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Electro-Weak Symmetry Breaking (EWSB) in the Triplet extension of The Standard Model

Master's Dissertation

presented by

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Abstract

The work for this dissertation is two-fold, First to understand the Standard Model(SM) and then look for physics beyond Standard Model. SM as the gauge theory and Electro-weak symmetry breaking in SM is discussed thoroughly.

We investigate the extension of the Standard Model (SM) with hypercharge zero(Y=0) Triplet in the scalar sector,i.e the Higgs Triplet Model (HTM) (SM Doublet + Y=0 Real Triplet). The ρ parameter highly constrains the model at tree level. Another feature of this extension is that the triplet field does not couple to fermions, which makes it hard to produce them at the colliders. The Dark matter in Inert Triplet Model (ITM), i.e. when triplet field does not take part in electroweak symmetry breaking is also discussed, the neutral component of triplet field can be a potential candidate for Dark matter when radiative corrections are taken into account.

We also address the issue of vacuum stability of the Higgs potential in the Standard Model and the Higgs Triplet Model. We studied the bounds on Higgs mass coming from Triviality, Vacuum stability and also investigated the perturbative scale of the theory in both Standard Model and Higgs Triplet Model. Extending the scalar sector also enhances the stability of the potential. The parameter space of HTM is constrained by the theoretical considerations like vacuum stability, perturbativity etc.

KEYWORDS: Beyond Standard Model, Dark matter,Stability

Contents

1 Introduction

The search for new physics beyond standard model has become a top priority task for High Energy Physics (HEP) Community. There are many theoretical issues which can be addressed only in beyond standard model scenarios, and we know that New Physics must exist but we have no definitive sign of this.

On the one hand SM, the gauge theory of electromagnetic, weak, and strong interactions of quarks and leptons, is extremely successful in describing the fundamental interactions of particles. The celebrated Higgs mechanism is a cornerstone in the electroweak region of the SM. As far as gauge theory is concerned, it does do not require the scalar sector to be minimal, and this inspires hope to see new physics through the scalar sector i.e. Higgs can also be present in some different representation. On the other hand, we know that the SM is not a complete description of every physical phenomenon. The reliable indications come from its failure to accommodate dark matter, to provide a viable explanation of the present baryon asymmetry in the Universe. The Yukawa sector i.e. the Yukawa term, the gauge invariant mass term for fermions is also just postulated as it is. Hierarchy problem, Naturalness etc. also remains an unsolved mystery in the SM. SM can't explain the non-zero and extremely small Neutrino mass. Although the Standard Model with a minimal doublet in scalar sector assumes neutrinos to be strictly massless, their non-zero neutrino masses can be accommodated by adding right-handed neutrinos and assuming they get their masses via the same Yukawa-type interactions as charged fermions. However this would be extremely ad-hoc, and many physicists believe this as a clear indication of New Physics. The stability of the vacuum on extrapolation of the self higgs quartic coupling up-to high energy is also not addressed in SM.

In short, many intriguing features seek for explanations, which the SM does not.One of the ways out is Model building based on extended Higgs sectors where we try to find explanations for the issues in SM by taking into account the SM results around the electroweak scale. The scalar sector, which we start to explore, is still very weakly constrained and it can hide new physics treasures waiting to be discovered. It is this hope of existence that drives most of the works on extended scalar sector model building.

2 Gauge Theory

A gauge theory is a theory where the action is invariant under a continuous group symmetry. When the symmetry group depends on spacetime, it is called a local symmetry. The continuous symmetry is called a gauge group and this transformation is called a gauge transformation. We know nature likes symmetries. Symmetry principles underlie most of the dynamics of the Standard Model. Symmetries have important consequences. They imply conserved currents and definite statements about the spectrum of the theory. The first consequence is given by Noether's theorem, which states that for any invariance of the action under a continuous global transformation of the fields there exists conserved currents. The theorem includes both internal and space-time symmetries.
Gauge degrees of freedom[1] can be thought as some additional fictional degrees of freedom in a hypothetical gauge space where you can move around in different directions specified by the parameters of the particle gauge group. In a nutshell, Gauge theory is the mathematical description of the system which is followed by Nature. Apart from this internal symmetry, there are many more other symmetries to explore, but the question that we ask ourselves is that whether these symmetries are exact? Do Nature exactly follows symmetry? This we will explore in the further discussion, and we will see that this is not the case and to explain some more fundamental concept like mass these symmetries must be broken. Gauge theory is the theory of massless particles, and we already know that in SM except for gluons and photons all other the particles are massive so there should be some way out which solves this problem.

2.1 Gauge Theory in Quantum Mechanics

As we all know, the phase of a wave function does not visibly affect the physics of a system. As an example, we choose to ignore(absorb into constants). This means that we can arbitrarily pick a phase to start with and as long as we carry this phase through the problem, no physical effects will be observed(the particle will end up in the correct location). Here the gauge transformation is regarded as the change in phase of a wave function.

2.2 Abelian Gauge Theory

Here we are concerned with the abelian gauge group i.e. a group in which the result of applying the group operation to two group elements does not depend on the order in which they are written. The gauge group that we will discuss is U(1).

2.2.1 Scalar QED

Scalar Quantum Electro Dynamics is the theory of photons coupled to the charged spinless field (complex scalar field). This is one of the most straightforward examples of interacting field theory based on gauge invariance. First, we will discuss the minimal coupling scheme i.e. how to couple matter with the massless field?

The coupling of matter with the massless field is done using Noether Procedure [2].The steps are given below :

Step-1: Linearize the symmetry transformation.
Step-2: Temporarily make the parameter(say α) local.
Step-3: Evaluate the first order change in action. $\delta I = \int d^4x[\partial_\mu \alpha(x) j^\mu]$
Step-4: For Every $\partial_\mu \alpha$ - Replace with the field term and add that term.

Consider the case of coupling of a charged complex scalar field with Maxwell field. Now we apply the Noether's Procedure to find the interaction term.

The Action is given by :

$$I(0) = \int d^4x((\partial^\mu \phi)^\dagger(\partial_\mu \phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}). \tag{1}$$

$$\delta I(0) = \int d^4x g(\partial_\mu \alpha(x) j^\mu(0)), \tag{2}$$

$$j^\mu(0) = i(\phi^\dagger \partial^\mu \phi - \partial^\mu \phi^\dagger \phi) \tag{3}$$

Now for every $\partial_\mu \alpha(x)$ replace by A_μ and add that term.
So the first term to be added is : $gA_\mu j^\mu(0)$.
This term can be decomposed into two terms on taking variation and one of the term cancels the first term.
Repeat this process till $\partial_\mu \alpha$ vanishes.
For this case we have to add two terms these are:

First Term(L_1) : $gA_\mu j^\mu(0)$.
Second Term(L_2) : $g^2 A_\mu A^\mu \phi^\dagger \phi$.

So our lagrangian can be written as:

$$L = L_0 + L_1 + L_2 \tag{4}$$

$$L = (\partial^\mu \phi)^\dagger (\partial_\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + g A_\mu j^\mu(0) + g^2 A_\mu A^\mu \phi^\dagger \phi \tag{5}$$

$$L = (D^\mu \phi)^\dagger (D_\mu \phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{6}$$

$$D_\mu \to \partial_\mu - igA_\mu$$

This is known as Minimal Coupling Scheme.

2.2.2 Quantum Electrodynamics

Quantum Electrodynamics(QED)[3][4] is the theory of photons interacting with matter. We will discuss how to couple electromagnetic field with the fermionic field. This is massless theory and was first given by Richard Feynman.
The lagrangian for QED can be written as

$$L = i\bar{\psi}\gamma^\mu \partial_\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e\bar{\psi}\gamma^\mu A_\mu \psi \tag{7}$$

$$L = i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{8}$$

$$\psi' \to exp(ie\alpha(x))\psi \tag{9}$$

$$A'_\mu = A_\mu - \partial_\mu \alpha(x) \tag{10}$$

Here in the seventh equation we have
First Term - Kinetic term for fermionic field
Second Term - Kinetic term for gauge field
Third Term - Interaction term

So we can replace all the derivatives by covariant derivatives so as to couple to a gauge field.

2.3 Non-Abelian Gauge Theory

Now the above group U(1) can be generalized and we have matrices as the group elements and now the group elements don't commute i.e they are non-abelian. The gauge group of Standard Model have Non-Abelian groups i.e SU(2), SU(3) and in bSM Physics this can be of an arbitrary dimension so we will discuss SU(N) group.

2.3.1 SU(N)

Let's consider the case of N independent complex scalar fields.

$$I(\Phi) = \int d^4x[(\partial^\mu \Phi)^\dagger(\partial_\mu \Phi) - m^2\Phi^\dagger\Phi - V(\Phi^\dagger\Phi) \tag{11}$$

$$\Phi' = U(x)\Phi \tag{12}$$

$$\partial_\mu \Phi' = U(x)\partial_\mu \Phi + \partial_\mu U \Phi \tag{13}$$

The second term in Eq. 13 is the non homogeneous term, So to make the Kinetic term gauge invariant, we need to replace the partial derivative by something else which would transform homogeneously under SU(N). i.e

$$D_\mu \Phi \rightarrow D'_\mu \Phi' = U(x)D_\mu \Phi \tag{14}$$

Now as we have seen earlier, $D_\mu = \partial_\mu + igA_\mu$
Now using the above equation We can find how gauge field transforms i.e

$$D'_\mu \Phi' = U(x)D_\mu \Phi \tag{15}$$

$$A'_\mu = UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \tag{16}$$

Where A_μ is a N X N Matrix, A_μ is itself a non tensor i.e. it transforms inhomogeneously under SU(N). The matrix valued Vector Field A_μ for SU(N) is a generalization of 4-Vector Potential of Electro Dynamics and is referred to as a Non-Abelian Gauge Field.
Now using Minimal Coupling let us couple the multicomponent Scalar Field to a matrix-valued Vector Field A_μ. So we have,

$$(D^\mu \Phi)^\dagger D_\mu \Phi = (\partial^\mu \Phi)^\dagger \partial_\mu \Phi - ig(\Phi^\dagger A^\dagger_\mu \partial^\mu \Phi - \partial_\mu \Phi^\dagger A^\mu \Phi) + g^2\Phi^\dagger A^\dagger_\mu A^\mu \Phi \tag{17}$$

The first term represents the kinetic term for scalars while the other two terms represent the coupling to gauge field.
Now let's try to construct Lagrangian for Non-Abelian case,
The matrix valued field Strength is given by $E_{\mu\nu}$ and it transforms as :-

$$E'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu \tag{18}$$

$$= UE_{\mu\nu}U^{-1} + [B_\mu, \bar{A}_\nu] - [B_\nu, \bar{A}_\mu] + \frac{i}{g}[B_\mu, B_\nu] \tag{19}$$

Where $B_\mu = (\partial_\mu U)U^{-1}$,and $\bar{A}_\mu = UA_\mu U^{-1}$. So this doesn't transform homogeneously, So we define the field strength using the covariant derivative as:

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \tag{20}$$

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \tag{21}$$

Now a little calculation shows that new field strength transform homogeneously, i.e

$$F_{\mu\nu} = UF_{\mu\nu}U^{-1}$$

Now we can construct our Lagrangian as :-

$$L = -\frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu}) \tag{22}$$

which is a gauge invariant term.

2.3.2 Yang-Mills Theory

Consider the case of internal Symmetry group SU(2) such as isospin under which ψ transform as a doublet.

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

The Free field lagrangian is given as

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi \tag{23}$$

Where $\bar{\psi}$ is a row vector and ψ is a column vector in isospin space. The transformation matrix for SU(2) is given as

$$U = exp(-ig\alpha^a T_a)$$

where g is the coupling constant, $\alpha_a = [\alpha_1, \alpha_2, \alpha_3]$ are the parameters and $T_a = [T_1, T_2, T_3]$ are the generators of the group and they follow the Lie Algebra.

$$[T_i, T_j] = \iota\epsilon_{ijk}T_k \tag{24}$$

The generators of this group $T_i = \sigma_i/2$ are given as :

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} and\, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Now we can make our Lagrangian locally gauge invariant by minimal coupling scheme by introducing the covariant derivative as :

$$L = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi\ , \tag{25}$$

$$D_\mu = \partial_\mu + igW_\mu^a T_a\ , \tag{26}$$

$$W_\mu^a = [\, W_\mu^1, W_\mu^2, W_\mu^3\,]\ . \tag{27}$$

Now the transformation law for W_μ and $W_{\mu\nu}$ can be derived from the SU(N) algebra as discussed before by taking N=2, taking the infinitesimal form of transformation matrix(U) and using the fact that,

$$(a.\tau)(b.\tau) = (a.b) + \iota\tau.(a \times b)$$

and we get,

$$W'_\mu = W_\mu + \partial_\mu \alpha + g(\alpha \times W_\mu), \tag{28}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu \tag{29}$$

So the most general Yang-Mills Lagrangian invariant under SU(2) local gauge symmetry is given by :

$$L_{YangMills} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} Tr(W_{\mu\nu} W^{\mu\nu}) \tag{30}$$

3 The Brout-Englert-Higgs mechanism

The Gauge theory predicts all the excitations of the fields to be massless, but we all know except gluons and photons all fundamental particles have mass. Higgs Mechanism is the mechanism that gives mass to these particles after electro-weak symmetry breaking. Goldstone's Theorem states that for every spontaneously broken generator, of a global symmetry group, there must be a corresponding Nambu-Goldstone boson. Whenever a local symmetry is broken the goldstone disappears ,and we get massive vector bosons. The words used to describe this are that the vector boson becomes massive by "eating" the would-be Nambu-Goldstone boson, which becomes its longitudinal polarization degree of freedom. This is called the Higgs mechanism.

To study higgs mechanism, Let's go back to our Scalar QED Lagrangian with a complex scalar field ϕ which transforms under a gauge transformation like:

$$\phi \to e^{\iota\theta(x)}\phi \tag{31}$$

$$\phi^\dagger \to \phi^\dagger e^{-\iota\theta(x)} \tag{32}$$

The Lagrangian density for Scalar QED is given as

$$L = (D_\mu\phi)^\dagger D^\mu\phi - V(\phi, \phi^\dagger) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{33}$$

where the covariant defined as discussed before is given by

$$D_\mu\phi = (\partial_\mu + \iota g A_\mu)\phi \tag{34}$$

$$\tag{35}$$

and potential is given as

$$V(\phi) = \mu^2 \mid \phi^\dagger \phi \mid + \lambda(\mid \phi^\dagger \phi \mid)^2 \tag{36}$$

where, $\lambda \equiv \text{scalar self coupling},\ \lambda > 0 \quad \text{and}\ \mu^2 \equiv \text{scalar mass parameter}\mu^2 < 0.$

$$< 0|\phi|0 >= \frac{v}{\sqrt{2}}, \tag{37}$$

In Unitary gauge We have :

$$D_\mu \phi = \frac{1}{\sqrt{2}}[\partial_\mu h + \iota g V_\mu (v + h)] e^{\frac{\iota G}{v}} \tag{38}$$

$$D_\mu \phi^\dagger = \frac{1}{\sqrt{2}}[\partial_\mu h - \iota g V_\mu (v + h)] e^{\frac{-\iota G}{v}} \tag{39}$$

$$V_\mu = A_\mu + \frac{1}{gv}\partial_\mu G \tag{40}$$

Now the vector field strength is written in terms of the new vector V_μ as :

$$\frac{1}{4}F^{\mu\nu}F_{\mu\nu} = \frac{1}{4}(\partial_\mu V_\nu - \partial_\nu V_\mu)(\partial^\mu V^\nu - \partial^\nu V^\mu) \tag{41}$$

After symmetry breaking the lagrangian becomes :

$$L = \frac{1}{2}[\partial^\mu h \partial_\mu h + g^2 (v + h)^2 V^\mu V_\mu] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \lambda(vh + \frac{h^2}{2})^2 \tag{42}$$

$$\tag{43}$$

Now as discussed before we see that there is a new V_μ field and G field disappears.There are quadratic terms for V_μ and h fields so excitations corresponding to these fields are massive. :

$$m_h^2 = 2\lambda v^2 \tag{44}$$

$$m_V^2 = g^2 v^2 \tag{45}$$

,so after the spontaneous breaking of a local gauge symmetry, we get a massive vector boson.

4 The Standard Model

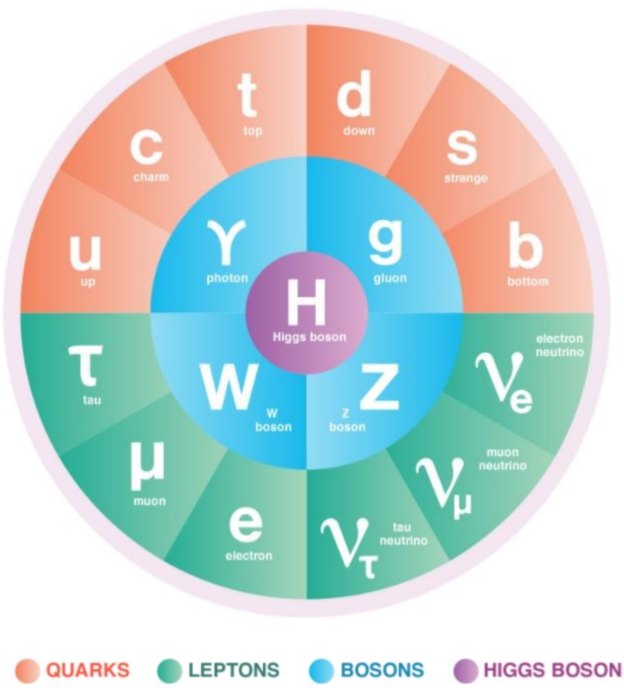
The Standard Model is local gauge theory described by the following gauge group:

$$SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y$$

- $SU(3)_C$ describes the rotations in color space. We have 8 d.o.f's that corresponds to 8 gluons.
- $SU(2)_L$ describes the rotation in weak isospin space. We have 3 d.o.f's that corresponds to three weak gauge bosons i.e. W^+, W^-, Z
- $U(1)_Y$ describes the rotation in hyper-charge space and we have one gauge boson.

$SU(3)_C$, $SU(2)_L$ are the non-abelian gauge groups while $U(1)_Y$ is the abelian gauge group.

4.1 Symmetries, gauge theory and particle content



The $SU(2)_L \bigotimes U(1)_Y$ forms the electro-weak model. The third component of $SU(2)_L$ and $U(1)_Y$ mixes to give Z boson and massless photon i.e. even after symmetry breaking there is electromagnetic symmetry, and the gauge group $U(1)_{EM}$ remains unbroken.

$$SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$$

The $SU(2)_L$ subgroup is known as weak isospin. Left-handed SM fermions are doublets under $SU(2)_L$:

$$\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}, \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

The right-handed fermions are all singlets under $SU(2)_L$.

$$e_R, \quad \mu_R, \quad \tau_R, \quad u_R, \quad d_R, \quad c_R, \quad s_R, \quad t_R, \quad b_R$$

4.2 The Electro-Weak Model

It is known as the unbroken $SU(2)_L \bigotimes U(1)_Y$.

This describes the weak and the electromagnetic interactions and the gauge bosons corresponding to these forces are massless in this model.

The Lagrangian for gauge field is given by:

$$L_{gauge} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{46}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu \tag{47}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{48}$$

The Lagrangian coupled with the fermionic field is :

$$L = -\frac{1}{4} Tr(W_{\mu\nu} W^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \iota\bar{\psi}\gamma^\mu D_\mu \psi \tag{49}$$

$$D_\mu \psi_L = [\partial_\mu + \iota g W^a_\mu T_a + \iota g' B_\mu Y_{lL}]\psi_L \tag{50}$$

$$D_\mu \psi_R = [\partial_\mu + \iota g' B_\mu Y_{lR}]\psi_R \tag{51}$$

The interactions of the electroweak gauge bosons with fermions are determined by the covariant derivative. For example, the covariant derivatives acting on the lepton fields are:

$$D_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} = [\partial_\mu + \iota g' B_\mu Y_{lL} + \iota g W^a_\mu T^a] \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \tag{52}$$

$$D_\mu e_R = [\partial_\mu + \iota g' B_\mu Y_{lR}] e_R \tag{53}$$

Where Y_{lL} and Y_{lR} are the weak hypercharges of left-handed leptons and right-handed leptons, and 2×2 unit matrices are understood to go with the ∂_μ and B_μ terms in above equation. A multiplicative factor can always be absorbed into the definition of the coupling g', so without loss of generality, it is traditional to set $Y_{lR} = Q_l = -1$. The weak hypercharges of all other fermions are then fixed. Using the explicit form of the $SU(2)_L$

generators in terms of Pauli matrices, the covariant derivative of left-handed leptons is:

$$D_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} = \partial_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} + \iota[g'Y_{lL} \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} + \frac{g}{2} \begin{pmatrix} W_\mu^3 & W_\mu^1 - \iota W_\mu^2 \\ W_\mu^1 + \iota W_\mu^2 & -W_\mu^3 \end{pmatrix}] \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \tag{54}$$

Therefore, the covariant derivatives of the lepton fields can be summarized as:

$$D_\mu \nu_e = \partial_\mu \nu_e + \iota(g'Y_{lL} + \frac{g}{2} W_\mu^3)\nu_e + \iota\frac{g}{2}(W_\mu^1 - \iota W_\mu^2)e_L \tag{55}$$

$$D_\mu e_L = \partial_\mu e_L + \iota(g'Y_{lL} - \frac{g}{2} W_\mu^3)e_L + \iota\frac{g}{2}(W_\mu^1 + \iota W_\mu^2)\nu_e \tag{56}$$

$$D_\mu e_R = \partial_\mu e_R - \iota g' B_\mu e_L \tag{57}$$

The covariant derivative of a field must carry the same electric charge as the field itself, for the charge to be conserved. Evidently, then, $W_\mu^1 - \iota W_\mu^2$ must carry electric charge +1 and $W_\mu^1 + \iota W_\mu^2$ must carry electric charge -1, so these must be identified with $W^\pm$ bosons of the weak interactions. Consider the interaction Lagrangian following from

$$L = \iota \begin{pmatrix} \bar{\nu_e} & \bar{e_L} \end{pmatrix} \gamma^\mu D_\mu \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \tag{58}$$

$$= -\frac{g}{2}\bar{\nu_e}\gamma^\mu e_L(W_\mu^1 - \iota W_\mu^2) - \frac{g}{2}\bar{e_L}\gamma^\mu \nu_e(W_\mu^1 + \iota W_\mu^2) + ... \tag{59}$$

$$W_\mu^+ = \frac{1}{\sqrt{2}}(W_\mu^1 - \iota W_\mu^2); \tag{60}$$

$$W_\mu^- = \frac{1}{\sqrt{2}}(W_\mu^1 + \iota W_\mu^2) \tag{61}$$

The vector bosons B_μ and W_μ^3 are both electrically neutral. As a result of spontaneous symmetry breaking, we will find that they mix. In other words, the fields with well-defined masses (”mass eigenstates”) are not B_μ and W_μ^3, but are orthogonal linear combinations of these two gauge eigenstate fields. One of the mass eigenstates is the photon field A_μ, and the other is the massive Z boson vector field, Z_μ. One can write the relation between the gauge eigenstate and mass eigenstate fields as a rotation in field space by an angle θ_W, known as the weak mixing angle or Weinberg angle:

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} cos\theta_W & sin\theta_W \\ -sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \tag{62}$$

with the inverse relation:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} cos\theta_W & -sin\theta_W \\ sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \tag{63}$$

We now require that the resulting theory has the correct photon coupling to fermions, by requiring that the field A_μ appears in the covariant derivatives in the way dictated by QED. The covariant derivative of the right-handed electron field can be written:

$$D_\mu e_R = \partial_\mu e_R - \iota g' cos\theta_W A_\mu e_R + \iota g' sin\theta_W Z_\mu e_R. \tag{64}$$

Comparing to $D_\mu e_R = \partial_\mu e_R - \iota e A_\mu e_R$ from QED, we conclude that:

$$g' cos\theta_W = e \tag{65}$$

$$D_\mu e_L = \partial_\mu e_L + \iota(g' Y_{lL} cos\theta_W - \frac{g}{2} sin\theta_W) A_\mu e_L + ... \tag{66}$$

Again comparing to the prediction of QED that $D_\mu e_L = \partial_\mu e_L - \iota e A_\mu e_L$, it must be that:

$$\frac{g}{2} sin\theta_W - g' Y_{lL} cos\theta_W = e \tag{67}$$

is the electromagnetic coupling. In the same way:

$$D_\mu \nu_e = \partial_\mu \nu_e + \iota(g' Y_{lL} cos\theta_W + \frac{g}{2} sin\theta_W) A_\mu \nu_e + ... \tag{68}$$

where the . . . represent W and Z terms. However, we know that the neutrino has no electric charge, and therefore its covariant derivative cannot involve the photon. So, the coefficient of A_μ must vanish:

$$\frac{g}{2} sin\theta_W + g' Y_{lL} cos\theta_W = 0. \tag{69}$$

$$Y_{lL} = -\frac{1}{2}, \tag{70}$$

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \tag{71}$$

$$tan\theta_W = \frac{g'}{g} \tag{72}$$

so that

$$sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}; \qquad cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{73}$$

These are requirements that will have to be satisfied by the spontaneous symmetry breaking mechanism. The present numerical values from experiment are approximately:

$$g = 0.652; \tag{74}$$

$$g' = 0.357; \tag{75}$$

$$e = 0.313; \tag{76}$$

$$sin^2\theta_W = 0.231. \tag{77}$$

4.3 Electro-weak symmetry breaking in SM

By higgs mechanism[5] all the fundamental particles get mass but photons and gluons remain massless as $U(1)_{EM}$ and $SU(3)_C$ remain as the symmetry of vacuum. At a basic level, one views higgs mechanism as a phase transition. One passes from the electroweak symmetric phase to the Higgs phase. In the case of Higgs phenomenon, the vacuum expectation value (*vev*) of the Higgs field plays the role of the order parameter. Because of this phase transition, weak force carrier particles and the Higgs acquire mass. Quarks and leptons gain mass through the postulated Yukawa term after EWSB.
The scalar part of the Lagrangian is given as

$$\mathcal{L}_s = (D^\mu\phi)^\dagger(D_\mu\phi) - \mu^2 \mid \phi^\dagger\phi \mid -\lambda(\mid \phi^\dagger\phi \mid)^2 \,, \tag{78}$$

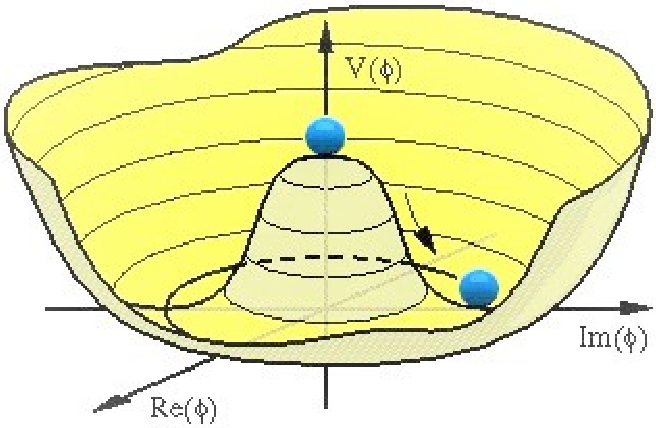
Where ϕ is a complex scalar $SU(2)_L$ doublet, i.e $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.
It couples to the gauge boson in a gauge invariant way, and the covariant derivative involving the gauge-fields is given by :

$$D_\mu = \partial_\mu - \iota\frac{g}{2}\tau \cdot W_\mu - \iota\frac{g'}{2}B_\mu Y \,. \tag{79}$$

where Y is the hypercharge and τ_a are the pauli matrices (generators of $SU(2)_L$) and W_μ is the gauge field corresponding to $SU(2)_L$ and B_μ is the gauge field corresponding to $U(1)_Y$. Higgs field gets non-zero vev at the minimum of the potential,

$$< v >= \sqrt{-\frac{\mu^2}{2\lambda}} \tag{80}$$

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h \end{pmatrix} \tag{81}$$



After EWSB the potential becomes a Mexican hat potential where the minima of the field is given by a circle in the complex plane and acquire a non-zero vev. If we choose a point in the minima and look around the potential is no more symmetric. Now we we look for fluctuation around the minima i.e. we expand the potential around the minima and try to see what happens. The field ϕ here is written in Unitary gauge, where we remove the goldstone bosons i.e the charged part ϕ^+ gives the pair of charged goldstones $G^\pm$ and the complex part of the neutral component becomes the neutral goldstone. Now solving the first part of the scalar part of the lagrangian we get,

$$(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{(v+h)^2}{4}[g^2W_\mu^+W^{-\mu} + \frac{1}{2}(gW_\mu^3 - g'B_\mu)(gW^{\mu 3} + g'B^\mu)] \quad (82)$$

The higgs doublet's contribution to the gauge boson masses-squared in matrix form in the basis of (W_1, W_2, W_3, B) as :

$$M_\phi^2 = \frac{v_\phi^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix} \quad (83)$$

This is a block diagonal matrix which is already diagonalized in the above 2 x 2 block and the lower 2 x 2 block is non-diagonal which tells us that there is mixing between the W_μ^3 and B_μ field. This is diagonalized by the weak mixing angle :

$$Sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}} \quad (84)$$

$$\begin{bmatrix} W_\mu^3 \\ B_\mu \end{bmatrix} = \begin{bmatrix} cos\theta_w & sin\theta_w \\ sin\theta_w & -cos\theta_w \end{bmatrix}\begin{bmatrix} Z_\mu \\ A_\mu \end{bmatrix} \quad (85)$$

This diagonalization gives the Z boson and (massless) photon eigenstates. In terms of W^+, W^-, Z we can write the first term as :

$$(D^\mu\Phi)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu h\partial_\mu h + \frac{(v+h)^2}{4}[g^2W^{-\mu}W_\mu^+ + \frac{1}{2}(g^2+g'^2)Z^\mu Z_\mu] \quad (86)$$

The masses of $W^\pm$, and Z boson are given as :

$$M_z^2 = \frac{(g^2+g'^2)v_{SM}^2}{4} \quad (87)$$

$$M_w^2 = \frac{g^2 v_{SM}^2}{4} \quad (88)$$

$$\rho = \frac{M_w^2}{cos^2\theta_w M_z^2} \quad (89)$$

The electro-weak parameter ρ is precisely measured experimentally and is

$$\rho = 1.0004 \pm 0.00024 \tag{90}$$

The ρ parameter is of utmost importance as when we will extend the higgs sector with a different higgs representation, the model has to respect this parameter and can be highly constraint due to ρ parameter. Now looking at the potential part of the scalar sector of the lagrangian

$$V(\Phi) = \mu^2 \mid \Phi^\dagger \Phi \mid + \lambda(\mid \Phi^\dagger \Phi \mid)^2 \tag{91}$$

After EWSB the potential becomes

$$V(\phi^\dagger \phi) = \lambda v^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^2 \tag{92}$$

The first term is the mass term for higgs field h while the third and the fourth term represents the three point and four point self-interaction of the higgs field.The mass of the fluctuation of the higgs field is :

$$M_h^2 = 2\lambda v^2 \tag{93}$$

This is the standard model higgs discovered at LHC around 125GeV.
The higgs potential and vacuum both respects the electromagnetic symmetry and after EWSB, the gauge group becomes :

$$SU(2)_L \bigotimes U(1)_Y \rightarrow U(1)_{EM} \tag{94}$$

The mass of fermions in the standard model is generated by postulated Yukawa term.The guiding principle for any term in lagrangian is that it should be real, gauge invariant and Lorentz invariant. The yukawa term follows all these principles and the minimal requirement for this term to be gauge invariant is a SU(2) higgs doublet.

In the Standard Model, we have three families of fermions which are shown as:

$$l_R^j = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \qquad l_L^j = \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \qquad \nu_i = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$d_R^j = \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \qquad d_L^j = \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \qquad u_L^j = \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \qquad u_R^j = \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

so the most general yukawa term is written as:

$$L_{e,\mu\tau} = -\begin{pmatrix} \bar{\nu_e}^i & \bar{l_L}^i \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y_{ei}^j l_R^j + h.c \tag{95}$$

$$L_{d,s,b} = -\begin{pmatrix} \bar{u_L}^i & \bar{d_L}^i \end{pmatrix} \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} y^j_{di} d^j_R + h.c \tag{96}$$

$$L_{u,c,t} = -\begin{pmatrix} \bar{u_L}^i & \bar{d_L}^i \end{pmatrix} \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix} y^j_{ui} u^j_R + h.c \tag{97}$$

where the i, j indices run over the three families.
In general, the Yukawa couplings $y^j_{ei}, y^j_{di}, y^j_{ui}$ are complex 3×3 matrices in gauge basis.After EWSB in unitary gauge the yukawa term is written as:

$$L = -(1+\frac{h}{v})(\bar{l}'^i_L m^j_{ei} l'_{Rj} + \bar{d}'^i_L m^j_{di} d'_{Rj} + \bar{u}'^i_L m^j_{ui} u'_{Rj}) + h.c \tag{98}$$

where

$$m^j_{fi} = \frac{v}{\sqrt{2}} y^j_{fi} \tag{99}$$

Now redefining the fields, we go in the mass basis where these matrices are diagonalized as :

$$l_{Li} = L^j_{Li} l_{Lj}; \qquad l_{Ri} = L^j_{Li} l_{Rj} \qquad d'_{Li} = D^j_{Li} d_{Lj}\ ;$$
$$d'_{Ri} = D^j_{Ri} d_{Rj}\ ; \quad u'_{Li} = U^j_{Li} u_{Lj};\ u'_{Ri} = U^j_{Ri} u_{Rj}$$

After diagonalization the matrices become :

$$L^\dagger_L m_e l_R = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} \tag{100}$$

$$D^\dagger_L m_d D_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \tag{101}$$

$$U^\dagger_L m_u U_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \tag{102}$$

ad the yukawa term becomes

$$L_{quarks} = -(1+\frac{h}{v})(\bar{u_L} m_u u_R + \bar{c_L} m_c c_R + \bar{t_L} m_t t_R + \bar{d_L} m_d d_R + \bar{s_L} m_s s_R + \bar{b_L} m_b b_R) \tag{103}$$

$$L_{leptons} = -(1+\frac{h}{v})(\bar{e}_L m_e e_R + \bar{\mu}_L m_\mu \mu_R + \bar{\tau}_L m_\tau \tau_R) \tag{104}$$

The first term in both the above expressions is the mass term for quarks and leptons while the second term represents the interaction of higgs with fermions.

5 Custodial Symmetry in the framework of SM

The custodial symmetry[6] is an approximate (accidental) global symmetry in the Higgs sector. The scalar part of Lagrangian is written as :

$$\mathcal{L}_s = Tr[(D^\mu\Phi)^\dagger(D_\mu\Phi)] - \mu^2 Tr[(\Phi^\dagger\Phi)] + \lambda(Tr[\Phi^\dagger\Phi])^2 \,, \tag{105}$$

where Φ is constructed as shown below :

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} : \qquad \epsilon\phi^* = \iota\sigma_2 \begin{pmatrix} \phi^{+*} \\ \phi^{0*} \end{pmatrix} ; \tag{106}$$

$$\epsilon\phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}, where \quad (\phi^- = \phi^{+*}) \tag{107}$$

We define a bi-doublet field

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \tag{108}$$

Now

$$L_{Higgs} = Tr((D^\mu\Phi)^\dagger D_\mu\Phi) - \mu^2 Tr(\Phi^\dagger\Phi) - \lambda(Tr(\Phi^\dagger\Phi))^2 \tag{109}$$

Under $\qquad SU(2)_L : \qquad \Phi \to L\Phi$

$$= Tr((D^\mu L\Phi)^\dagger(D_\mu L\Phi)) - \mu^2 Tr(L\Phi)^\dagger(L\Phi) - \lambda(Tr((L\Phi)^\dagger)L\Phi)))^2 \tag{110}$$

$$= Tr((D^\mu\Phi)^\dagger L^\dagger L D_\mu\Phi) - \mu^2 Tr(\Phi^\dagger L^\dagger L\Phi) - \lambda(Tr(\Phi^\dagger L^\dagger L\Phi))^2 \tag{111}$$

$$= Tr((D^\mu\Phi)^\dagger D_\mu\Phi) - \mu^2 Tr(\Phi^\dagger\Phi) - \lambda(Tr(\Phi^\dagger\Phi))^2 \tag{112}$$

is invariant under $SU(2)_L$. More precisely we can see as:

$$D_\mu\Phi = \partial_\mu\Phi - \frac{\iota}{2} g W_\mu^a \sigma^a \Phi 0 - \frac{\iota}{2} g' B_\mu \sigma_3 \tag{113}$$

So,

$$Tr(\partial^\mu\Phi^\dagger\partial_\mu\Phi) \Rightarrow Tr(\partial^\mu\Phi^\dagger L^\dagger L\partial_\mu\Phi) \Rightarrow Tr(\partial^\mu\Phi^\dagger\partial_\mu\Phi) \tag{114}$$

$$Tr(\partial^\mu\Phi^\dagger W_\mu^a\sigma^a\Phi) \Rightarrow Tr(\partial^\mu\Phi^\dagger L^\dagger L W_\mu^a\sigma^a L^\dagger L\Phi) \Rightarrow Tr(\partial^\mu\Phi^\dagger W_\mu^a\sigma^a\Phi) \tag{115}$$

$$Tr(\partial_\mu\Phi^\dagger B^\mu\Phi\sigma_3) \Rightarrow Tr(\partial_\mu\Phi^\dagger L^\dagger L B^\mu L^\dagger L\Phi\sigma_3) \Rightarrow Tr(\partial_\mu\Phi^\dagger B^\mu\Phi\sigma_3) \tag{116}$$

Now due to the trace, we have another symmetry
Under $\qquad SU(2)_R : \qquad \Phi \to \Phi R^\dagger$

$$Tr(\partial^\mu\Phi^\dagger\partial_\mu\Phi) \Rightarrow Tr(R\partial^\mu\Phi^\dagger\partial_\mu\Phi R^\dagger) \Rightarrow Tr(\partial^\mu\Phi^\dagger\partial_\mu\Phi) \tag{117}$$

$$Tr(\partial^\mu\Phi^\dagger W_\mu^a\sigma^a\Phi) \Rightarrow Tr(R\partial^\mu\Phi^\dagger W_\mu^a\sigma^a\Phi R^\dagger) \Rightarrow Tr(\partial^\mu\Phi^\dagger W_\mu^a\sigma^a\Phi) \tag{118}$$

$$Tr(\partial_\mu\Phi^\dagger B^\mu\Phi\sigma_3) \Rightarrow Tr(R\partial_\mu\Phi^\dagger B^\mu\Phi R^\dagger\sigma_3) \Rightarrow Tr(\partial_\mu\Phi^\dagger B^\mu\Phi R^\dagger\sigma_3 R) \tag{119}$$

Since $R^\dagger \sigma_3 R \neq \sigma_3$. So the last term is not invariant but if we impose $g' = 0$ the $SU(2)_R$ symmetry is restored as the term which spoils will not be there.
Thus in limit $g' = 0$, the higgs sector of Standard Model has global symmetry

$$SU(2)_L \times SU(2)_R$$

$$SU(2)_L \rightarrow \text{a global version of gauge symmetry.}$$
$$SU(2)_R \rightarrow \text{approximate(accidental)global symmetry.}$$

Under $SU(2)_L \times SU(2)_R$ the field Φ transforms as :

$$\Phi \rightarrow L\Phi R^\dagger$$

Now when Higgs field acquires a vacuum expectation value

$$< \Phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \tag{120}$$

This breaks both $SU(2)_L$ and $SU(2)_R$

$$L < \Phi > \neq < \Phi >, \qquad < \Phi > R^\dagger \neq < \Phi > \tag{121}$$

but leaves unbroken the subgroup $SU(2)_{L+R}$.

$$SU(2)_{L+R} : L\Phi L^\dagger \qquad i.e. \quad when \quad L = R \tag{122}$$
$$L < \Phi > L^\dagger = < \Phi > \tag{123}$$

Thus the Higgs vacuum expectation value breaks the gloabal symmetry as

$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R} \tag{124}$$

This is called custodial symmetry.

- Number of broken generators= 3+3-3=3.
- So, by Goldstone theorem, this gives rise to three massless Goldstone bosons, which are then eaten by Higgs mechanism to provide mass to $W^\pm$ and Z bosons.

The ρ parameter is given by :

$$\rho = \frac{M_W^2}{M_Z^2 cos^2\theta_W} = \frac{g^2}{g^2 + g'^2}\ , \tag{125}$$

Thus in limit $g' \rightarrow 0$ $W^\pm$, Z form a triplet of an unbroken global symmetry. This explains why $M_W = M_Z$. Custodial symmetry also helps us understand properties of the theory beyond tree level. Due to the unbroken $SU(2)_{L+R}$ in the $g' \rightarrow 0$ limit, radiative corrections to the ρ parameter due to gauge and Higgs bosons must be proportional to g', and this helps in maintaining $\rho = 1$.

6 Near Criticality of the EWSB

6.1 Triviality

In the renormalization group approach, the scalar self-coupling λ is turned into a running coupling $\lambda(\mu)$ varying with the momentum scale μ characteristic of the process considered. The study of one-loop radiative corrections allows us to compute to lowest order (in λ) the evolution of $\lambda(\mu)$ with the scale μ, i.e. its beta function
(Detailed calculation given in Appendix-I):

$$\mu\frac{d\lambda}{d\mu} = \frac{3}{2\pi^2}\lambda^2 \tag{126}$$

We see that the coupling $\lambda(\mu)$ is monotonically increasing. If we want the theory to make sense all the way up to the scale Λ, we must impose that $\lambda(\mu) < \infty$ for scales $\mu < \Lambda$. If Λ is known, this imposes some bound on the value of λ . For example, if we send Λ to infinity, this imposes $\lambda(v) = 0$. This is why a theory described by an action which would valid at all energy scales is known as trivial, i.e. is a free field theory in the low energy regime. In practice, this only means that at some scale Λ smaller than the scale Λ_{Landau} where the coupling would hit the Landau pole. Thus, solving forλ the differential equation, we get

$$\lambda^{-1}(\mu) = \lambda^{-1} - \frac{3}{2\pi^2}ln\frac{\mu}{v}, \tag{127}$$

where $\lambda = \lambda(v)$, one obtains, using $\lambda^{-1}(\Lambda_{Landau}) = 0$,

$$\Lambda_{Landau} \sim ve^{\frac{2\pi^2}{(3\lambda)}}. \tag{128}$$

Since λ can be expressed in terms of m_h itself, this is used to put an upper bound, a triviality bound, on the scale of new physics.

$$\Lambda < ve^{\frac{4\pi^2v^2}{(3m_h^2)}} = \Lambda_T(m_h). \tag{129}$$

The right-hand side is a monotonically decreasing function of m_h. Alternatively, one may say that, for a given value of Λ, the Higgs mass is bounded by

$$m_h^2 < \frac{4\pi^2v^2}{3ln\frac{\Lambda}{v}}, \tag{130}$$

a decreasing function of Λ.
From the above equation, we can calculate bound on higgs mass at particular scale. The bound on variation of higgs mass with Scale is plotted in figure 1

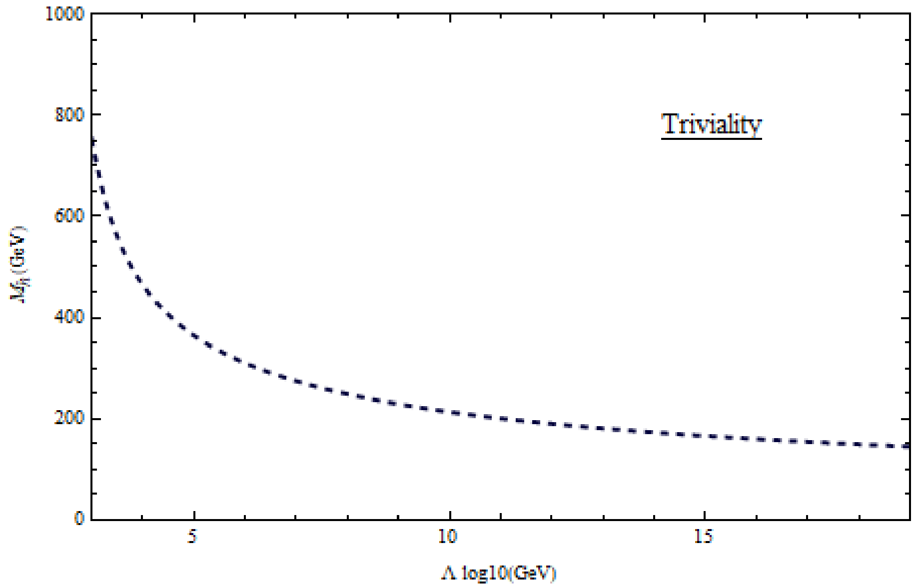


Figure 1: Bound on higgs mass from Triviality

We see that mass of higgs decreases with increasing scale. This means that if we say our theory is valid till Planck scale i.e 10^{19} GeV then it sets an upper limit on the mass of Higgs as shown in figure 1. So theoretically triviality sets an upper bound on Higgs mass when we demand that our theory is valid up-to Planck scale.

6.2 Perturbativity

There are two scenarios in which perturbation theory can break down. First, the Higgs self-coupling can be nonperturbative for large values of M_h. Secondly, the Higgs running coupling can be nonperturbative when scale μ is very large. Increasing the scale μ in Eq. (131), $\lambda(\mu)$ eventually becomes infinite, indicating the one-loop Landau pole. Perturbation theory ceases to be reliable long before reaching the location of this pole.
Our theory is perturbative up to some scale up to when λ is of $\mathcal{O}(\lambda)\sim 1$. So We can set $\lambda(\mu)$ to be maximum of 1 to calculate the scale up to which our theory is perturbative.

$$\lambda(\mu) = \frac{\lambda(v)}{1 - \frac{3}{2\pi^2}\lambda(v)\ln\frac{\mu}{v}} \tag{131}$$

We illustrate the scale up to which theory is perturbative for different values of $\lambda(\mu)$ in Table 1.For the maximum value of $\lambda(\mu) = 1$ we get our theory is perturbative up to a scale 1.38×10^{22} GeV.

S.No.	Value of $\lambda(\mu)$	Perturbative Scale Λ
1	0.1	0.000264
2	0.2	5.132×10^{10}
3	0.3	2.971×10^{15}
4	0.4	7.148×10^{17}
5	0.5	1.918×10^{19}
6	0.6	1.719×10^{20}
7	0.7	8.239×10^{20}
8	0.8	2.667×10^{21}
9	0.9	6.653×10^{21}
10	1	1.382×10^{22}

Table 1: Scale up to which theory is Perturbative, for different values of $\lambda(\mu)$

6.3 Vacuum Stability

An ever-increasing $\lambda(\mu)$ coupling leads to the constraints just discussed. An ever decreasing $\lambda(\mu)$ coupling leads to difficulties of another kind: as soon as the quartic coupling $\lambda(\mu)$ turns negative, the potential becomes unbounded from below at low values of the field ϕ and the theory suffers from an instability. For example, if the Standard Model Higgs is light, the dominant term comes from the top Yukawa interaction:

$$\mu\frac{d\lambda}{d\mu} = -\frac{3}{8\pi^2}y_t^4 \tag{132}$$

If we neglect gauge interactions, then the top Yukawa coupling $y_t(\mu)$ does not run and $\lambda(\mu)$ is solved as:

$$\lambda(\mu) = \lambda - \frac{3}{8\pi^2}y_t^4 ln\frac{\mu}{v}. \tag{133}$$

Defining the scale Λ_U at which the instability appears by the condition $\lambda(\Lambda_U) = 0$, we get

$$\Lambda_U = ve^{\frac{8\pi^2\lambda}{(3y_t^4)}}. \tag{134}$$

Then the scale of new physics Λ is constrained by

$$\Lambda < \Lambda_U = ve^{\frac{\pi^2 m_h^2 v^2}{(3m_t^4)}} \tag{135}$$

Where we have used $m_t^2 = \frac{y_t^2 v^2}{2}$. Alternatively, for a given value of Λ, we have

$$m_h^2 > \frac{3m_t^4}{\pi^2 v^2} ln \frac{\Lambda}{v}. \tag{136}$$

Again, this formula just shows a trend (for example that the vacuum stability bound increases with Λ, as well as with the top mass). For large enough Λ, it is not possible to neglect the running of λ_t due to gauge interactions. This strengthens the bound on m_h. From the Eq. 136, we can calculate bound on higgs mass at particular scale. The bound on variation of higgs mass with Scale is plotted in figure 2

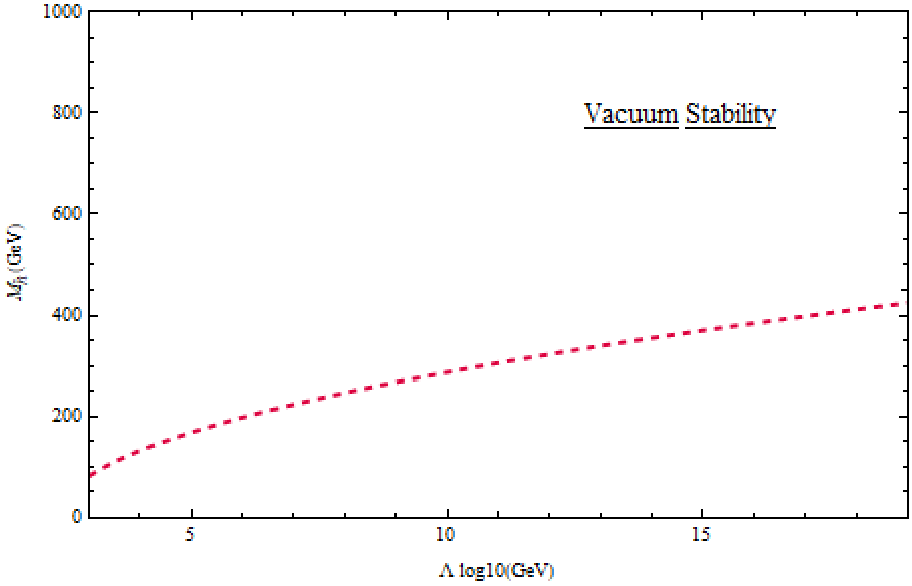


Figure 2: Bound on higgs mass from Vacuum Stability

In figure 2 we have ignored the contribution of gauge interactions on running of self Higgs quartic coupling, this provides a fair idea of how bound on Higgs mass changes with scale.

We see that mass of higgs increases with increasing scale. This means that if we say our theory is valid till Planck scale i.e 10^{19} GeV then it sets a lower limit on the mass of Higgs as shown in figure 2. So theoretically Vacuum Stability sets an lower bound on Higgs mass when we demand that our theory is valid up to Planck scale.

6.4 Running of Couplings

In Standard model, we have three gauge couplings and nine yukawa couplings corresponding to each quark and lepton. In the scalar sector we have two parameters, self higgs quartic coupling and mass parameter. At tree level, all these couplings are constant. When we look for the quantum corrections in our theory, we get ultraviolet divergences and to remove these divergences we renormalize our theory (our results should be physical).In doing so the divergences are absorbed in the parameters which are not physical and also our theory become scale dependent. The parameters of the renormalized theory are no longer constant but scale dependent e.g $\lambda(\mu)$ and that is why we call them as running couplings[7].

The running of couplings is given by the beta function β, The β function is the rate of change of renormalized coupling at the a particular scale corresponding to a fixed bare coupling.A positive sign of β function indicates a renormalized coupling that increases at large momenta and decreases at small momenta and vice versa.
(The derivation of beta function in detail is given in Appendix-II)

SARAH is a Mathematica package that gives us the beta function for all the coupling constants of the Standard model and other extensions of the standard model at two loop level. We learnt SARAH, how to load, initialize and perform the calculations etc. We developed a code to solve the coupled differential equations and plot the running of couplings.
(Beta function[8] for all couplings of Standard model are given in Appendix - III)

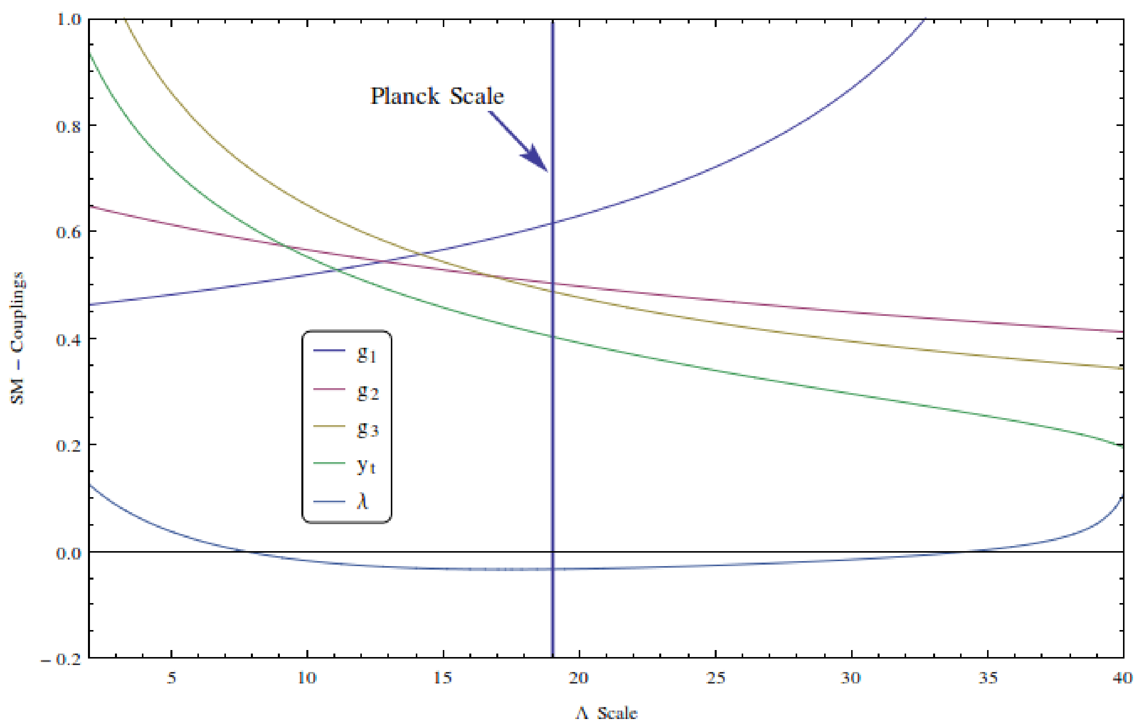


Figure 3: Running of gauge couplings(blue, purple ,olive), the top yukawa(green) and higgs self coupling(light blue) in standard model

In figure 3 we plot the running of top yukawa coupling y_t (green) and the SM gauge couplings g_1, g_2, g_3 and the self higgs quartic coupling λ_h at one-loop order.The vertical dark line shows the Planck scale. At low energies the running of λ_h is entirely dominated by the running of y_t, at scale around 10^{22}GeV, the gauge and top yukawa contributions have opposite signs and cancel each other and running is governed by λ_h itself, while at very high energy the running of λ_h is dominated by g_1.
The λ_h turns negative at a scale of 10^{8}GeV, destabilizing the vacuum. (this value is quite sensitive to the initial values of couplings) The figure 3 also shows the apparent unification of all SM couplings very close to Planck scale. The exact behaviour of the λ_h at one-loop order would in principle lead to an exact expression for the one loop effective potential in the Standard Model which shows a hint that we live in a metastable vacuum very close to critical line of vacuum decay.

7 Higgs Triplet Model

7.1 Introduction

Even though the Standard Model (SM) of the strong and electroweak interactions has proven enormously successful, it need not be the case that the Higgs sector consists solely of a higgs doublet field, that is a field with total weak isospin T = $\frac{1}{2}$ having two members with T_3 = +$\frac{1}{2}$ and T_3 = -$\frac{1}{2}$ and U(1) hypercharge Y = 1. The inclusion of additional doublets, singlets (fields with T = Y = 0), $SU(2)_L$ triplet representations (fields with T = 1, three-component field with T_3 = +1, 0,-1 members) or several of them is a frequently considered possibility very well consistent with the gauge theory. The purpose of this section is to review the phenomenology of a Higgs sector which contains both doublet and $SU(2)_L$ triplet fields. These yield many exotic features and unusual experimental signatures. In exploring the physics of electroweak symmetry breaking at future colliders, it will be important to consider the alternative possibilities characteristic of this and other non-minimal Higgs sectors. Our discussion here will focus primarily on models in which only the Higgs sector of the SM is extended via the addition of triplet representation with hypercharge Y = 0. We review various theoretical bounds on the parameter space and try to identify the allowed regions of parameter space.

7.2 Basic Framework

The scalar sector of the SM is extended using $SU(2)_L$ Triplet .The HTM consists of two fields SM Doublet and a Real Y=0 Triplet. Both the doublet and triplet field take part in electro-weak symmetry breaking i.e both gets vev at the minima of the potential. After EWSB one of the linear combinations of charged scalar fields of doublet and the triplet is eaten by W boson which then becomes massive, other orthogonal combinations of these fields become massive charged scalar fields. Similarly, the pseudo-scalar of the doublet become the longitudinal part of the massive Z gauge boson, and the real neutral components mix to give two neutral physical fields.

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad \xi = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}T_+ \\ \sqrt{2}T_- & -T_0 \end{pmatrix}$$

The kinetic part of the Lagrangian is given by,

$$\mathcal{L}_k = (D^\mu\Phi)^\dagger(D_\mu\Phi) + Tr[(D^\mu\xi)^\dagger(D_\mu\xi)] \tag{137}$$

where the covariant derivative involving the gauge-fields is given by,

$$D_\mu\Phi = (\partial_\mu - igT_aW_\mu^a - ig'B_\mu Y)\Phi, \tag{138}$$

$$D_\mu\xi = (\partial_\mu - igT_aW_\mu^a)\xi \tag{139}$$

The tree-level potential invariant under $SU(2)_LxU(1)_Y$ transformation is given by,

$$V(H,\xi) = m_h^2\Phi^\dagger\Phi + m_t^2Tr(\xi^\dagger\xi) + \lambda_h|\Phi^\dagger\Phi|^2 + \lambda_t(Tr|\xi^\dagger\xi|)^2 + \lambda_{ht}\Phi^\dagger\Phi Tr(\xi^\dagger\xi) + A_{ht}\Phi^\dagger\xi\ \Phi \tag{140}$$

7.3 EWSB in HTM

The neutral component of doublet and and triplet gets vev as

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v_h+h_1+\iota\eta}{\sqrt{2}} \end{pmatrix}, \qquad \xi = \frac{1}{2}\begin{pmatrix} v_t+h_2 & \sqrt{2}T_+ \\ \sqrt{2}T_- & -v_t-h_2 \end{pmatrix}$$

Recipe: To obtain Goldstones and Physical mass eigenstates

- Expand the potential around the minima.
- Using Minimization condition remove some of the parameters.
- Collect all the bilinear terms for the fields present in the model.
- Obtain the mixing matrices.
- Diagonalize the mixing matrices to get into mass basis.

Using minimization conditions :

$$m_h = \frac{1}{2}\left(A_{\rm ht}v_t - 2\lambda_h v_h^2 - \lambda_{\rm ht}v_t^2\right) \tag{141}$$

$$m_t = \frac{\frac{1}{2}A_{\rm ht}v_h^2 - v_h^2\lambda_{\rm ht}v_t - 2\lambda_t v_t^3}{2v_t} \tag{142}$$

The kinetic part for the triplet field is solved so as to obtain the contribution of triplet field to the mass of gauge bosons.The kinetic part for triplet field is given by,

$$\mathcal{L}_k = Tr[(D^\mu\xi)^\dagger(D_\mu\xi)] \tag{143}$$

Triplet's contribution to the gauge boson masses-squared in matrix form in the basis of (W_1, W_2, W_3, B) as :

$$M_\xi^2 = v_t^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{144}$$

This is also a block diagonal matrix and both the lower and upper 2 x 2 blocks are already diagonalized and the lower block is zero in this case so the Y = 0 triplet contributes to the $W_\pm$ mass and does not contribute to the Z or photon masses.The contribution from M_ξ^2 will not change the value of the mixing angle θ_w between the Z and the photon. and the value of θ_w is set entirely by the gauge couplings g and g'. The masses of $W^\pm$, Z

$$M_w^2 = \frac{g^2}{4}(v_h^2 + 4v_t^2) \tag{145}$$

$$M_z^2 = \frac{(g^2 + g'^2)}{4}v_h^2 \tag{146}$$

$$\rho = \frac{M_w^2}{cos^2\theta_w M_z^2} \tag{147}$$

$$\rho = 1 + \frac{4v_t^2}{v_h^2} \tag{148}$$

The vev v_h of the doublet field and v_t of the Y= 0 triplet field are related to SM vev by $v_{SM} = \sqrt{v_h^2 + v_t^2}$. Now,the experimental value of ρ is 1.004 $\pm$ 0.00024 at 1σ. This puts a stringent constraint on v_2 and we get v_2 should be less than 4 GeV.

Solving the potential part of the HTM and following the above-mentioned steps we get the mixing mass matrices. The mixing Matrix for Charged Higgs and the neutral higgs are respectively given below :

$$M_1 = \begin{pmatrix} A_{\rm ht} v_t & \frac{A_{\rm ht} v_h}{2} \\ \frac{A_{\rm ht} v_h}{2} & \frac{A_{\rm ht} v_h^2}{4 v_t} \end{pmatrix} \tag{149}$$

$$M_2 = \begin{pmatrix} 2 v_h^2 \lambda_h & v_h v_t \lambda_{\rm ht} - \frac{A_{\rm ht} v_h}{2} \\ v_h v_t \lambda_{\rm ht} - \frac{A_{\rm ht} v_h}{2} & \frac{A_{\rm ht} v_h^2}{4 v_t} + 2 v_t^2 \lambda_t \end{pmatrix} \tag{150}$$

The Mixing between the doublet and triplet in the charged and CP-even scalar are respectively given by :

$$\begin{pmatrix} \phi_\pm \\ T_\pm \end{pmatrix} = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} . \begin{pmatrix} G_\pm \\ H_\pm \end{pmatrix} \tag{151}$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} c_\gamma & -s_\gamma \\ s_\gamma & c_\gamma \end{pmatrix} . \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \tag{152}$$

After diagonalization of matrices M_1 and M_2 we get the mass eigenstates as :
Mass of Charged Higgs's $H^\pm$ and Neutral Higgs's H_1,H_2 are :

$$M_{H^\pm}^2 = \frac{A_{\rm ht} \left(v_h^2 + 4 v_t^2 \right)}{2 v_t} \tag{153}$$

$$M_{H1}^2 = \frac{A - \sqrt{(-A)^2 - 4B}}{4 v_t} \tag{154}$$

$$M_{H2}^2 = \frac{\sqrt{(-A)^2 - 4B} + A}{4 v_t} \tag{155}$$

where $A = A_{\rm ht} v_h^2 + 8 \lambda_h v_h^2 v_t + 8 \lambda_t v_t^3$,

$B = 8 A_{\rm ht} \lambda_h v_h^4 v_t + 16 A_{\rm ht} v_h^2 \lambda_{\rm ht} v_t^3 - 4 A_{\rm ht}^2 v_h^2 v_t^2 - 16 v_h^2 \lambda_{\rm ht}^2 v_t^4 + 64 \lambda_h v_h^2 \lambda_t v_t^4$
$G^\pm$, $G_0 = \eta$ are the goldstone bosons and $H^\pm$, H_1, H_2 are the physical mass eigenstates.The SM gauge symmetry prohibits direct couping of Sm fermions with the triplet. The HTM predicts the existence of a charged Higgs pair.

7.4 Theoretical bounds in HTM

For the stability of the vacuum, the scalar potential must be bounded from below i.e. it should not blow up to negative infinity along any direction of the field at large field values. At tree-level scalar potential $V(H, \Xi)$ is stable if

$$\lambda_h \geq 0, \qquad \lambda_t \geq 0, \qquad \lambda_{ht} \geq -\sqrt{\lambda_h \lambda_t}, \tag{156}$$

Perturbativity and Vacuum Stability put theoretical constraints on the allowed parameter space. We compute the RG evolution of all quartic couplings up to Planck scale using

RGEs at one loop level and try to identify the allowed values of parameters. We developed a code to solve the coupled differential equations and plot the running of couplings. (Beta function for all couplings of Higgs Triplet model are given in Appendix - IV)

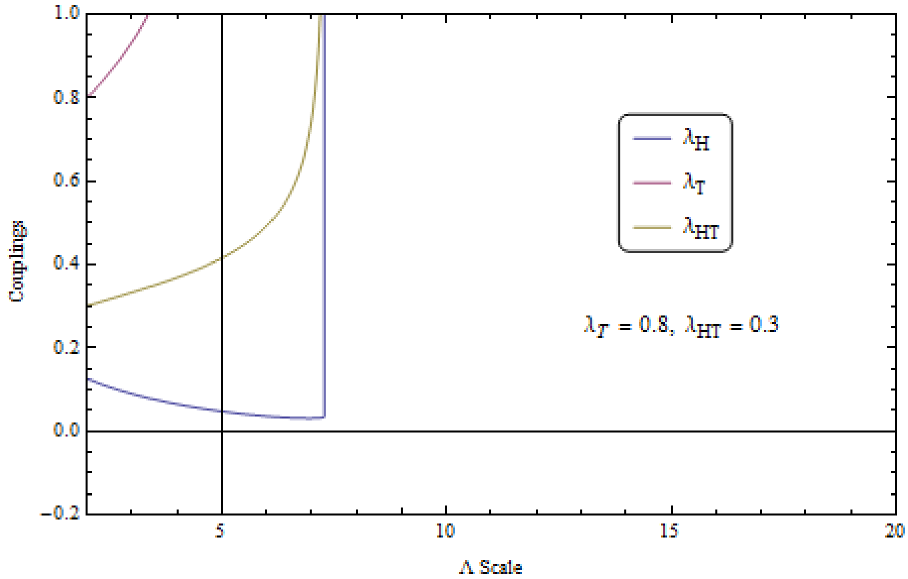


Figure 4: Running of quartic couplings in HTM, ($\lambda_h = 0.1264, \lambda_t = 0.8, \lambda_{ht} = 0.3$). Details are included in [9].

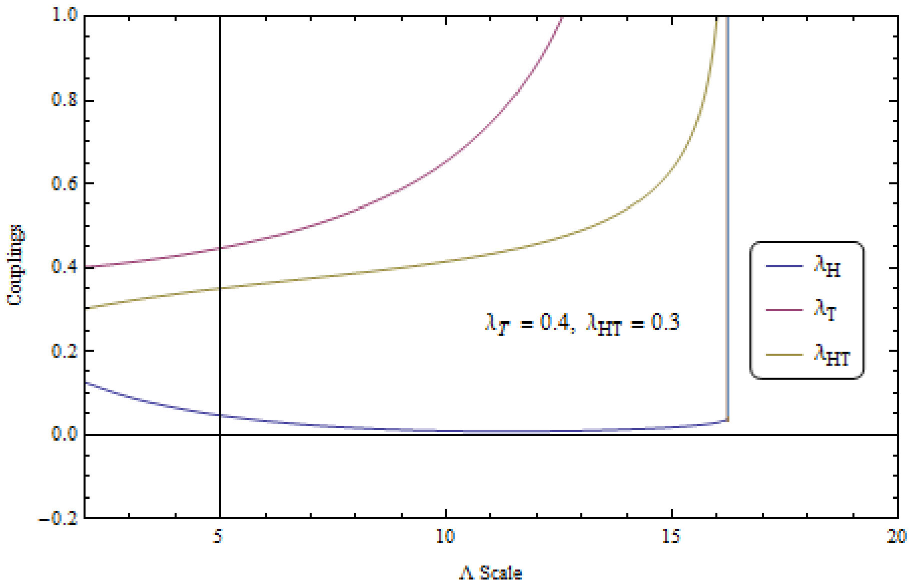


Figure 5: Running of quartic couplings in HTM ($\lambda_h = 0.1264, \lambda_t = 0.4, \lambda_{ht} = 0.3$). Details are included in [9].

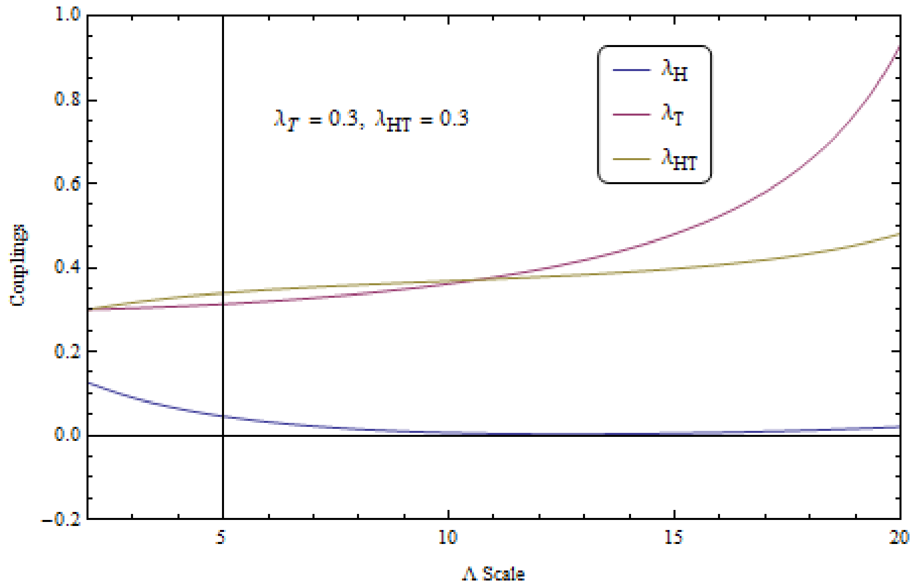


Figure 6: Running of quartic couplings in HTM ($\lambda_h = 0.1264, \lambda_t = 0.3, \lambda_{ht} = 0.3$). Details are included in [9].

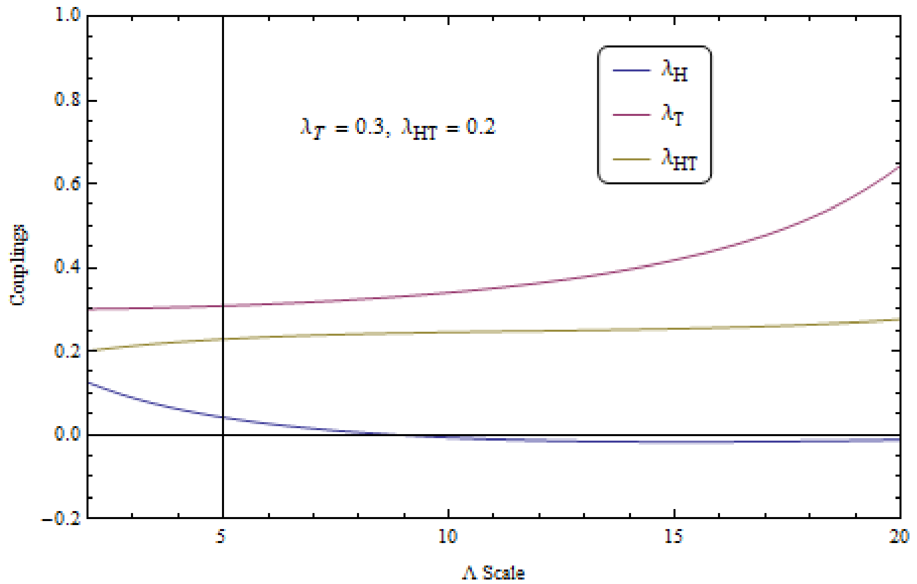


Figure 7: Running of quartic couplings in HTM ($\lambda_h = 0.1264, \lambda_t = 0.3, \lambda_{ht} = 0.2$). Details are included in [9].

As shown in figure 4,5, 6 that on increasing the value of λ_t the theory is perturbative up-to very low scale. So we can identify the allowed value of λ_t by demanding that our theory is perturbative up-to Planck scale.

S.No.	Value of λ_t	Perturbative ScaleΛ
1	0.3	6.9×10^{23}
2	0.4	1.7×10^{16}
3	0.5	1.5×10^{12}
4	0.6	6.8×10^{9}
5	0.7	2.1×10^{8}
6	0.8	1.9×10^{7}

Table 2: Scale up-to which theory is Perturbative, for different values of λ_t

Table 2 shows that the allowed values of λ_t is between 0.3 and 0.4 so that our theory is perturbative up-to Planck scale. We also $\lambda_t = 0.3$ and $\lambda_{ht} = 0.3$ stability is enhanced as shown in figure 6 and the vacuum is stable while if we take We also $\lambda_t = 0.3$ and $\lambda_{ht} = 0.2$ it again tells us that our vacuum is metastable as λ_h becomes negative at around 10^9 GeV which hints the presence of another global minima.

8 Dark Matter in Inert Triplet Model (ITM)

8.1 Introduction

Astronomical and cosmological observations gives a direct proof of the existence of dark matter (DM). SM don't say anything about the presence of DM so we have to go beyond the standard model (SM). In principle the DM candidate can be a scalar, a fermion or a vector. If the dark matter is a scalar particle , it first of all cannot develop a vacuum expectation value (vev) and will not take part in EWSB. Otherwise, the DM candidate will have mixing through SM Higgs portal interaction and eventually decay into the SM particles. Furthermore, the DM scalar should also not have any Yukawa couplings with the SM fermions. Therefore, to exclude the decay or any direct coupling with SM species we invoke a Z2 discrete symmetry, under which the DM scalar is assigned odd while the SM fields are even. Such DM scalar is naturally termed as an inert field.

8.2 Basic Framework

In addition to SM particles, we introduce an $SU(2)_L$ triplet scalar with Y=0. The triplet field don't take part in EWSB i.e $v_t = 0$.

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \xi = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}T_+ \\ \sqrt{2}T_- & -T_0 \end{pmatrix}$$

The Lagrangian for Y=0 ITM is given as :

$$\mathcal{L}_k = (D^\mu \Phi)^\dagger (D_\mu \Phi) + Tr[(D^\mu \xi)^\dagger (D_\mu \xi)] - V(\Phi, \xi) \tag{157}$$

where the covariant derivative involving the gauge-fields is given by,

$$D_\mu \Phi = (\partial_\mu - igT_a W_\mu^a - ig' B_\mu Y)\Phi, \tag{158}$$

$$D_\mu \xi = (\partial_\mu - igT_a W_\mu^a)\xi \tag{159}$$

Now we impose an additional Z_2 symmetry in which triplet is assigned odd and other fields are even. After imposing Z2 symmetry we get the potential of ITM as follow :

$$V = m_h^2 \Phi^\dagger \Phi + m_t^2 Tr(\xi^\dagger \xi) + \lambda_h |\Phi^\dagger \Phi|^2 + \lambda_t (Tr|\xi^\dagger \xi|)^2 + \lambda_{ht} \Phi^\dagger \Phi Tr(\xi^\dagger \xi) \tag{160}$$

8.3 EWSB in ITM

In Inert Triplet model triplet field don't take part in EWSB $v_t = 0$ and only doublet gets vev as :

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{v_h + h + \iota\eta}{\sqrt{2}} \end{pmatrix}, \qquad \xi = \frac{1}{2} \begin{pmatrix} T_0 & \sqrt{2}T_+ \\ \sqrt{2}T_- & -T_0 \end{pmatrix}$$

Using minimization conditions :

$$m_h = -\lambda_h v_h^2 \tag{161}$$

Triplet field do not contribute to mass of any of the SM particle
The masses of $W^\pm$,Z are given as

$$M_w^2 = \frac{g^2}{4} v_h^2 \tag{162}$$

$$M_z^2 = \frac{(g^2 + g'^2)}{4} v_h^2 \tag{163}$$

$$\rho = \frac{M_w^2}{cos^2\theta_w M_z^2} \tag{164}$$

Solving the potential part of the ITM we get the mass of the standard model higgs and triplet fields as shown below :

$$M_h^2 = 2\lambda v_h^2 \tag{165}$$

$$M_{T_0}^2 = \frac{1}{2} v_h^2 \lambda_{\rm ht} + m_t \tag{166}$$

$$M_{T_\pm}^2 = \frac{1}{2} v_h^2 \lambda_{\rm ht} + m_t \tag{167}$$

where m_t and λ_{ht} are the parameters as shown in the expression of the potential and here we have :

- $\phi^{\pm}$, η are the goldstone bosons.
- h,T_0, $T_{\pm}$ are the physical mass eigenstates
- Triplet and doublet don't mix at all
- At tree level the mass of neutral and charged component of triplet field are same

Now, since the mass of the neutral and charged component of triplet field are same at the tree level we have to look for the quantum correction. Quantum corrections will lead to mass splitting [10] between these and we will be able to identify the Dark Matter component(the lightest one). As we know that T_0 do not couple to Z and γ and only couples to $W^{\pm}$ but $T^{\pm}$ couples to all of them so we will have some extra bosonic loop contribution as shown in figure 8 and 9

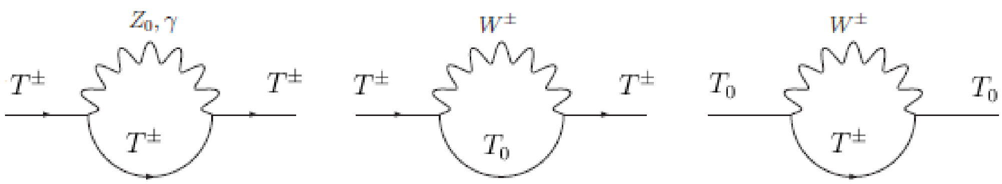


Figure 8: Self energy induced by three-point gauge interactions

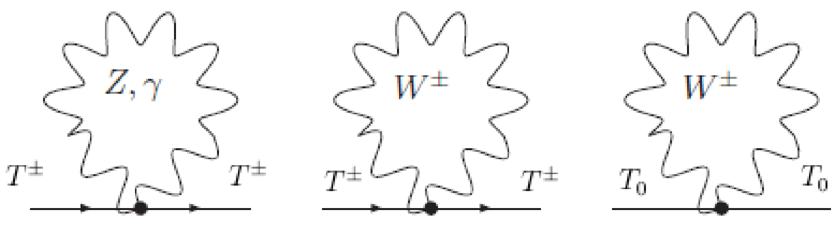


Figure 9: Self energy induced by four-point gauge interactions

Now since these are the bosonic loops, they will always have a positive contribution and there is mass splitting between mass of T_0 and $T^{\pm}$ as:

$$M_{T^{\pm}} = M_{T_0} + 166MeV \tag{168}$$

So T_0 is the lightest one and potential candidate for Dark Matter.

We developed the Mathematica code to solve the Standard Model(SM), Higgs Triplet Model(HTM) and Inert Triplet Model (ITM) which are given in Appendix-V.

9 Summary and Conclusion

In Section 2,3,4 we studied the Standard Model as a successful gauge theory describing the fundamental interactions. Now the experimental measurement of the higgs mass allows us to compute the self higgs quartic coupling at electro-weak scale. We extrapolate λ_h to high energies and studied the SM in its full perturbative validity range up to the Landau pole. At low energies, the running of λ_h is governed by top yukawa coupling which turns the λ_h negative at scale of around 10^8 GeV destabilizing the potential.
We analyzed that the SM coupling constants, remain perturbative in the entire energy domain between the Fermi and the Planck scales.

The critical condition for stability is also one of the important observation and motivates to extend the SM in scalar sector.We observed that adding a scalar enhances the stability. Theoretically, the parameter space of the extended SM with a Real Y=0 get bounds from Perturbativity, Vacuum Stability etc. We studied the perturbative validity range of the Higgs triplet model by scanning the parameter space. The allowed values of the parameters lead to two different scenarios,the first one where the EW Vacuum is stable and the other where it is still metastable.

We have also shown that the Inert Triplet model provides a Dark Matter candidate. The neutral component of the triplet can be the dark matter because when radiative correction are taken into account for two-point function there is mass splitting between charged and neutral components of triplet field and the neutral component arises as the lightest one.

A Appendix I : Running of λ (Four point function)

Renormalization of ϕ^4 theory[1]

Lagrangian for ϕ^4 theory is defined as:

$$L = \frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

$$S = \int d^4x[\frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4]$$

We will use Dimensional Regularization so we go into $4-\epsilon$ dimension and perform the calculations.

$$S = \int d^{4-\epsilon}x[\frac{1}{2}\partial_\mu\phi_0\partial^\mu\phi_0 - \frac{1}{2}m_0^2\phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4] \tag{169}$$

where ϕ_0, m_0 and λ_0 are known as bare parameters, We define the renormalized parameters ϕ_R, m_R and λ_R as follows :

$$\phi_0 = \tilde{Z}_\phi^{\frac{1}{2}}\phi_R = \tilde{Z}_\phi^{\frac{1}{2}}(\lambda_R, m_R)\phi_R \tag{170}$$

$$m_0 = Z_m m_R = Z_m(\lambda_R, m_R)m_R \tag{171}$$

$$\lambda_0 = Z_\lambda\mu\lambda_R = Z_\lambda(\lambda_R, m_R)\lambda_R \tag{172}$$

where the divergences(will come for some of the loop integrals)can be absorbed in $Z_m, Z_\lambda, \tilde{Z}_\phi$. Doing a little bit of algebra we can write our Lagrangian into two parts : one part having renormalized fields and the other we call as counter terms(kind of new interaction terms).

$$L = \frac{1}{2}\partial_\mu\phi_R\partial^\mu\phi_R - \frac{1}{2}m_R^2\phi_R^2 - \frac{1}{4!}\mu^\epsilon\lambda_R\phi_R^4 + \frac{1}{2}(\tilde{Z}_\phi - 1)\partial_\mu\phi_R\partial^\mu\phi_R - \frac{1}{2}(\tilde{Z}_\phi Z_m^2 - 1)m_R^2\phi_R^2$$
$$-\frac{1}{4!}\mu^\epsilon\lambda_R(\tilde{Z}_\phi^2 Z_\lambda - 1)\phi_R^4$$

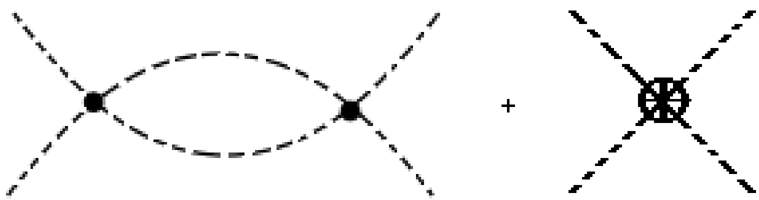


Figure 10: Four point function (left : s-channel one loop diagram right: counter term diagram)

Actually there are three diagrams for one-loop correction of four-point function, these are s, t and u-channel respectively. These can be transformed to each other just by interchanging the Mandelstam variables. The loop diagram as shown in figure 10 is logarithmically divergent(UV Divergent) and is given as :

$$= (\frac{-\iota\lambda_R}{4!})^2 \int \frac{d^4l}{(2\pi)^4} \frac{\iota}{l^2-m^2+\iota\epsilon} \frac{\iota}{(l-p)^2-m^2+\iota\epsilon} \times 8 \times 4 \times 3 \times 3 \times 2$$

We use dimensional regularization to get rid of UV divergence so we do the integral in 4-ϵ dimension.

$$=\frac{(\lambda_R)^2}{2}\mu^{4-D} \int \frac{d^Dl}{(2\pi)^D} \frac{\iota}{l^2-m^2} \frac{\iota}{(l-p)^2-m^2}$$

Now, using the Feynman parametrization:

$$\frac{1}{ab} = \int_0^1 \frac{dx}{[xa+(1-x)b]^2}$$

which after some algebra reduces our denominator as:

$$\frac{1}{(l^2-m^2)((l-p)^2-m^2)} = \int_0^1 \frac{dx}{(x(l^2-m^2)+(1-x)((l-p)^2-m^2))^2} \tag{173}$$

so our integral becomes

$$=\frac{(\lambda_R)^2}{2}\mu^{4-D} \int \frac{d^Dl}{(2\pi)^D} \int_0^1 \frac{dx}{((l-(1-x)p)^2-m^2+p^2x(1-X))^2}.$$

Since our integrals are now convergent after the regularization, we can switch the order of integration and shift the integration parameter as $l \to \tilde{l} = l - p(1-x)$, which leaves the measure dl unchanged. This gives us

$$= \frac{(\lambda_R)^2}{2}\mu^{4-D} \int_0^1 dx \int \frac{d^D\tilde{l}}{(2\pi)^D[\tilde{l}^2-m^2+p^2x(1-x)]^2}$$

The momentum integral is calculated by , first performing the Wick rotation

$$=\frac{(\lambda_R)^2}{4!}\mu^{4-D} \int_0^1 \int \frac{d^D\tilde{l}_E}{(2\pi)^D[-\tilde{l}_E^2-m^2+p^2x(1-x)]^2}$$

Then carrying out the angular integrals

$$=\frac{(\lambda_R)^2}{4!}\mu^{4-D} \frac{2\pi^{\frac{D}{2}}}{\Gamma(\frac{D}{2})} \int_0^1 \int_0^\infty \frac{d\bar{l}_E\tilde{l}_E^{D-1}}{[\tilde{l}_E^2+m^2-p^2x(1-x)]^2}$$

First we write

$$\tilde{l}_E^{D-1}d\tilde{l}_E = \frac{1}{2}\tilde{l}_E^{D-2}d\tilde{l}_E^2 = \frac{1}{2}(\tilde{l}_E^2)^{\frac{D}{2}-1}d\tilde{l}_E^2$$

and we are only left to calculate the integral over $\tilde{l}_E$. This can be done using the following result derived using the Euler beta function:

$$\int_0^\infty \frac{x^{\alpha-1}}{(x+a)^\beta}dx = a^{\alpha-\beta}\frac{\Gamma(\alpha)\Gamma(\beta-\alpha)}{\Gamma(\beta)}$$

Using Euler Beta function, the integral becomes;

$$=\frac{\iota(\lambda_R)^2}{4!}\mu^{4-D}\frac{\pi^{\frac{D}{2}}}{(2\pi)^D}\frac{\Gamma(\frac{D}{2})\Gamma(2-\frac{D}{2})}{\Gamma(2)} \int_0^1 dx[m^2-p^2(1-x)]^{\frac{D}{2}-2}$$

$$=\frac{\iota(\lambda_R)^2}{16\pi^2}\mu^{4-D} \int_0^1 dx[\frac{4\pi\mu^2}{m^2-p^2x(1-x)}]^{\frac{4-D}{2}}\Gamma(2-\frac{D}{2})$$

$$=\frac{\iota(\lambda_R)^2}{16\pi^2}\mu^\epsilon e^{\frac{\epsilon}{2}ln\int_0^1 dx\frac{4\pi\mu^2}{m^2-p^2x(1-x)}}$$

Taking the limit $\epsilon \to 0$ we get the correction to the four-point function from the s-channel loop as :

$$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon)$$

$$= \frac{\iota(\lambda_R)^2}{32\pi^2}[(\frac{2}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon))(1 + \frac{\epsilon}{2}\int_0^1 dx ln(\frac{4\pi\mu^2}{m^2 - p^2 x(1-x)}))]$$

$$= \frac{\iota\lambda_0^2}{16\pi^2}(\frac{1}{\epsilon} - \frac{\gamma}{2} + \frac{1}{2}ln(4\pi\frac{\mu^2}{m_R^2}) - \frac{1}{2}\int_0^1 dx ln[1 - \frac{p^2}{m_R^2}x(1-x)]) \tag{174}$$

Now for our results to be physical the sum of the diagrams in figure 10 should be finite.

$$(Z_\lambda \tilde{Z}_\phi^2 - 1) - \frac{3\lambda_0^2}{16\pi^2}(\frac{1}{\epsilon} - \frac{\gamma}{2} + \frac{1}{2}ln(4\pi\frac{\mu^2}{m_R^2}) - \frac{1}{2}\int_0^1 dx ln[1 - \frac{p^2}{m_R^2}x(1-x)]) = finite \tag{175}$$

In order to have cancel the divergence we can absorb the divergence in Z_λ where $Z_\phi = 1$

$$Z_\lambda = 1 + \frac{3\lambda_0^2}{16\pi^2}\frac{1}{\epsilon} \tag{176}$$

Now calculating the beta function for self higgs quartic couping :

$$\beta(\lambda_R) = \mu\frac{\partial}{\partial\mu}(\lambda_0\mu^{-\epsilon}Z_\lambda^{-1}) \tag{177}$$

where

$$Z_\lambda = 1 + \frac{3\lambda^2}{16\pi^2}\frac{1}{\epsilon} \tag{178}$$

$$\beta(\lambda_R) = \mu\frac{\partial}{\partial\mu}[\lambda\mu^{-\epsilon}(1 - \frac{3\lambda^2}{16\pi^2}\frac{1}{\epsilon})] \tag{179}$$

$$\beta(\lambda_R) = -\epsilon\lambda\mu^{-\epsilon} + \frac{3\lambda^2}{16\pi^2}\mu^{-\epsilon} \tag{180}$$

Now taking the limit $\epsilon \to 0$ we get

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} \tag{181}$$

Note : The result differs a little from what we stated above in section 6.1 because the normalization factor self higgs quartic term is different form what we have taken here in ϕ^4 theory.

B Appendix II : Callan-Symanzik Equation

The Callan-Symanzik Equation [3]

In the renormalization conditions, the renormalization scale M is arbitrary. We could just as well have defined the same theory at a different scale M'. By "the same theory", we mean a theory whose bare Green's functions,

$$< \Omega|T\phi_0(x_1)\phi_0(x_2)........\phi_0(x_n)|\Omega >, \tag{182}$$

are given by the same functions of the bare coupling constant λ_0 and the cutoff Λ. These functions make no reference to M . The dependence on M enters only when we remove the cutoff dependence by rescaling the fields and eliminating λ_0 in favor of the renormalized coupling λ. The renormalized Green's functions are numerically equal to the bare Green's functions, up to a rescaling by powers of the field strength renormalization Z:

$$< \Omega|T\phi(x_1)\phi(x_2)........\phi(x_n)|\Omega >= Z^{\frac{-n}{2}} < \Omega|T\phi_0(x_1)\phi_0(x_2)........\phi_0(x_n)|\Omega > . \tag{183}$$

The renormalized Green's functions could be defined equally well at another scale M', using a new renormalized coupling λ' and a new rescaling factor Z'.

Let us write more explicitly the effect of an infinitesimal shift of M . Let $G^{(n)}(x_1, x_2, ..., x_n)$ be the connected n-point function, computed in renormalized perturbation theory:

$$G^{(n)}(x_1, x_2, ..., x_n) =< \Omega|T\phi(x_1).....\phi(x_n)|\Omega >_{connected} \tag{184}$$

Now suppose that we shift M by δM. There is a corresponding shift in the coupling constant and the field strength such that the bare Green's functions remain fixed:

$$M \to M + \delta M, \tag{185}$$

$$\lambda \to \lambda + \delta\lambda, \tag{186}$$

$$\phi \to (1 + \delta\eta)\phi. \tag{187}$$

Then the shift in any renormalized Green's function is simply that induced by the field rescaling,

$$G^{(n)} \to (1 + n\delta\eta)G^{(n)}. \tag{188}$$

If we think of $G^{(n)}$ as a function of M and λ, we can write this transformation as

$$dG^{(n)} = \frac{\partial G^{(n)}}{\partial M}\delta M + \frac{\partial G^{(n)}}{\partial \lambda}\delta\lambda = n\delta\eta G^{(n)}. \tag{189}$$

Rather than writing this relation in terms of $\delta\lambda$ and $\delta\eta$, it is conventional to define the dimensionless parameters

$$\beta = \frac{M}{\delta M}\delta\lambda; \qquad \gamma = -\frac{M}{\delta M}\delta\eta \tag{190}$$

Making these substitutions and multiplying through by $\frac{M}{\delta M}$, we obtain

$$[M\frac{\partial}{\partial M}+\beta\frac{\partial}{\partial\lambda}+\eta\gamma]G^{(n)}(x_1,........x_n;M,\lambda)=0 \tag{191}$$

The parameters β and γ are the same for every n , and must be independent of the x_i . Since the Green's function $G^{(n)}$ is renormalized, β and γ cannot depend on the cutoff, and hence, by dimensional analysis, these functions cannot depend on M . Therefore they are functions only of the dimensionless variable λ. We conclude that any Green's function of massless ϕ^4 theory must satisfy

$$[M\frac{\partial}{\partial M}+\beta(\lambda)\frac{\partial}{\partial\lambda}+\eta\gamma(\lambda)]G^{(n)}(x_1,........x_n;M,\lambda)=0 \tag{192}$$

$\beta(\lambda)$ is known as Callan-Symanzik beta-function and $\gamma(\lambda)$ is known as Anomalous dimension.

C Appendix III : SM RGEs at One-Loop

- $g_1'(t)-\frac{41}{160\pi^2}g_1(t)^3\log(10)=0$
- $g_2'(t)+\frac{19}{96\pi^2}g_2(t)^3\log(10)=0$
- $g_3'(t)+\frac{7}{16\pi^2}g_3(t)^3\log(10)=0$
- $y'(t)-\frac{1}{16\pi^2}[-\frac{17}{20}g_1(t)^2y(t)-\frac{9}{4}g_2(t)^2y(t)-8g_3(t)^2y(t)+\frac{9y(t)^3}{2}]\log(10)=0$
- $\lambda'(t)-\frac{1}{16\pi^2}[-9g_2(t)^2\lambda(t)-\frac{9}{5}g_1(t)^2\lambda(t)+\frac{27}{200}g_1(t)^4+\frac{9}{8}g_2(t)^4+\frac{9}{20}g_1(t)^2g_2(t)^2+24\lambda(t)^2+12\lambda(t)y(t)^2-6y(t)^4]\log(10)=0$

D Appendix IV : HTM RGEs at One-Loop

- $\lambda_T'(t)-\frac{1}{16\pi^2}[-12g_2(t)^2\lambda_T(t)+3g_2(t)^4+\lambda_{\mathrm{HT}}(t)^2+11\lambda_T(t)^2]\log(10)=0$
- $A'(t)-\frac{1}{16\pi^2}[\frac{21}{2}A(t)g_2(t)^2-\frac{9}{10}A(t)g_1(t)^2+4A(t)\lambda_H(t)+4A(t)\lambda_{\mathrm{HT}}(t)+6A(t)y(t)^2]\log(10)=0$
- $\lambda_{\mathrm{HT}}'(t)-\frac{1}{16\pi^2}[-\frac{33}{2}g_2(t)^2\lambda_{\mathrm{HT}}(t)-\frac{9}{10}g_1(t)^2\lambda_{\mathrm{HT}}(t)+3g_2(t)^4+12\lambda_H(t)\lambda_{\mathrm{HT}}(t)+4\lambda_{\mathrm{HT}}(t)^2+6\lambda_{\mathrm{HT}}(t)\lambda_T(t)+6y(t)^2\lambda_{\mathrm{HT}}(t)]\log(10)=0$
- $\lambda_H'(t)-\frac{1}{16\pi^2}[-9g_2(t)^2\lambda_H(t)-\frac{9}{5}g_1(t)^2\lambda_H(t)+\frac{27}{200}g_1(t)^4+\frac{9}{8}g_2(t)^4+\frac{9}{20}g_1(t)^2g_2(t)^2+24\lambda_H(t)^2+12y(t)^2\lambda_H(t)+\frac{3}{2}\lambda_{\mathrm{HT}}(t)^2-6y(t)^4]\log(10)=0$

E Appendix V : Mathematica Codes

Kinetic Part of the Standard Model

$$\text{Dmu}\Phi = \frac{i}{2\sqrt{2}} * \left(g_1 * \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix} . \Phi + g_2 * \begin{pmatrix} W_{\mu 3} & W_{\mu^+} \\ W_{\mu^-} & -W_{\mu 3} \end{pmatrix} . \Phi\right);$$

$$\Phi = \begin{pmatrix} 0 \\ v + h \end{pmatrix}; \quad \Phi^+ = (\phi_- \;\; \phi_0); \quad \text{(* Definition of Doublet*)}$$

$$\text{Dmu}\Phi^+ = \left\{\left\{\frac{-i\,(v+h)\,g_2\,W^{\mu^-}}{2\sqrt{2}}, \frac{-i\,\left((h+v)\,B^\mu\,g_1 - (v+h)\,g_2\,W^{\mu 3}\right)}{2\sqrt{2}}\right\}\right\};$$

```
r = Simplify[DmuΦ+ . DmuΦ];
```

$$\text{y1} = \text{Solve}\left[c_{\theta_w} == \frac{g_2}{\sqrt{g_1{}^2 + g_2{}^2}}\right];$$

$$\text{y2} = \text{Solve}\left[s_{\theta_w} == \frac{g_1}{\sqrt{g_1{}^2 + g_2{}^2}}\right];$$

$$\text{x1} = \text{Solve}\left[\begin{pmatrix} W^{\mu 3} \\ B^\mu \end{pmatrix} == \begin{pmatrix} c_{\theta_w} & s_{\theta_w} \\ -s_{\theta_w} & c_{\theta_w} \end{pmatrix} . \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}\right] \;/.\; \text{y1} \;/.\; \text{y2};$$

$$\text{x2} = \text{Solve}\left[\begin{pmatrix} W_{\mu 3} \\ B_\mu \end{pmatrix} == \begin{pmatrix} c_{\theta_w} & s_{\theta_w} \\ -s_{\theta_w} & c_{\theta_w} \end{pmatrix} . \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}\right] \;/.\; \text{y1} \;/.\; \text{y2};$$

```
Simplify[r /. x1 /. x2]
```

$$\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{\frac{1}{8}\,(h+v)^2\,\left(Z^\mu\,g_1^2\,Z_\mu + g_2^2\,(W^{\mu^-}\,W_{\mu^+} + Z^\mu\,Z_\mu)\right)\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}$$

$$\text{U4} = \frac{1}{8}\,(h+v)^2\,\left(Z^\mu\,g_1^2\,Z_\mu + g_2^2\,\left(W^{\mu^-}\,W_{\mu^+} + Z^\mu\,Z_\mu\right)\right);$$

$$\text{var} = \left\{Z^\mu, Z_\mu, A_\mu, A^\mu, W_{\mu^+}, W^{\mu^+}, W^{\mu^-}, W_{\mu^-}, h\right\};$$

```
U5 = FromCoefficientRules[
  Select[CoefficientRules[U4, var], Total@#[[1]] == 2 &], var ]
```

$$\frac{1}{8}\,v^2\,W^{\mu^-}\,g_2^2\,W_{\mu^+} + Z^\mu\left(\frac{1}{8}\,v^2\,g_1^2 + \frac{1}{8}\,v^2\,g_2^2\right)Z_\mu$$

Potential of Standard Model

$G = m_h\, H^+ H + \lambda_h * (H^+ H)^2;\; H^+ = H = \frac{v_h}{\sqrt{2}};\; G1 = \mathrm{Simplify}[G];$

$x1 = \mathrm{Solve}[D[G1, v_h] == 0, m_h];$

$U2 = m_h * (\Phi_+ . \Phi) + \lambda_h * (\Phi_+ . \Phi)^2;$

$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix};\quad \Phi_+ = (\phi_-\ \ \phi_0);$

$\phi_0 = \frac{v_h + h_1 + i\,\eta_1}{\sqrt{2}};\qquad \phi_0 = \frac{v_h + h_1 - i\,\eta_1}{\sqrt{2}};$

$U3 = \mathrm{Expand}[U2]\quad /.\ x1;$

Interaction Terms

In[14]:= $U4 = \phi_-^2\, \phi_+^2\, \lambda_h + \phi_-\, \phi_+\, h_1^2\, \lambda_h + \frac{1}{4} h_1^4\, \lambda_h + 2\, \phi_-\, \phi_+\, h_1\, v_h\, \lambda_h + h_1^3\, v_h\, \lambda_h + h_1^2\, v_h^2\, \lambda_h - \frac{1}{4} v_h^4\, \lambda_h + \phi_-\, \phi_+\, \eta_1^2\, \lambda_h + \frac{1}{2} h_1^2\, \eta_1^2\, \lambda_h + h_1\, v_h\, \eta_1^2\, \lambda_h + \frac{1}{4} \eta_1^4\, \lambda_h;$

$vari = \{\phi_+, \phi_-, T_+, T_-, h_1, \eta_1\};$

```
U5 = FromCoefficientRules[
  Select[CoefficientRules[U4, vari], Total@#[[1]] == 2 &], vari ]
```

Out[16]= $h_1^2\, v_h^2\, \lambda_h$

$\mathrm{Solve}\left[\frac{m_h}{2} == v_h^2\, \lambda_h\right]$

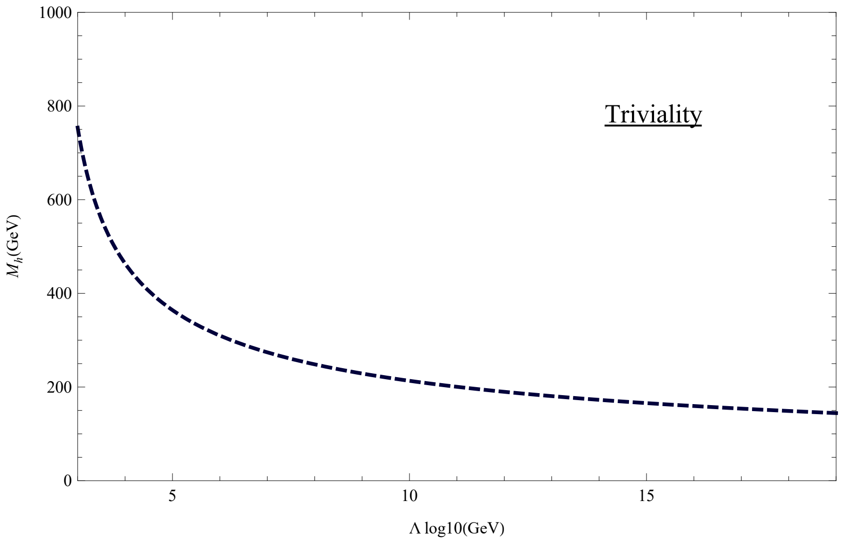
Out[12]= $h_1^2\, v_h^2\, \lambda_h$

Out[13]= $\{\{m_h \to 2\, v_h^2\, \lambda_h\}\}$

Theoretical Bounds : Standard Model

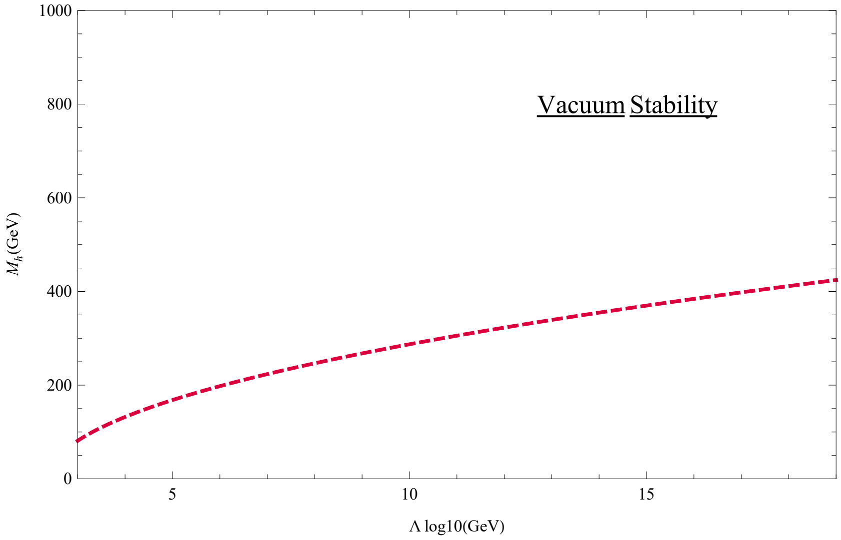
```
Plot[Sqrt[N[(4 × 22 × 22 × 246^2)/(3 × 7 × 7 Log[N[10^x/246]])]], {x, 3, 19},
 PlotStyle → Directive[RGBColor[0., 0., 0.23], AbsoluteThickness[2.], Dashed],
 PlotRange → {{3, 19}, {0, 1000}}, Axes → True, AxesOrigin → {3, 1},
 FrameLabel → {log10 [GeV] Λ, M_h [GeV]}, Frame → True]
```

Plot :Upper bound fromTriviality




```
Plot[Sqrt[N[(3 * 175^4)/(π^2 * 246^2) Log[N[10^x/246]]]], {x, 3, 19},
 PlotStyle → Directive[RGBColor[0.86, 0., 0.24], AbsoluteThickness[2.],
   Dashing[{Small, Small}]], PlotRange → {{3, 19}, {0, 1000}}, Axes → True,
 AxesOrigin → {3, 1}, FrameLabel → {log10[GeV] Λ, M_h [GeV]}, Frame → True]
```

Plot :Lower bound from Vacuum Stability

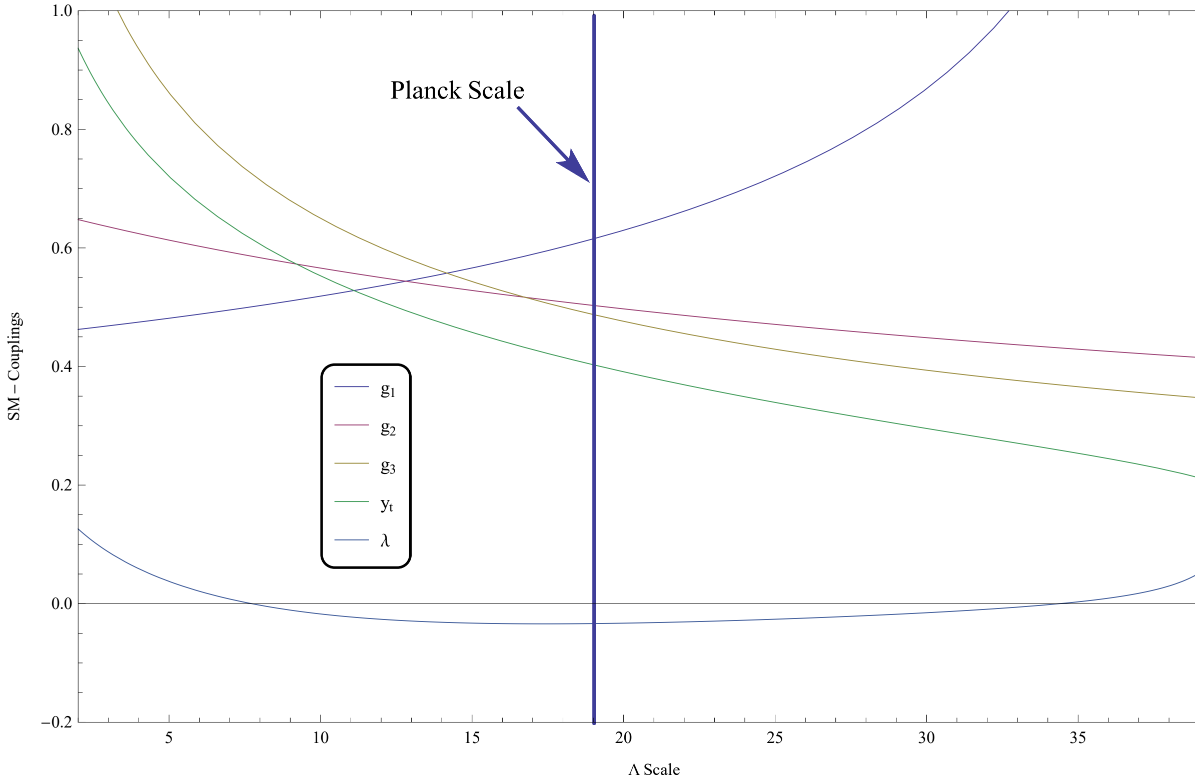


]

Running of Couplings

```
s1 = NDSolve[{g1'[t] - 41/(16*π*π*10) * g1[t]^3 * Log[10] == 0,
    g2'[t] + 19/(16*π*π*6) * g2[t]^3 * Log[10] == 0,
    g3'[t] + 7/(16*π*π) * g3[t]^3 * Log[10] == 0, y'[t] - 1/(16*π*π) *
      (9/2 y[t]^3 - 8 g3[t]^2 * y[t] - 9/4 * g2[t]^2 * y[t] - 17/20 g1[t]^2 * y[t]) * Log[10] == 0
    , λ'[t] - 1/(16*π*π) * (27 g1[t]^4/200 + 9 g1[t]^2 g2[t]^2/20 + 9 g2[t]^4/8 - 9 g1[t]^2 λ[t]/5 -
        9 g2[t]^2 λ[t] + 24 λ[t]^2 + 12 λ[t] y[t]^2 - 6 y[t]^4) * Log[10] == 0, g1[2] == 0.46256,
    g2[2] == 0.64779, g3[2] == 1.16666, y[2] == 0.93690, λ[2] == 0.12604},
   {g1[t], g2[t], g3[t], y[t], λ[t]}, {t, 2, 40}];
s = Plot[Evaluate[{{g1[t] /. s1}, {g2[t] /. s1}, {g3[t] /. s1}, {y[t] /. s1},
     {λ[t] /. s1}}], {t, 2, 40}, Frame → True, FrameLabel → {Scale Λ , SM - Couplings },
  PlotRange → {{2, 40}, {-0.2, 1}}, Axes → True, PlotLegends → Placed[
    LineLegend[{"g1", "g2", "g3", "yt", "λ"}, LegendFunction → Frame], {0.25, 0.36}]]
```

Plot of running Couplings



Higgs Triplet Model : Minimization conditions, Mixing Matrices , Mass matrices and Interactions

In[35]:=

```
(* Triplet Extension of Standard Model(HTM) : SM Doublet + Y=
0 Real Triplet *)
(* Info : Scroll down to end to see useful results *)

(* Minimization Conditions*)

ClearAll[x, ϕ, ξ, M, hc, H, T]; (*Clear all the Definitions *)

U2 = m_h * (Φ^+ .Φ) + m_t * Tr[(ξ^+ .ξ)] + λ_h * (Φ^+ .Φ)^2 + λ_t * (Tr[(ξ^+ .ξ)])^2 +
    λ_ht * (Φ^+ .Φ) * Tr[(ξ^+ .ξ)] + A_ht * (Φ^+ .ξ.Φ); (*Potential of HTM*)

Φ = {{ϕ_+}, {ϕ_0}}; Φ^+ = (ϕ_- ϕ_O); (* Definition of Doublet*)

ξ = 1/2 {{T_0, Sqrt[2] T_+}, {Sqrt[2] T_-, -T_0}};
ξ^+ = 1/2 {{T_0, Sqrt[2] T_+}, {Sqrt[2] T_-, -T_0}};          (* Definition of Triplet *)

ϕ_0 = h_1 ;   T_0 = h_2;        ϕ_O = h_1 ;    T_O = h_2;
U3 = Expand[U2] ;
U4 = ϕ_- T_+ A_ht h_1/Sqrt[2] + T_- ϕ_+ A_ht h_1/Sqrt[2] + 1/2 ϕ_- ϕ_+ A_ht h_2 - 1/2 A_ht h_1^2 h_2 + ϕ_- ϕ_+ m_h +
    h_1^2 m_h + T_- T_+ m_t + 1/2 h_2^2 m_t + ϕ_-^2 ϕ_+^2 λ_h + 2 ϕ_- ϕ_+ h_1^2 λ_h + h_1^4 λ_h + T_- ϕ_- T_+ ϕ_+ λ_ht +
    T_- T_+ h_1^2 λ_ht + 1/2 ϕ_- ϕ_+ h_2^2 λ_ht + 1/2 h_1^2 h_2^2 λ_ht + T_-^2 T_+^2 λ_t + T_- T_+ h_2^2 λ_t + 1/4 h_2^4 λ_t;
t1 = Solve[T_+ == 0]  ;
t2 = Solve[T_- == 0] ;
t3 = Solve[ϕ_+ == 0] ;
t4 = Solve[ϕ_- == 0] ;
v1 = Solve[h_1 == v_h/Sqrt[2]]  ;
v2 = Solve[h_2 == v_t]  ;
Expand[U4] /. t1 /. t2 /. t3 /. t4;
V4 = -1/2 A_ht h_1^2 h_2 + h_1^2 m_h + 1/2 h_2^2 m_t + h_1^4 λ_h + 1/2 h_1^2 h_2^2 λ_ht + 1/4 h_2^4 λ_t;
y1 = Solve[D[V4, h_1] == 0, m_h]  /. v1 /. v2
y2 = Solve[D[V4, h_2] == 0, m_t] /. v1 /. v2
```

Using Minimization conditions

Out[50]= $\{\{\{\{m_h \to \frac{1}{2}(A_{ht}\, v_t - 2\, v_h^2\, \lambda_h - v_t^2\, \lambda_{ht})\}\}\}\}$

Out[51]= $\{\{\{\{m_t \to \frac{\frac{1}{2} A_{ht}\, v_h^2 - v_h^2\, v_t\, \lambda_{ht} - 2\, v_t^3\, \lambda_t}{2\, v_t}\}\}\}\}$

In[80]:= ClearAll[x, ϕ, ξ, M, hc, H, T]; (*Clear all the Definitions *)

$U2 = m_h * (\Phi^+ . \Phi) + m_t * Tr[(\xi^+ . \xi)] + \lambda_h * (\Phi^+ . \Phi)^2 + \lambda_t * (Tr[(\xi^+ . \xi)])^2 + \lambda_{ht} * (\Phi^+ . \Phi) * Tr[(\xi^+ . \xi)] + A_{ht} * (\Phi^+ . \xi . \Phi)$; (*Potential of HTM*)

$\Phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$; $\Phi^+ = (\phi_- \quad \phi_O)$; (* Definition of Doublet*)

$\xi = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}\, T_+ \\ \sqrt{2}\, T_- & -T_0 \end{pmatrix}$;

$\xi^+ = \frac{1}{2}\begin{pmatrix} T_O & \sqrt{2}\, T_+ \\ \sqrt{2}\, T_- & -T_0 \end{pmatrix}$; (* Definition of Triplet *)

$\phi_0 = \frac{v_h + h_1 + i\, \eta_1}{\sqrt{2}}$; $T_0 = v_t + h_2$; $\phi_O = \frac{v_h + h_1 - i\, \eta_1}{\sqrt{2}}$;

$T_O = v_t + h_2$; $\eta_1 = 0$; (* Fluctuation around minima*)

U3 = Expand[Expand[U2] /. y1 /. y2]

(* Potential after Expansion *)

$$U4 = \frac{1}{2}\phi_- T_+ A_{ht} h_1 + \frac{1}{2} T_- \phi_+ A_{ht} h_1 + \frac{1}{2}\phi_- \phi_+ A_{ht} h_2 - \frac{1}{4} A_{ht} h_1^2 h_2 + \frac{1}{2}\phi_- T_+ A_{ht} v_h + \frac{1}{2} T_- \phi_+ A_{ht} v_h - \frac{1}{2} A_{ht} h_1 h_2 v_h + \frac{T_- T_+ A_{ht} v_h^2}{4 v_t} + \frac{A_{ht} h_2^2 v_h^2}{8 v_t} + \phi_- \phi_+ A_{ht} v_t + \frac{1}{8} A_{ht} v_h^2 v_t + \phi_-^2 \phi_+^2 \lambda_h + \phi_- \phi_+ h_1^2 \lambda_h + \frac{1}{4} h_1^4 \lambda_h + 2\phi_- \phi_+ h_1 v_h \lambda_h + h_1^3 v_h \lambda_h + h_1^2 v_h^2 \lambda_h - \frac{1}{4} v_h^4 \lambda_h + T_- \phi_- T_+ \phi_+ \lambda_{ht} + \frac{1}{2} T_- T_+ h_1^2 \lambda_{ht} + \frac{1}{2}\phi_- \phi_+ h_2^2 \lambda_{ht} + \frac{1}{4} h_1^2 h_2^2 \lambda_{ht} + T_- T_+ h_1 v_h \lambda_{ht} + \frac{1}{2} h_1 h_2^2 v_h \lambda_{ht} + \phi_- \phi_+ h_2 v_t \lambda_{ht} + \frac{1}{2} h_1^2 h_2 v_t \lambda_{ht} + h_1 h_2 v_h v_t \lambda_{ht} - \frac{1}{4} v_h^2 v_t^2 \lambda_{ht} + T_-^2 T_+^2 \lambda_t + T_- T_+ h_2^2 \lambda_t + \frac{1}{4} h_2^4 \lambda_t + 2 T_- T_+ h_2 v_t \lambda_t + h_2^3 v_t \lambda_t + h_2^2 v_t^2 \lambda_t - \frac{1}{4} v_t^4 \lambda_t;$$

Interaction Terms in HTM

Out[85]=
$$\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{\left\{\frac{1}{2}\phi_- T_+ A_{ht} h_1 + \frac{1}{2} T_- \phi_+ A_{ht} h_1 + \frac{1}{2}\phi_- \phi_+ A_{ht} h_2 - \frac{1}{4} A_{ht} h_1^2 h_2 + \frac{1}{2}\phi_- T_+ A_{ht} v_h + \frac{1}{2} T_- \phi_+ A_{ht} v_h - \frac{1}{2} A_{ht} h_1 h_2 v_h + \frac{T_- T_+ A_{ht} v_h^2}{4 v_t} + \frac{A_{ht} h_2^2 v_h^2}{8 v_t} + \phi_- \phi_+ A_{ht} v_t + \frac{1}{8} A_{ht} v_h^2 v_t + \phi_-^2 \phi_+^2 \lambda_h + \phi_- \phi_+ h_1^2 \lambda_h + \frac{1}{4} h_1^4 \lambda_h + 2\phi_- \phi_+ h_1 v_h \lambda_h + h_1^3 v_h \lambda_h + h_1^2 v_h^2 \lambda_h - \frac{1}{4} v_h^4 \lambda_h + T_- \phi_- T_+ \phi_+ \lambda_{ht} + \frac{1}{2} T_- T_+ h_1^2 \lambda_{ht} + \frac{1}{2}\phi_- \phi_+ h_2^2 \lambda_{ht} + \frac{1}{4} h_1^2 h_2^2 \lambda_{ht} + T_- T_+ h_1 v_h \lambda_{ht} + \frac{1}{2} h_1 h_2^2 v_h \lambda_{ht} + \phi_- \phi_+ h_2 v_t \lambda_{ht} + \frac{1}{2} h_1^2 h_2 v_t \lambda_{ht} + h_1 h_2 v_h v_t \lambda_{ht} - \frac{1}{4} v_h^2 v_t^2 \lambda_{ht} + T_-^2 T_+^2 \lambda_t + T_- T_+ h_2^2 \lambda_t + \frac{1}{4} h_2^4 \lambda_t + 2 T_- T_+ h_2 v_t \lambda_t + h_2^3 v_t \lambda_t + h_2^2 v_t^2 \lambda_t - \frac{1}{4} v_t^4 \lambda_t\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}\right\}$$

In[113]:=
```
vari = {ϕ₊, ϕ₋, T₊, T₋, h₁, h₂};

U5 = FromCoefficientRules[Select[CoefficientRules[U4, vari], Total@#[[1]] == 2 &],
  vari ] (*Collecting the bilinear terms
*)
```

All the quadratic terms

Out[114]=
$$\frac{1}{2}\phi_- T_+ A_{ht} v_h + \frac{1}{2} T_- \phi_+ A_{ht} v_h + \frac{T_- T_+ A_{ht} v_h^2}{4 v_t} + \phi_- \phi_+ A_{ht} v_t + h_1^2 v_h^2 \lambda_h + h_1 h_2 \left(-\frac{1}{2} A_{ht} v_h + v_h v_t \lambda_{ht}\right) + h_2^2 \left(\frac{A_{ht} v_h^2}{8 v_t} + v_t^2 \lambda_t\right)$$

In[134]:=
```
                                          (* Mixing of Charged Higgs *)
z1 = Solve[h₁ == 0, h₁];
z2 = Solve[h₂ == 0, h₂];
e₁₁ = D[D[U5, ϕ₊], ϕ₋];
e₁₂ = D[D[U5, ϕ₊], T₋];
e₂₁ = D[D[U5, T₊], ϕ₋];
e₂₂ = D[D[U5, T₊], T₋];
M₁ = Simplify[(e₁₁ e₁₂
               e₂₁ e₂₂)] // MatrixForm
```

Mixing Matrix for Charged Higgs

Out[140]//MatrixForm=
$$\begin{pmatrix} A_{ht} v_t & \frac{A_{ht} v_h}{2} \\ \frac{A_{ht} v_h}{2} & \frac{A_{ht} v_h^2}{4 v_t} \end{pmatrix}$$

In[147]:= `Eigenvalues[{{A_ht v_t, A_ht v_h/2}, {A_ht v_h/2, A_ht v_h^2/(4 v_t)}}]`

$$\text{Eigenvalues}\left[\left\{\left\{A_{ht}\, v_t,\ \frac{A_{ht}\, v_h}{2}\right\},\ \left\{\frac{A_{ht}\, v_h}{2},\ \frac{A_{ht}\, v_h^2}{4\, v_t}\right\}\right\}\right]$$

Mass Spectrum

Out[147]=
$$\left\{0,\ \frac{A_{ht}\left(v_h^2 + 4\, v_t^2\right)}{4\, v_t}\right\}$$

(* Mixing of Neutral Components *)

In[141]:=

$$g_{11} = D[D[U5, h_1], h_1];$$
$$g_{12} = D[D[U5, h_1], h_2];$$
$$g_{21} = D[D[U5, h_2], h_1];$$
$$g_{22} = D[D[U5, h_2], h_2];$$
$$M_2 = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \text{ // MatrixForm}$$

Mixing Matrix for Neutral Higgs

Out[145]//MatrixForm=

$$\begin{pmatrix} 2\, v_h^2\, \lambda_h & -\frac{1}{2} A_{ht}\, v_h + v_h\, v_t\, \lambda_{ht} \\ -\frac{1}{2} A_{ht}\, v_h + v_h\, v_t\, \lambda_{ht} & 2\left(\frac{A_{ht}\, v_h^2}{8\, v_t} + v_t^2\, \lambda_t\right) \end{pmatrix}$$

Mass Spectrum

Eigenvalues[%420]

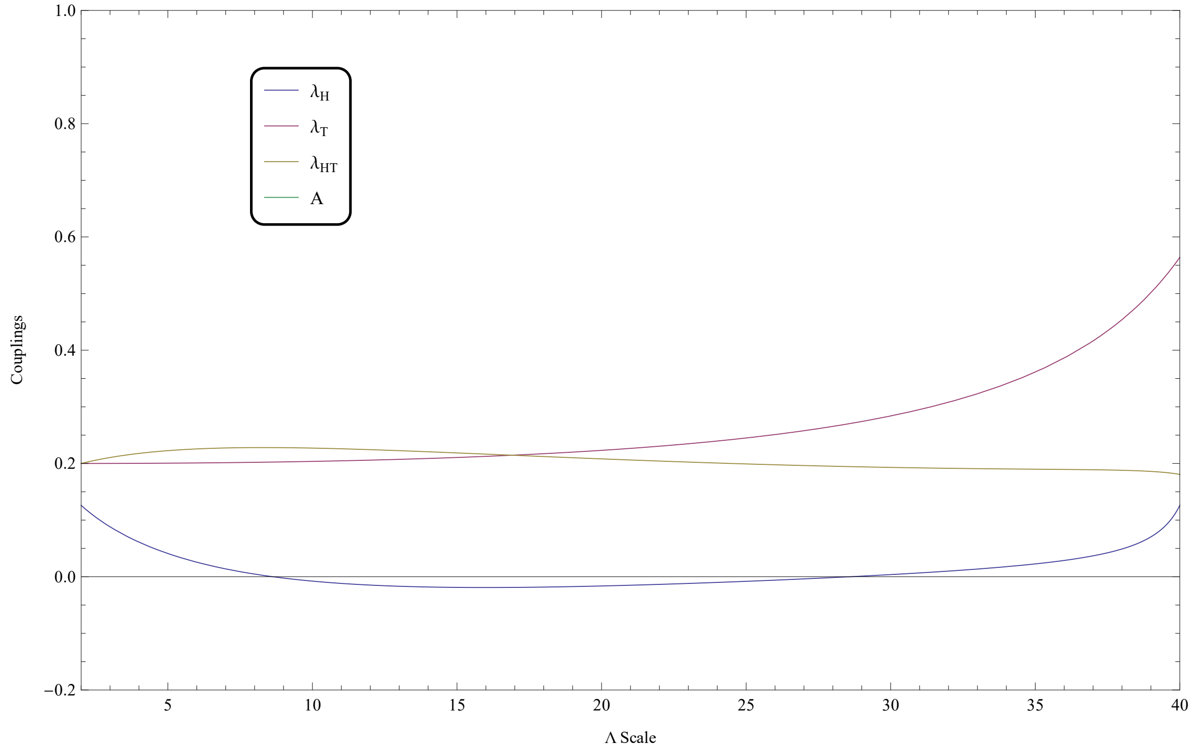
$$\left\{\frac{1}{8\, v_t}\left(A_{ht}\, v_h^2 + 8\, v_h^2\, v_t\, \lambda_h + 8\, v_t^3\, \lambda_t - \sqrt{\left(\left(-A_{ht}\, v_h^2 - 8\, v_h^2\, v_t\, \lambda_h - 8\, v_t^3\, \lambda_t\right)^2 - 4\left(-4\, A_{ht}^2\, v_h^2\, v_t^2 + 8\, A_{ht}\, v_h^4\, v_t\, \lambda_h + 16\, A_{ht}\, v_h^2\, v_t^3\, \lambda_{ht} - 16\, v_h^2\, v_t^4\, \lambda_{ht}^2 + 64\, v_h^2\, v_t^4\, \lambda_h\, \lambda_t\right)\right)}\right),\right.$$
$$\left.\frac{1}{8\, v_t}\left(A_{ht}\, v_h^2 + 8\, v_h^2\, v_t\, \lambda_h + 8\, v_t^3\, \lambda_t + \sqrt{\left(\left(-A_{ht}\, v_h^2 - 8\, v_h^2\, v_t\, \lambda_h - 8\, v_t^3\, \lambda_t\right)^2 - 4\left(-4\, A_{ht}^2\, v_h^2\, v_t^2 + 8\, A_{ht}\, v_h^4\, v_t\, \lambda_h + 16\, A_{ht}\, v_h^2\, v_t^3\, \lambda_{ht} - 16\, v_h^2\, v_t^4\, \lambda_{ht}^2 + 64\, v_h^2\, v_t^4\, \lambda_h\, \lambda_t\right)\right)}\right)\right\}$$

Running Couplings in Higgs Triplet Model

```
s1 = NDSolve[{g1'[t] - (41/(16 * π * π * 10)) * g1[t]^3 * Log[10] == 0,
    g2'[t] + (19/(16 * π * π * 6)) * g2[t]^3 * Log[10] == 0,
    g3'[t] + (7/(16 * π * π)) * g3[t]^3 * Log[10] == 0, y'[t] - (1/(16 * π * π)) *
      ((9/2) y[t]^3 - 8 g3[t]^2 * y[t] - (9/4) * g2[t]^2 * y[t] - (17/20) g1[t]^2 * y[t]) * Log[10] == 0,
    λH'[t] - (1/(16 * π * π)) * ((27 g1[t]^4)/200 + (9 g1[t]^2 g2[t]^2)/20 + (9 g2[t]^4)/8 - (9 g1[t]^2 λH[t])/5 -
         9 g2[t]^2 λH[t] + 24 λH[t]^2 + 12 λH[t] y[t]^2 - 6 * y[t]^4 + (3/2) λHT[t]^2) * Log[10] == 0,
    λHT'[t] - (1/(16 * π * π)) * (3 g2[t]^4 - (9 g1[t]^2 λHT[t])/10 - (33 g2[t]^2 λHT[t])/2 +
         12 λH[t] λHT[t] + 4 λHT[t]^2 + 6 λHT[t] λT[t] + 6 λHT[t] y[t]^2) * Log[10] == 0,
    λT'[t] - (2/(16 * π * π)) * (3 g2[t]^4 + λHT[t]^2 - 12 g2[t]^2 λT[t] + 11 λT[t]^2) * Log[10] == 0,
    A'[t] - (1/(16 * π * π)) * ((21/2) g2[t]^2 A[t] + 4 λH[t] A[t] +
         4 λHT[t] A[t] + 6 y[t]^2 A[t] - (9/10) g1[t]^2 A[t]) * Log[10] == 0,
    g1[2] == 0.46256, g2[2] == 0.64779, g3[2] == 1.1666, y[2] == 0.93690,
    λH[2] == 0.12604, λT[2] == 0.2, λHT[2] == 0.2, A[2] == 0.2},
   {g1[t], g2[t], g3[t], y[t], λH[t], λT[t], λHT[t], A[t],}, {t, 2, 40}];
```

```
s = Plot[Evaluate[{{λH[t] /. s1}, {λT[t] /. s1}, {λHT[t] /. s1}, (*{A[t]/.s1}*)}],
  {t, 2, 40}, Frame → True, FrameLabel → {Scale Λ, Couplings },
  PlotRange → {{2, 40}, {-0.2, 1}}, Axes → True, PlotLegends →
   Placed[LineLegend[{"λH", "λT", "λHT", "A"}, LegendFunction → Frame], {0.2, 0.80}]]
```

HTM Running Lambdas



Inert Triplet Model : Minimization conditions, Mass Spectrum

In[17]:=

```
                        (* Triplet Extension of Standard Model(HTM) : SM Doublet + Y=
 0 Real Triplet *)
(* Info : Scroll down to end to see useful results *)

(* Minimization Conditions*)

ClearAll[x, ϕ, ξ, M, hc, H, T]; (*Clear all the Definitions *)

U2 = m_h * (Φ⁺ .Φ) + m_t  * Tr[(ξ⁺.ξ)] + λ_h * (Φ⁺ .Φ)^2 +
    λ_t * (Tr[(ξ⁺.ξ)])^2 + λ_ht * (Φ⁺ .Φ) * Tr[(ξ⁺.ξ)]; (*Potential of HTM*)
```

$\Phi = \begin{pmatrix} 0 \\ \phi_0 \end{pmatrix}$; $\Phi^+ = (0 \;\; \phi_0)$; (* Definition of Doublet*)

$\xi = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}\,T_+ \\ \sqrt{2}\,T_- & -T_0 \end{pmatrix}$;

$\xi^+ = \frac{1}{2}\begin{pmatrix} T_0 & \sqrt{2}\,T_+ \\ \sqrt{2}\,T_- & -T_0 \end{pmatrix}$;   (* Definition of Triplet *)

$\phi_0 = h_1$; $\phi_0 = h_1$;   (* Fluctuation around minima*)

U3 = Expand[U2] ;

U4 = $h_1^2\, m_h + T_- T_+ m_t + \frac{1}{2} m_t T_0^2 + h_1^4 \lambda_h + T_- T_+ h_1^2 \lambda_{ht} + \frac{1}{2} h_1^2 T_0^2 \lambda_{ht} + T_-^2 T_+^2 \lambda_t + T_- T_+ T_0^2 \lambda_t + \frac{1}{4} T_0^4 \lambda_t$;

t1 = Solve[T_+ == 0] ;
t2 = Solve[T_- == 0] ;
t3 = Solve[ϕ_+ == 0] ;
t4 = Solve[ϕ_- == 0] ;
v1 = Solve[$h_1 == \frac{v_h}{\sqrt{2}}$] ;
t5 = Solve[T_0 == 0] ;
Expand[U4] /. t1 /. t2 /. t3 /. t4 /. t5;
V4 = $h_1^2 m_h + h_1^4 \lambda_h$;
y1 = Solve[D[V4, h_1] == 0, m_h] /. v1

Out[32]= $\{\{\{m_h \to -v_h^2 \lambda_h\}\}\}$

Using minimization condition :

$\{\{\{\{m_h \to -v_h^2 \lambda_h\}\}\}\}$

In[61]:=
```
ClearAll[x, ϕ, ξ, M, hc, H, T]; (*Clear all the Definitions *)

U2 = m_h * (Φ⁺ .Φ) + m_t * Tr[(ξ⁺.ξ)] + λ_h * (Φ⁺ .Φ)^2 +
    λ_t * (Tr[(ξ⁺.ξ)])^2 + λ_ht * (Φ⁺ .Φ) * Tr[(ξ⁺.ξ)]; (*Potential of HTM*)

Φ = (0, ϕ_0); Φ⁺ = (0 ϕ_0); (* Definition of Doublet*)

ξ = 1/2 {{T_0, √2 T_+}, {√2 T_-, -T_0}};

ξ⁺ = 1/2 {{T_0, √2 T_+}, {√2 T_-, -T_0}};          (* Definition of Triplet *)

ϕ_0 = (v_h + h_1)/√2 ;          ϕ_0 = (v_h + h_1)/√2 ;          (* Fluctuation around minima*)

U3 = Expand[Expand[U2] /. y1]

                                          (* Potential after Expansion *)
```

$$U4 = T_- T_+ m_t + \frac{1}{2} m_t T_0^2 + \frac{1}{4} h_1^4 \lambda_h + h_1^3 v_h \lambda_h + h_1^2 v_h^2 \lambda_h - \frac{1}{4} v_h^4 \lambda_h + \frac{1}{2} T_- T_+ h_1^2 \lambda_{ht} + \frac{1}{4} h_1^2 T_0^2 \lambda_{ht} + T_- T_+ h_1 v_h \lambda_{ht} + \frac{1}{2} h_1 T_0^2 v_h \lambda_{ht} + \frac{1}{2} T_- T_+ v_h^2 \lambda_{ht} + \frac{1}{4} T_0^2 v_h^2 \lambda_{ht} + T_-^2 T_+^2 \lambda_t + T_- T_+ T_0^2 \lambda_t + \frac{1}{4} T_0^4 \lambda_t;$$

Interaction Terms

Out[66]=

$$\left\{\left\{\left\{\left\{T_- T_+ m_t + \frac{1}{2} m_t T_0^2 + \frac{1}{4} h_1^4 \lambda_h + h_1^3 v_h \lambda_h + h_1^2 v_h^2 \lambda_h - \frac{1}{4} v_h^4 \lambda_h + \frac{1}{2} T_- T_+ h_1^2 \lambda_{ht} + \frac{1}{4} h_1^2 T_0^2 \lambda_{ht} + T_- T_+ h_1 v_h \lambda_{ht} + \frac{1}{2} h_1 T_0^2 v_h \lambda_{ht} + \frac{1}{2} T_- T_+ v_h^2 \lambda_{ht} + \frac{1}{4} T_0^2 v_h^2 \lambda_{ht} + T_-^2 T_+^2 \lambda_t + T_- T_+ T_0^2 \lambda_t + \frac{1}{4} T_0^4 \lambda_t\right\}\right\}\right\}\right\}$$

⋮

```
vari = {T_+, T_-, h_1, T_0};

U5 = FromCoefficientRules[Select[CoefficientRules[U4, vari], Total@#[[1]] == 2 &],
  vari ] (*Collecting the bilinear terms
*)
```

All the quadratic terms: Mass terms

$$h_1^2 v_h^2 \lambda_h + T_0^2 \left(\frac{m_t}{2} + \frac{1}{4} v_h^2 \lambda_{ht}\right) + T_- T_+ \left(m_t + \frac{1}{2} v_h^2 \lambda_{ht}\right)$$

$$h_1^2 v_h^2 \lambda_h + T_0^2 \left(\frac{m_t}{2} + \frac{1}{4} v_h^2 \lambda_{ht}\right) + T_- T_+ \left(m_t + \frac{1}{2} v_h^2 \lambda_{ht}\right)$$

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