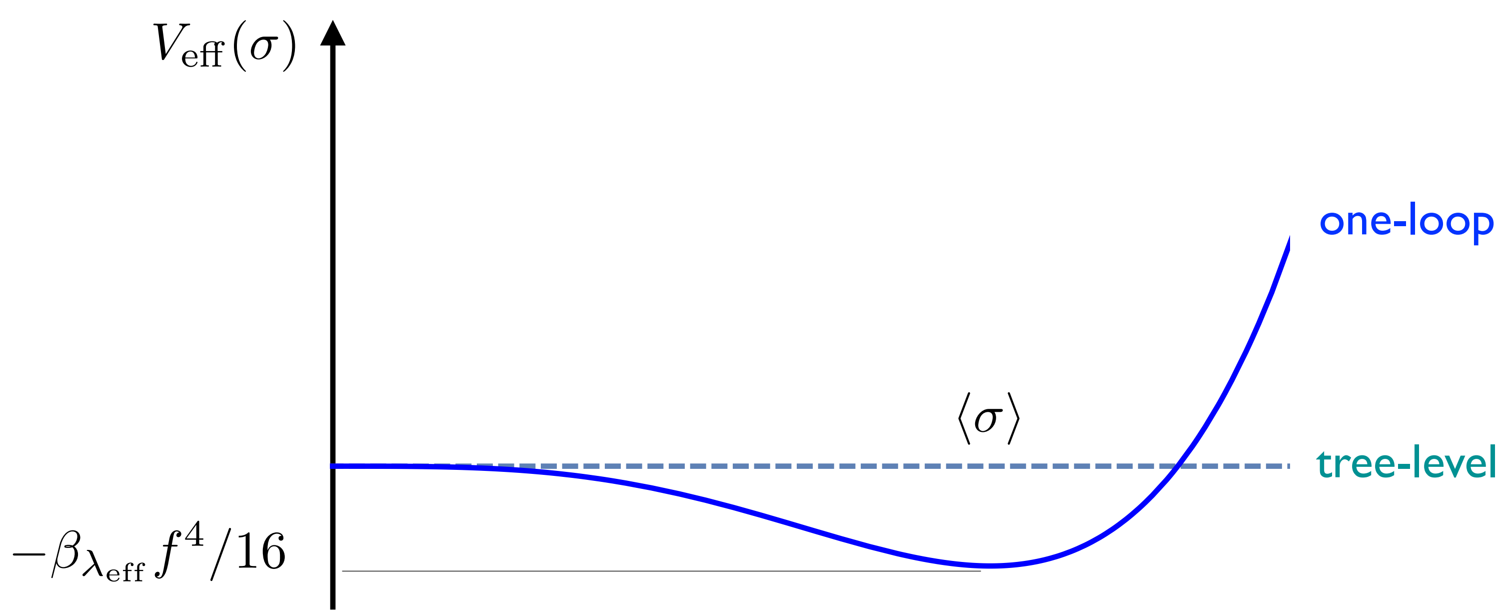
Dynamics mainly controlled by field σ

Radiative PQ breaking

Radiative corrections can lift the flat direction and stabilize the field

$$V_{\rm eff}(\sigma) \approx \frac{\beta_{\lambda_{\rm eff}}}{4}\sigma^4\left(\log\frac{\sigma}{\langle\sigma\rangle} - \frac{1}{4}\right) \qquad \langle\sigma\rangle \approx \Lambda$$

- beta function needs to be positive at the reference scale



Thermal corrections

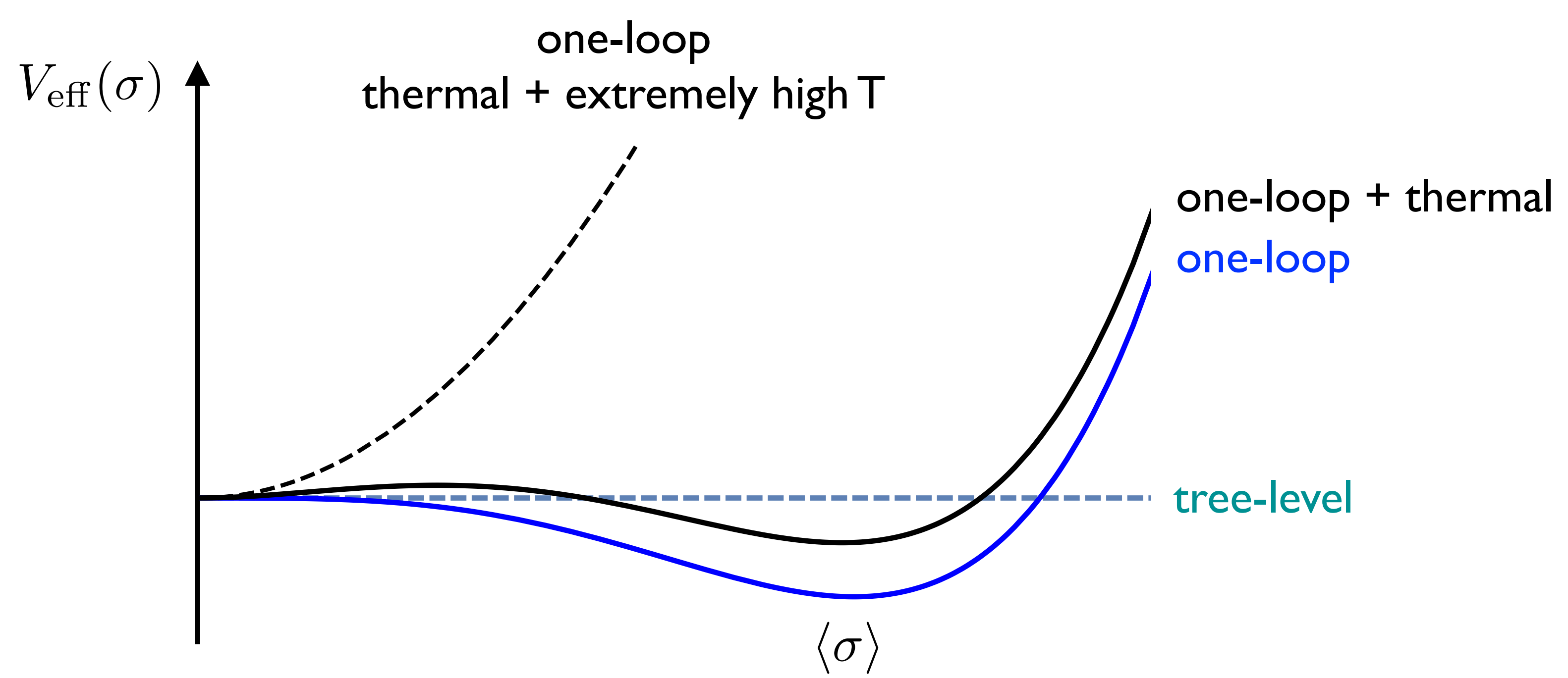
Due to flatness of the potential thermal corrections are always important

[Witten '81]

$$F(\sigma;T) \simeq \frac{N}{24}\hat{g}^2\sigma^2T^2 + \sum_i \frac{m_i^4}{64\pi^2}\log\frac{T^2}{m_i^2} + V_{\rm eff}(\sigma)$$

even for $T \ll f$
one can formally expand at hight-T
close to the origin

$m_i \sim \hat{g}\sigma$
where $\hat{g}$ is a typical coupling of σ
to other light fields



barrier lasts for arbitrarily low temperatures!

Nucleation and supercooling

Due to small deviation from conformal invariance we expect **significant supercooling**

- the integral of the bounce action can be done exactly [Brézin, Parisi '78]

$$\frac{S_3}{T} \approx 18.9 \frac{\sqrt{N/12}}{\hat{g}^3} \frac{16\pi^2/b_{\text{eff}}}{\log(M/T)}, \qquad \beta \equiv b_{\text{eff}}\hat{g}^4/(16\pi^2)$$

S_3/T scales logarithmically with the temperature

- given the peculiar form of the bounce action $S_3/T = \#/\log(M/T)$ we find **lower bound** on the nucleation temperature

$$T_n \gtrsim \sqrt{M H_I} \sim 0.1 f \left(\frac{f}{M_{\text{Pl}}}\right)^{\frac{1}{2}}$$

- the beta parameter in minimized for large supercooling

$$\beta/H = \#/\log^2(M/T)$$

this scenario has the maximal effect on the amplitude of gravitational wave power spectrum generated during the bubble collisions

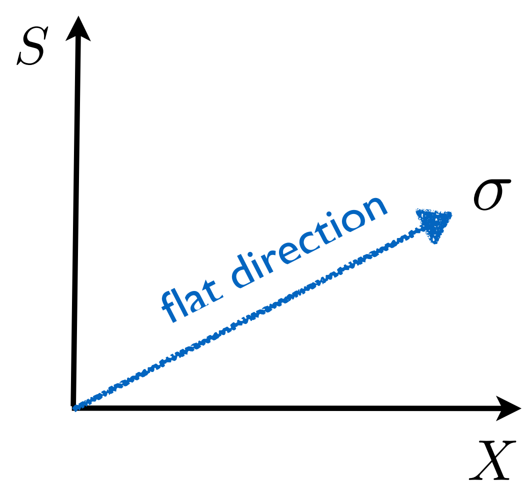
An explicit realisation

Two complex scalars: one charged under PQ and one with U(1) gauge charge

$$\mathcal{L} = -\frac{1}{4g^2}F^2 + |D_\mu S|^2 + |\partial_\mu X|^2 + (yXQQ^c + \text{h.c.}) - \lambda_S|S|^4 - \lambda_X|X|^4 - \lambda_{XS}|S|^2|X|^2$$

[see related Hambye, Strumia, Teresi '18]

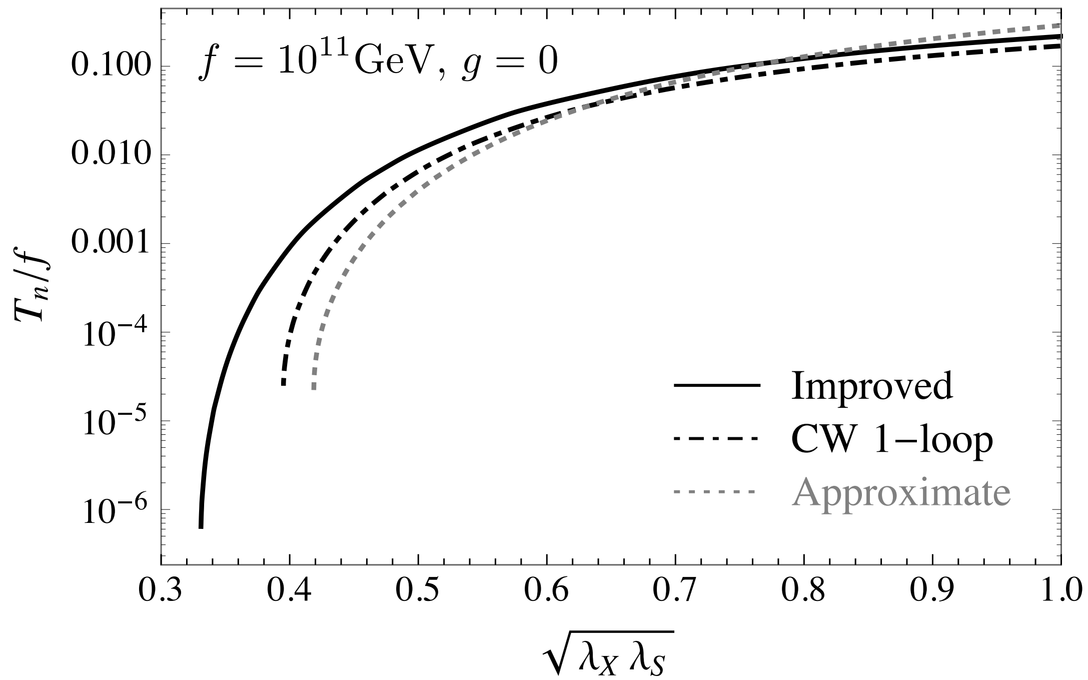
A tree-level flat direction is realized for $\lambda_{XS} = -2\sqrt{\lambda_S\lambda_X}$

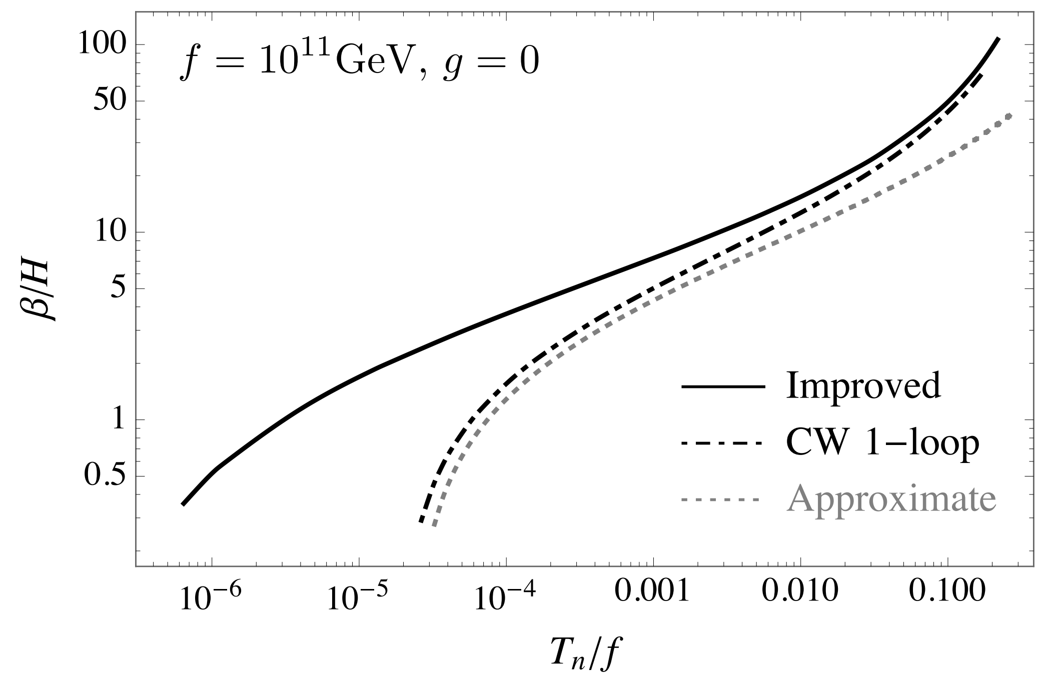


… lifted by the running induced by the quartic couplings and by the gauge interactions

An explicit realisation: results

Results with quartic coupling dominance:

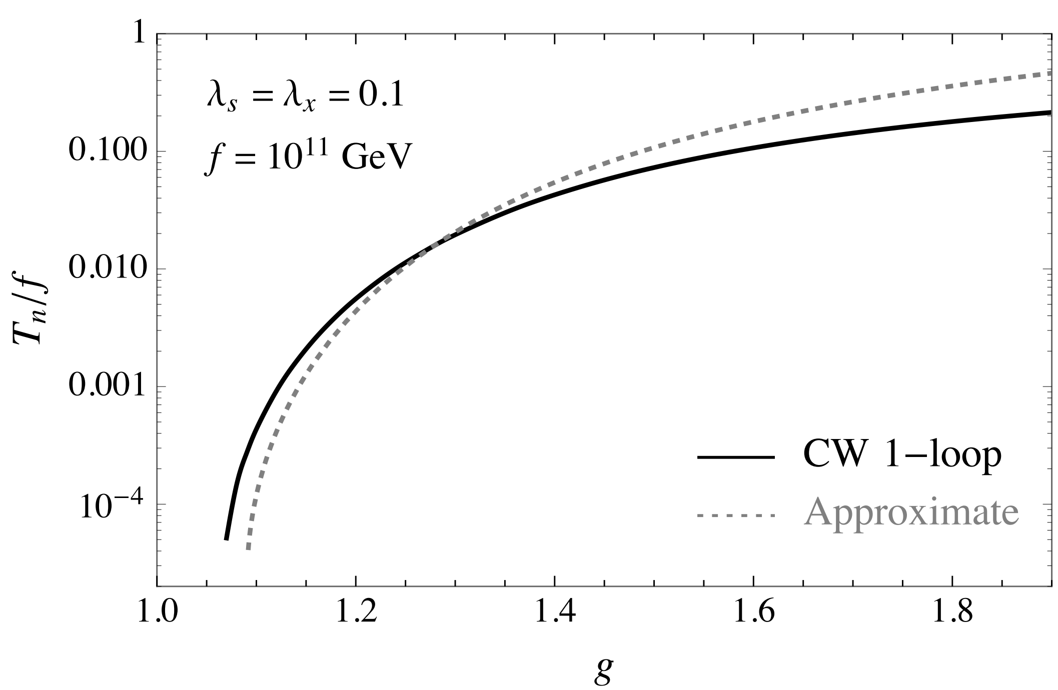


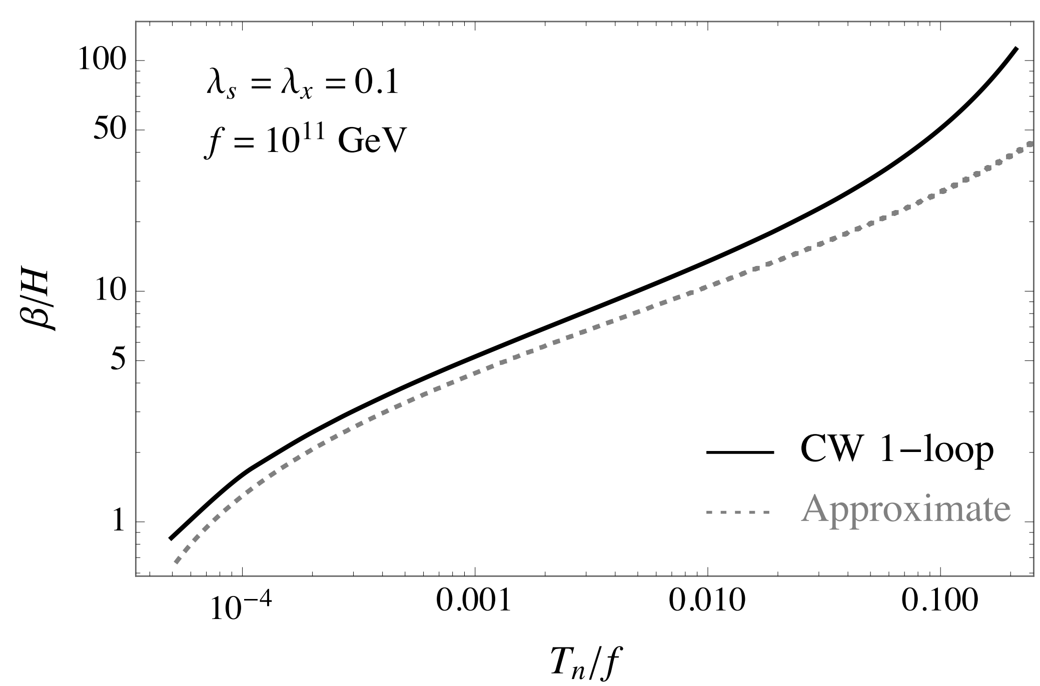


a sizable region wit large supercooling and $\beta/H \sim few$

approximate analytic results work remarkably well!

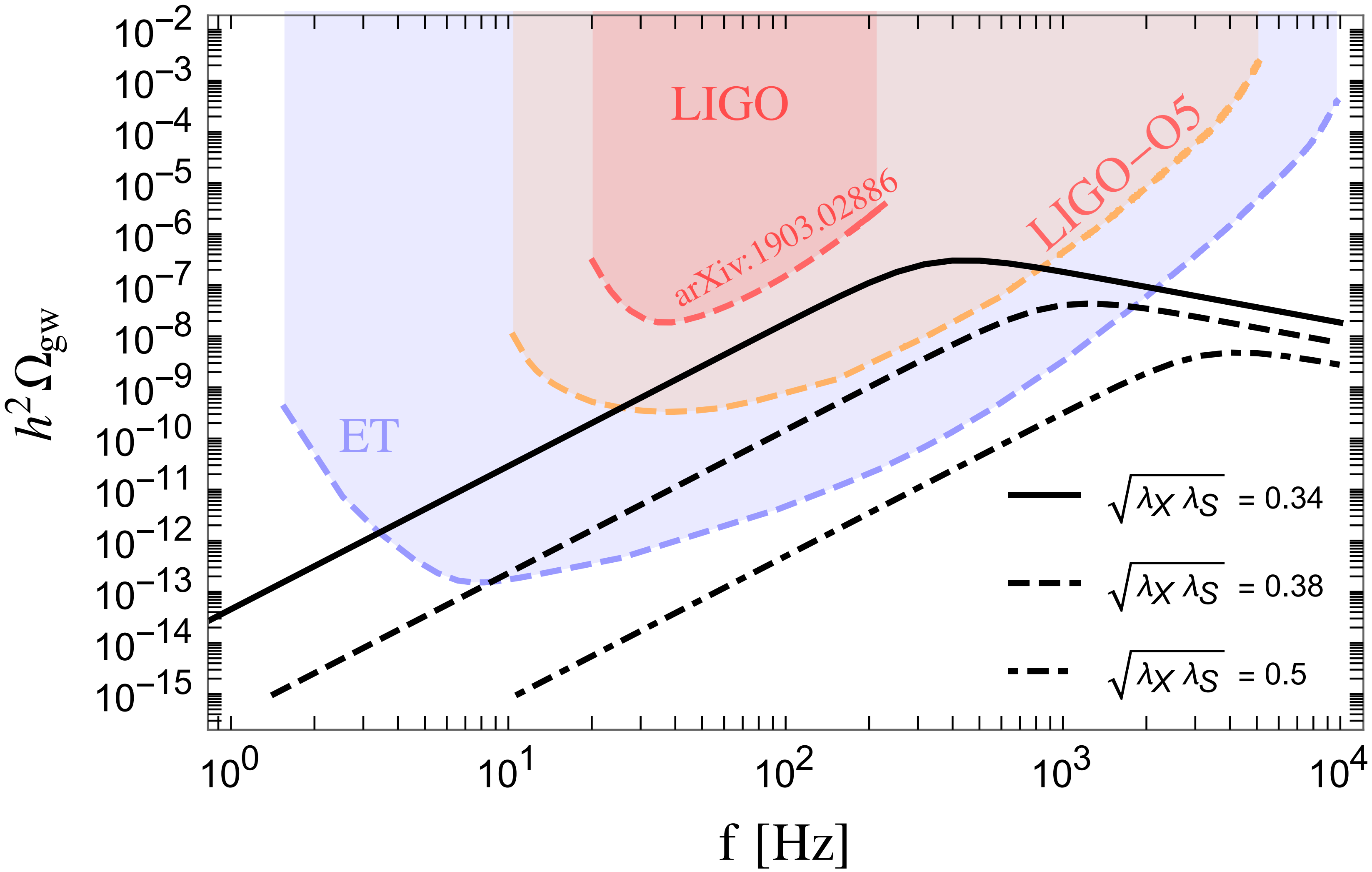
Results with gauge coupling dominance:





results insensitive to improvement (small running)

Gravitational waves



For large supercooling spectrum within the range of ground based experiments

Portion of the parameter space accessible at LIGO

$$h^2\Omega_{\rm gw}\big|_{\rm peak} \simeq 1.27\times 10^{-10}\left(\frac{100}{\beta/H}\right)^2 \qquad f_{\rm peak}\simeq 3.83\times 10^5\,{\rm Hz}\left(\frac{\beta/H}{100}\right)\left(\frac{T}{10^{11}{\rm GeV}}\right)$$

Radiative PQ breaking at strong coupling

Confinement phase transition

We consider a model with the **axion** together with a **dilaton**:
PQ breaking linked to **confinement PhT**

strongly coupled large-N CFT at finite temperature with global Peccei-Quinn U(1)

tiny deviation from scale invariance realises a 1st order phase transition with a large amount of supercooling
(in the same spirit as in the weakly coupled case)

breaking of scaling invariance at a scale f also triggers PQ breaking

$$\langle 0 | j^{\mu}_{\rm PQ}(p) | a \rangle \sim \frac{N}{4\pi} f \, p^{\mu}$$

Explicit realization in 5D through AdS/CFT duality

[Creminelli, Nicolis, Rattazzi; Randall, Servant;…]

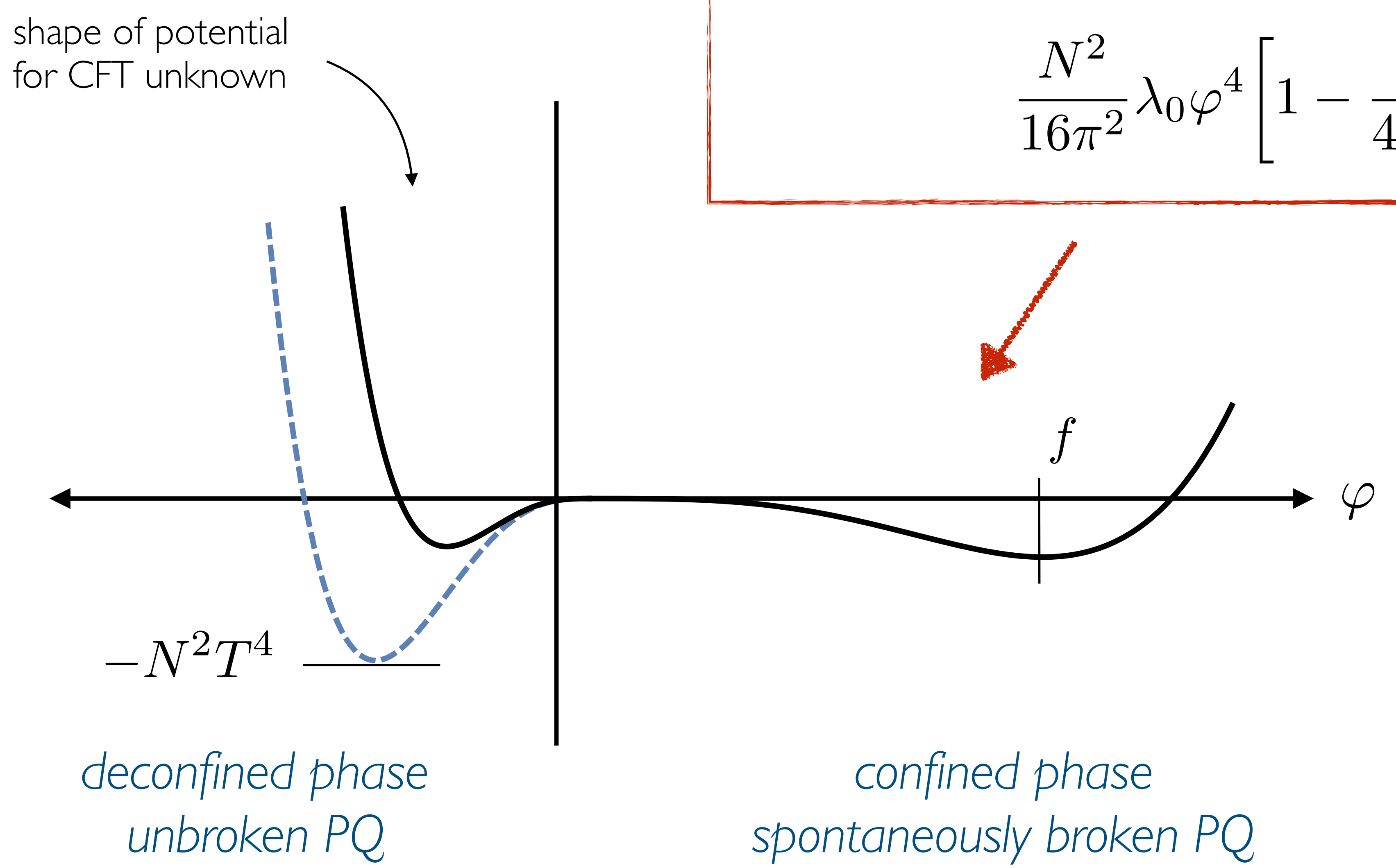
The dilaton potential

CFT explicitly broken by (almost) marginal deformation

$$\mathrm{CFT} + \frac{g}{\Lambda^\epsilon}\mathcal{O} \quad \longrightarrow \quad \beta_g = \epsilon g + a\, N \frac{g^3}{16\pi^2} + \ldots$$

Dilaton potential from running of quartic coupling

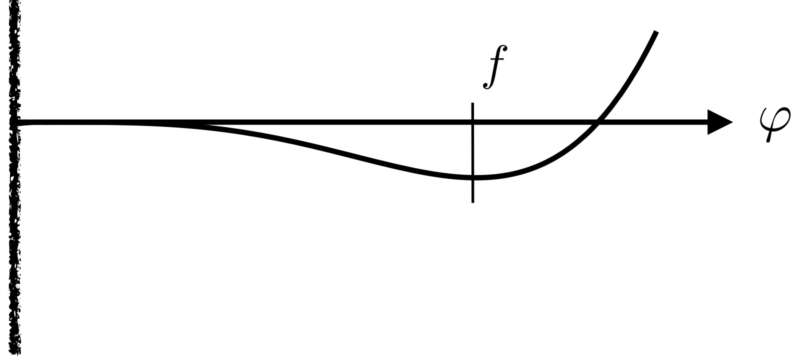
$$\frac{N^2}{16\pi^2}\lambda_0 \varphi^4 \left[1 - \frac{4}{4+\epsilon}\left(\frac{\varphi}{f}\right)^\epsilon\right]$$



Analytic approximations

At large supercooling tunnelling happens very close to the origin

$$\frac{N^2}{16\pi^2}\lambda_0\varphi^4\left[1-\frac{4}{4+\epsilon}\left(\frac{\varphi}{f}\right)^{\epsilon}\right] \longrightarrow \frac{N^2}{16\pi^2}\phi^4\lambda_0\left(1-\frac{4}{4+\epsilon}\left[\frac{T}{c|\lambda_0/(16\pi^2)|^{1/4}f}\right]^{\epsilon}\right)$$

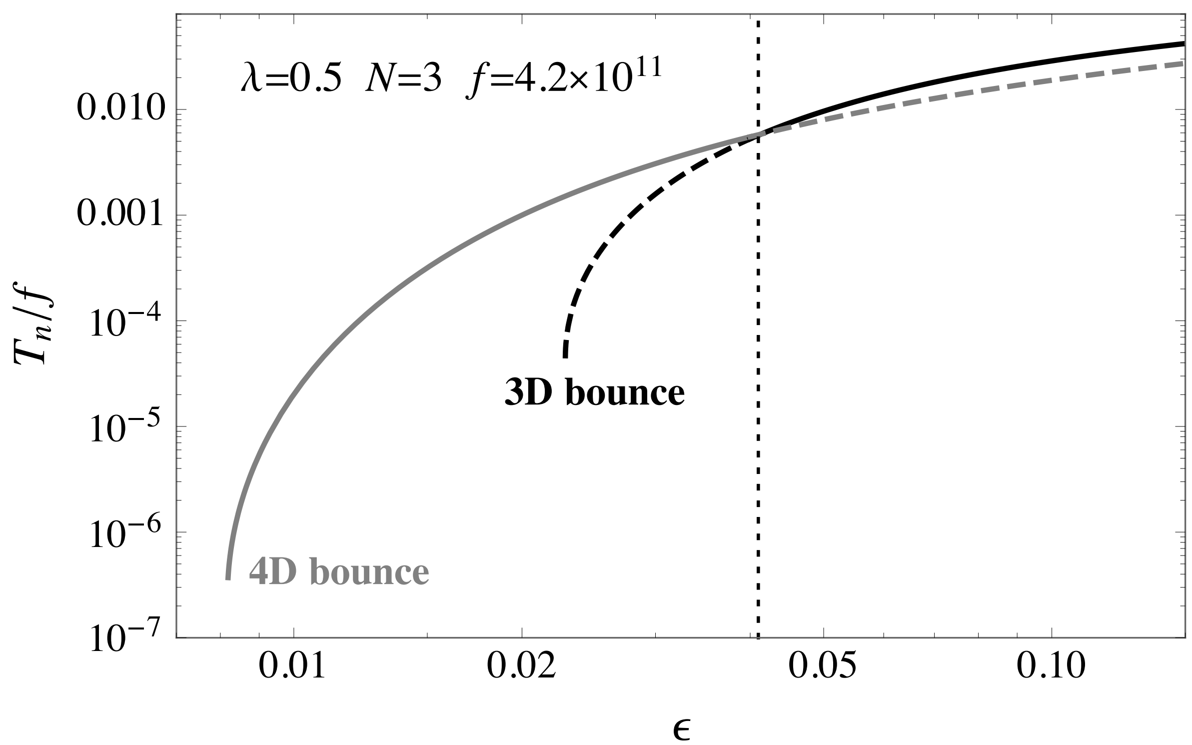


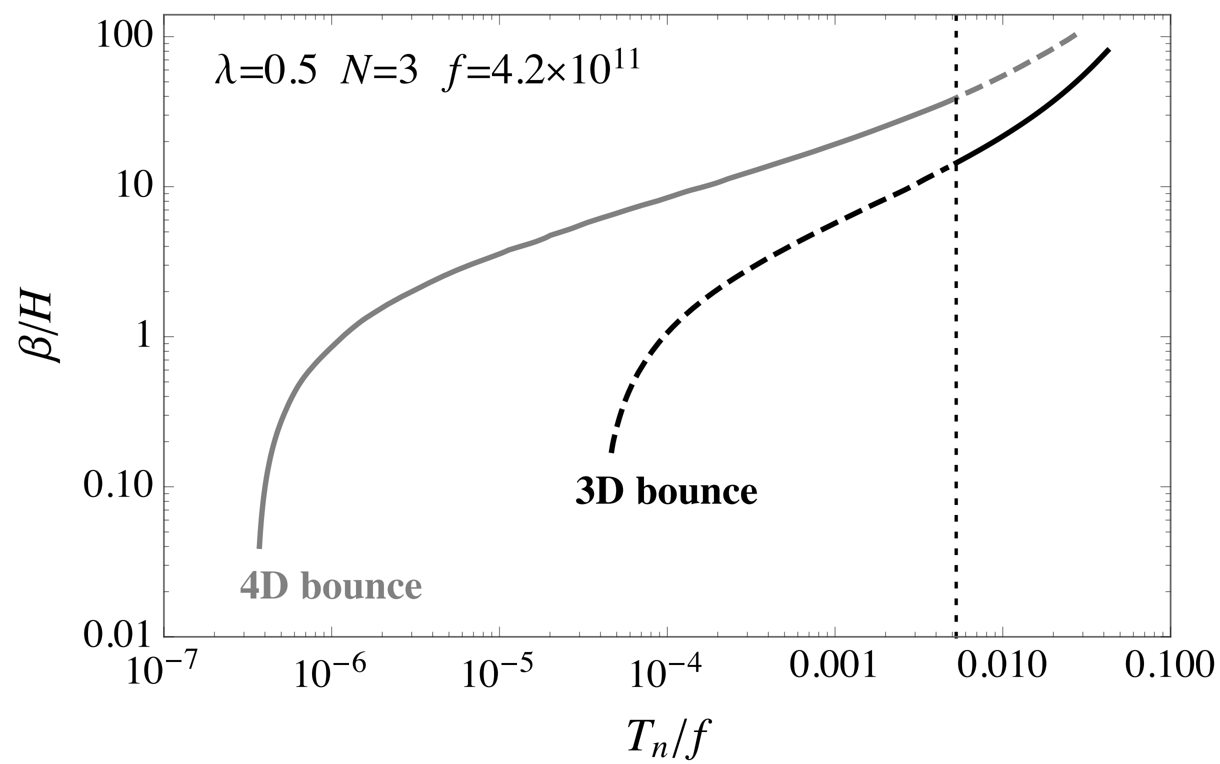
simple evolution of the quartic coupling with the temperature

- the 3D bounce action is given by $$\frac{S_3}{T}=28.5\,\frac{N^2}{16\pi^2}\times\frac{(16\pi^2)^{1/4}}{|\lambda_0|^{3/4}}\times\frac{1}{|g(T,\epsilon)|^{3/4}}$$
- 4D bounce can also be relevant (dominant at low T) $$S_4\sim 26\,\frac{N^2}{16\pi^2}\times\frac{1}{|\lambda_0|}\frac{1}{|g(T,\epsilon)|}$$

Properties of the phase transition

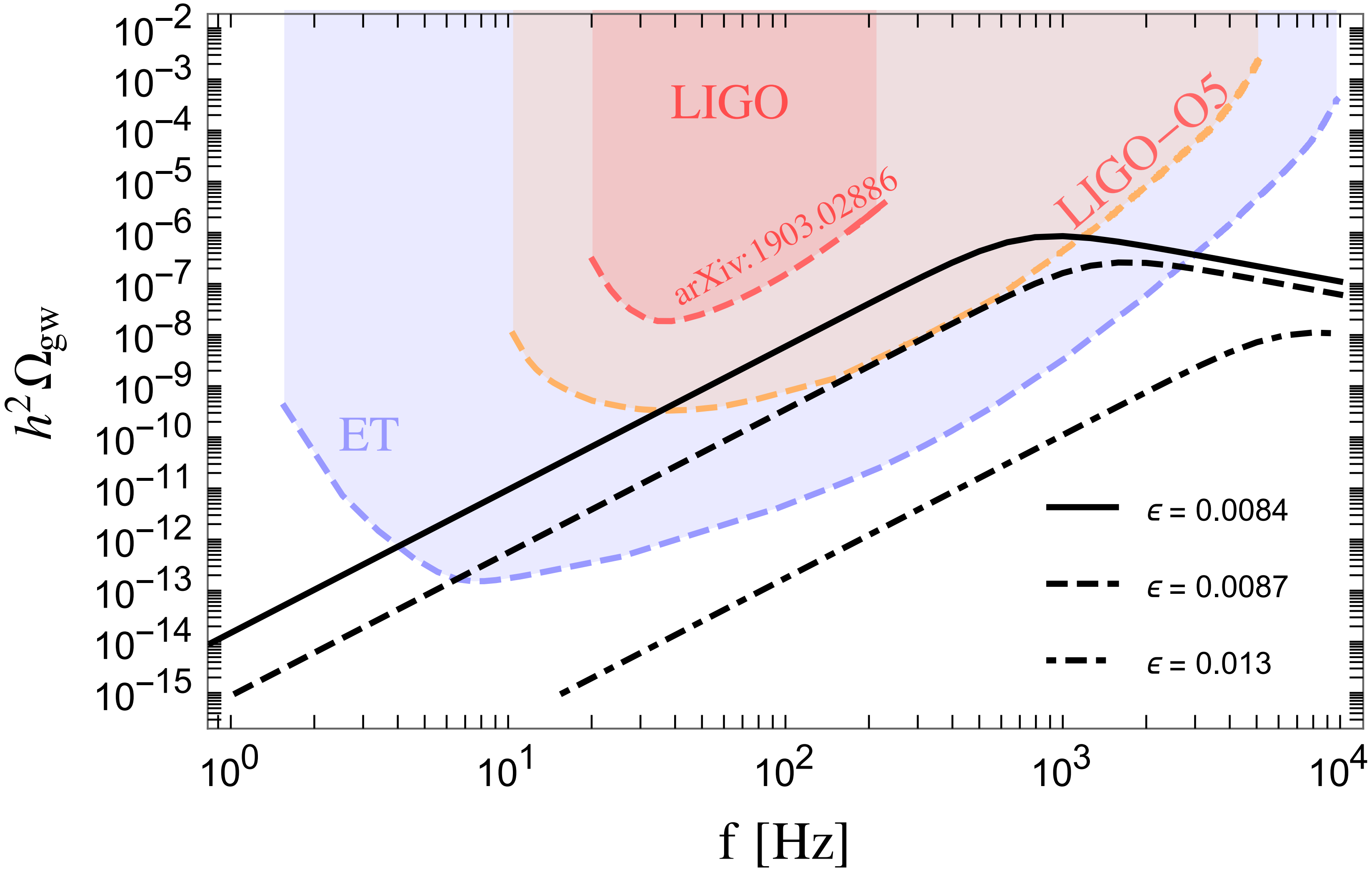
Most of the effects controlled by the size of the free energy
(shape of the CFT potential almost irrelevant)





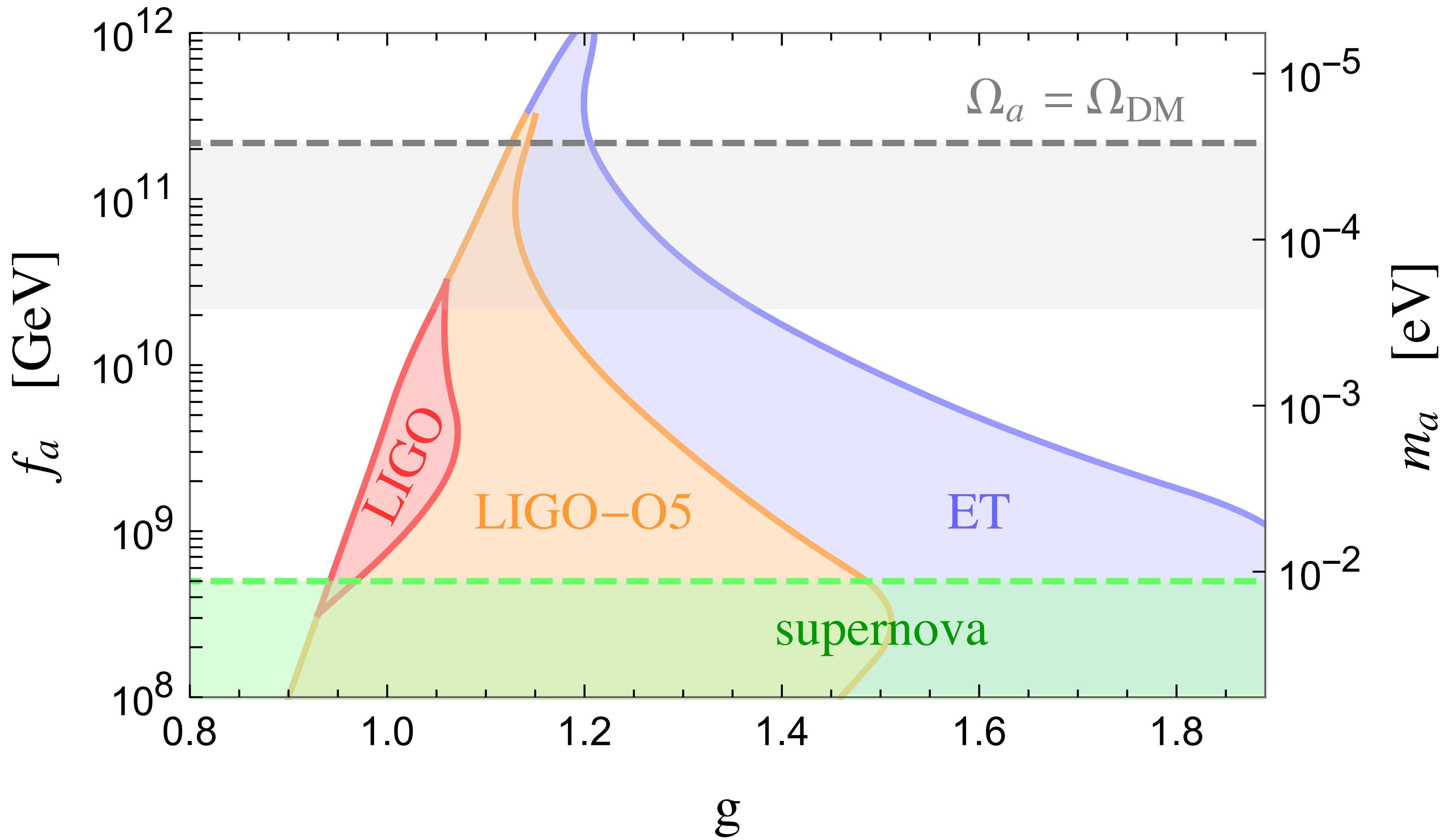
- $\beta/H \sim few$ can be obtained but only in small portion of the parameter space

Gravitational waves



Portion of the parameter space accessible at LIGO

Gravitational waves



Conclusions

Phase transitions are important events in the evolution of the Universe

New physics can significantly modify the SM predictions and open appealing scenarios:

- strong first-order **EW phase transition** from extended Higgs sector
  - possibility to achieve EW baryogenesys
  - collider signatures (at future machines)
  - detectable gravitational wave signal (at space-based interferometers)
- **Peccei-Quinn phase transition**
  - minimal scenarios predict second-order transition
  - possible first order for axion + scalar and axion + dilaton systems
  - detectable gravitational wave signal (at ground-based interferometers)

- Baryon and Lepton numbers are *classically* conserved in the SM
- Conservation is spoiled by quantum corrections

$$\partial_\mu j^\mu_B = \partial_\mu j^\mu_L = N_f \left(\frac{g^2}{32\pi^2} W\widetilde{W} - \frac{g'^2}{32\pi^2} Y\widetilde{Y} \right)$$

- B-L is conserved while B+L is anomalous

$$\Delta B = \Delta L = N_f \Delta N_{CS}$$

$$N_{CS} = \frac{g^2}{32\pi^2} \int d^3x \epsilon^{ijk} \left(W^a_{ij} W^a_k - \frac{g}{3} \epsilon_{abc} W^a_i W^b_j W^c_k \right)$$

We want to compute ΔB between two configurations of gauge fields with vanishing field strength tensor.

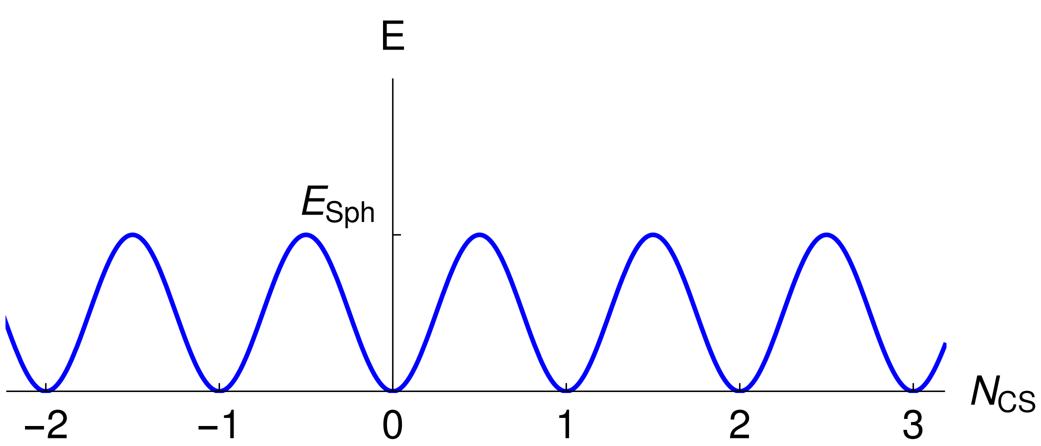
the corresponding potential are not necessarily zero but can be pure gauge fields

$$W_\mu = -\frac{i}{g} U(x) \partial_\mu U^{-1}(x)$$

There are two classes of gauge transformations that keep $W_{\mu\nu} = 0$

- trivial continuous transformations of the potential with $\Delta N_{CS} = 0$
- continuous transformations of the potential with $\Delta N_{CS} \neq 0$ must enter regions where $W_{\mu\nu} \neq 0$

vacuum states with different topological charge are separated by a barrier!



- transition rate for barrier penetration (instanton)

$$\Gamma \sim \exp\left(-\frac{4\pi}{\alpha_W}\right) \sim 10^{-162}$$

- transition rate for "jumping over" the barrier (sphaleron)

static and unstable solutions of the eom

in the symmetric phase

$$\Gamma = k(\alpha_W T)^4$$

in the broken phase

$$\Gamma \sim 2.8 \times 10^5 T^4 \left(\frac{\alpha_W}{4\pi}\right)^4 \kappa \exp\left(-\frac{E_{sph}(T)}{T}\right) \qquad E_{sph}(T) = \frac{2m_W(T)}{\alpha_W} B(\lambda/g_W^2)$$