Tire
Brush
Model
Group 9
Introduction

An analysis of the mechanism of tire contact force generation under transient conditions is presented. The model consists of independent bristles, in which the state of each bristle at any instant of time depends on the state of the same bristle at a previous time step. Friction between the tire and the ground follows an experimentally verified stick–slip law. Simulation results reveal show how transient friction force generation may differ substantially from steady state predictions.
CAD Model of vehicle
CAD Model of Tire
Terminology

Mass distribution along tread, \( C = \frac{dm}{dx} \)

Longitudinal velocity of bristle base w.r.t. ground, \( V_{sx} \)

Stiffness coefficients per unit tread length, \( K_{xx} = \frac{\frac{dF}{dx}}{dx} \)

Longitudinal velocity of bristle base w.r.t. ground, \( V_{sy} \)

Stiffness coefficients per unit tread length, \( K_{yx} = \frac{\frac{dF}{dy}}{dx} \)

Sliding velocity of the mass, \( u \)

Stiffness coefficients per unit tread length, \( D_{xx} = \frac{\frac{dF}{du_x}}{dx} \)

Rate of change in deformation of bristles, \( u_x \) and \( u_y \)

Stiffness coefficients per unit tread length, \( D_{yx} = \frac{\frac{dF}{du_y}}{dx} \)
Methodology

• There is evidence that the length of the contact patch does not change significantly with an increase in the forward velocity.

• If a constant forward velocity is considered, the use of a model with a constant contact patch length and parabolic pressure distribution can serve as a good starting point for the analysis.

• Tire tread is modelled as a one-dimensional series of bristles distributed on the tire periphery. The bristles incorporate anisotropic stiffness and damping in the lateral and longitudinal directions, and the distributed tread mass on the tire periphery is also taken into account by attaching an infinitesimal mass to the end of each bristle.
• The bristle, connecting the mass to the wheel periphery, is deformed laterally, as well as longitudinally, and the mass may or may not be sliding on the ground, depending on the viscoelastic restoring forces applied by the bristle, the normal force at the specific position, and the coefficient of friction. The normal force distribution throughout the length of the contact patch is given by the parabolic equation –

\[ F_{vertical} = \frac{3F_z}{4\alpha} \left(1 - \left(\frac{x}{a}\right)^2\right) \]

where, \(F_z\) is the total vertical force applied on the wheel hub and \(2a\) is the total length of the contact patch.

• The global frame of reference (OXYZ) is attached to the ground, while a second frame of reference (oxyz) has its origin on the point in the contact patch where the vertical line from the center of the wheel plane meets the ground.
• Point b, where the bristle is connected to the tire periphery (i.e. the bristle base), enters the contact patch at coordinates
\[(x, y, z) = (a, 0,0)\]
on the moving frame of reference and travels throughout the contact patch with velocity,
\[V_d = \omega R_d\]
where \(R_d\) is the radius of the vertically loaded tire under pure rolling condition.

• When the vertical force results in the generation of a high enough frictional force, the infinitesimal mass dm sticks on the ground. In any other case, the mass moves with respect to the ground with a sliding velocity \(u\).

• Irrespective of whether the tire is slipping or not, the velocity of travel of point \(b\) throughout the length of the contact patch is \(V_d\). Thus, the vertical force on point \(b\) varies according to following relationship
\[F_{vertical} = \frac{3F_z}{4\alpha} \left(1 - \left(\frac{x_d}{a}\right)^2\right)\]
\[x_d = V_d\]
During simulation, the stick–slip conditions described by above relations are checked using a velocity transition threshold. For velocities below the value of the threshold, the infinitesimal mass is considered to be stationary, while sliding occurs for velocities greater than the threshold.

If the sticking condition is satisfied, then the magnitude of the friction force equates to the magnitude of the forces applied by the bristle. When the mass is sliding on the ground, the magnitude of the friction force becomes:

$$|f| = \mu_k \frac{3F_z}{4\alpha} \left(1 - \left(\frac{x_d}{a}\right)^2\right) dx$$

Friction between the tread and the road follows a simplified stick–slip law derived from experimental measurements by Braghin et al. Here, $F_{\text{external}}$ denotes force applied by bristle on mass, $\mu_k$ is the coefficient of kinetic friction, $F_{\text{max}}$ is the maximum friction force.

$$f = \begin{cases} -F_{\text{external}} & \text{for } u = 0 \\ F_{\text{external}} \leq F_{\max} = \text{factor} \cdot \mu_k \cdot F_{\text{vertical}} = \mu F_{\text{vertical}} & \text{AND} \\ f = \mu_k F_{\text{vertical}} & \text{for } |u| > 0 \\ |F_{\text{external}}| > \mu F_{\text{vertical}} & \text{OR} \end{cases}$$
The components of \( f \) in the longitudinal and lateral directions are –

\[
f_x = \frac{u_x}{\sqrt{u_x^2 + u_y^2}} \mu_k \frac{3F_z}{4\alpha} \left(1 - \frac{x_d}{a}\right)^2 dx
\]

\[
f_y = \frac{u_y}{\sqrt{u_x^2 + u_y^2}} \mu_k \frac{3F_z}{4\alpha} \left(1 - \frac{x_d}{a}\right)^2 dx
\]

The differential equations, describing the motion of the mass in longitudinal and lateral directions are written as -

\[
\dot{u}_x C dx = (x_s - x)K_{xx} dx + (V_{sx} - u_x)D_x dx - f_x
\]

\[
\dot{u}_y C dx = (y_s - y)K_{yx} dx + (V_{sy} - u_y)D_y dx - f_y
\]

\[
\dot{x}_s = V_{sx}
\]

\[
\dot{x}_s = V_{sy}
\]
Simulation

In the steady state model, the motion of the infinitesimal mass is followed throughout the contact patch and is representative of the motion of all such elements in contact with the ground. The transient model is run for purely cornering conditions, so that $V_{sx}$ vanishes. While $V_{sy}$ is constant in the steady state model, it changes in each time step in the transient model. The state of a mass $dm$ at $t + dt$ results from the state of the same mass at $t$. 

In order to solve the problem, the vectors of the state variables, positions, and velocities of all infinitesimal masses forming the contact patch have to be defined. If the length of the contact patch is $2a = ndx$, then $n$ infinitesimal masses are involved in the problem. At a random operating point, for example at time $t$, each mass is characterized by its velocity and position in the $oxy$ plane. At time $t + dt$, every mass has moved one place towards the end of the contact patch, travelling a distance of $dx = Vdt$. This sequential switching is also reflected in the state vectors, so that the state of the $(i + 1)^{th}$ mass at time $t + dt$ can be calculated by using the state of the $i^{th}$ mass at time $t$. In order for each mass exactly to take the place of the one adjacent to it, the time step has to be constant and the number of masses has to be set according to the relationship –

$$n = \frac{2a}{Vdt}$$
Integration procedure for the transient tyre model

New entry with initial conditions $0, V_{xy}$

Time

$t$

$t + dt$

State Vector at time $t$

Single Step Range Kutta

$y_{i-1}, u_{yi-1}$

$y_i, u_{yi}$

$y_{i+1}, u_{yi+1}$

Single Step Range Kutta

$y_{i+1}, u_{yi+1}$

$y_{i+2}, u_{yi+2}$

State Vector at time $t + dt$

Last entry leaves the Vector

1

$i$

$i + 1$

$i + 2$

$n$

$n$
The transient response of a tire in a lateral maneuver is chosen as the case study for this paper. The wheel is moving forward with a constant velocity of 10 m/s, while an increasing lateral velocity is imposed on the wheel rim. The result is a transient increase in lateral slip at a constant rate of 30.96 deg/s. Practically, this maneuver is approximately equivalent to the transient increase in slip angle of the rear tires of a car as a result of oversteering behavior, when the driver ceases upon acceleration mid-way through a tight corner.

<table>
<thead>
<tr>
<th>$l$ (m)</th>
<th>$R$ (m)</th>
<th>$b$ (m)</th>
<th>$F_x$ (N)</th>
<th>$K_y$ (N/m$^2$)</th>
<th>$K_{\text{inter}}$ (N/m$^2$)</th>
<th>$D_y$ (N s/m$^2$)</th>
<th>$D_{\text{inter}}$ (N s/m$^2$)</th>
<th>$\mu_k$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.065</td>
<td>0.29</td>
<td>0.18</td>
<td>4150</td>
<td>$9 \times 10^6$</td>
<td>$3.6 \times 10^6$</td>
<td>$8 \times 10^2$</td>
<td>$3.2 \times 10^2$</td>
<td>0.9</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Time step = $1 \times 10^{-4}$ s, velocity transition threshold = 0.012 m/s.
Results & Discussion

- Fluctuations in self-aligning moment are not only evident, but also magnified.
- While, in the beginning, steady state and transient responses are almost identical, multiplication of the lateral force distribution with the corresponding distances from the vertical axis results in more intense oscillations, which are even more pronounced after the peak value of self-aligning moment in the range of slip ratios between 0.045 and 0.14.
- The transient curve smoothens out towards the end of the graph, inside the region of saturated operation.
• As clear from the figure, both models yield similar results in the low, linear range of the force–slip diagram.
• As the slip ratio increases and the graphs enter the non-linear region of operation, microscopic stick–slip action between the tread elements and the road leads to minor fluctuations, captured by the transient model. With a further increase in the slip ratio, higher amplitudes of oscillation are predicted by the transient model (about 10% of the total lateral force).
• The three sequential drops in lateral force predicted by the transient model could alter the response of a vehicle significantly.
• As the slip ratio increases further, the period and amplitude of oscillations decrease continually, and finally the response smoothens completely at the saturated area of operation.
Model Integration Using MSC Adams

• MSC ADAMS can be used for driver input and can be integrated with MATLAB using ADAMS/Control and resultant forces and moments can be sent back to ADAMS.

• A general procedure for such a control integration is shown here.

• Another tire model, namely Pacejka 89 is used to demonstrate the results obtained for such transient maneuvers.
Vehicle and Tire Parameters in ADAMS

<table>
<thead>
<tr>
<th>Components</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total vehicle mass</td>
<td>kg</td>
<td>980</td>
</tr>
<tr>
<td>Wheelbase</td>
<td>m</td>
<td>2.443</td>
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<tr>
<td>Gravity center height</td>
<td>m</td>
<td>0.53</td>
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<tr>
<td>Rear axle to gravity center</td>
<td>m</td>
<td>1.460</td>
</tr>
<tr>
<td>Tire radius</td>
<td>mm</td>
<td>340.6</td>
</tr>
<tr>
<td>Tire width</td>
<td>mm</td>
<td>255</td>
</tr>
</tbody>
</table>
MATLAB Proposed Integration with Adams using ADAMS/Control
References


• Braghin, F., Cheli, F., and Resta, F. Friction law identification for rubber compounds on rough surfaces at medium sliding speeds. 3rd AIMETA International Tribology Conference, Salerno, Italy, 18–20 September 2002.

Thank you!