# DYNAMIC FRICTION MODEL


**GROUP 8**

ADITYA HALGEKAR – ME17BTECH11001

ANIRUTHAN R –ME17BTECH11007

ANSHUL MESHRAM – ME17BTECH11008

ARVIND PRAJAPAT – ME17BTECH11011

CHANDRASHEKHAR DHANGAR – ME17BTECH11016

# INTRODUCTION

Vehicle motion is primarily determined by the friction forces transferred to the road by the tires. Therefore, a proper tire friction model is one of the key elements of a complete vehicle model intended for the use in different vehicle dynamics simulation studies for handling applications.

Static friction models are appropriate when we have steady-state conditions for the linear and angular velocities. Dynamic friction models attempt to capture the transient behavior of the tire-road contact forces under time-varying velocity conditions.

# DAHL MODEL

Dahl's model starts with the stress-strain curve in classical solid mechanics. When subject to stress, the friction force increases gradually until rupture occurs.

SIMULATION MODEL :

$$\frac{dF}{dx} = \sigma_0 (1 - \frac{F}{F_c} sgn(v_r))^\beta$$

Where $x$ = relative displacement, $\sigma_0$ = stiffness coefficient, $F_c$ = maximum friction force (Coulomb force) and $\beta$ is a parameter that determines the shape of the stress-strain curve.

$v_r = {}^{dx}/_{dt}$ is the relative velocity. ($v_r = r\omega - v$)

For time-domain model,

$$\frac{dF}{dt} = \frac{dF}{dx}\frac{dx}{dt} = \frac{dF}{dx}v_r = \sigma_0(1 - \frac{F}{F_c}sgn(v_r))^{\beta}v_r$$

The most commonly used value of $\beta$ is 1. Therefore,

$$\frac{dF}{dt} = \sigma_0\left(v_r - \frac{F|v_r|}{F_c}\right)$$

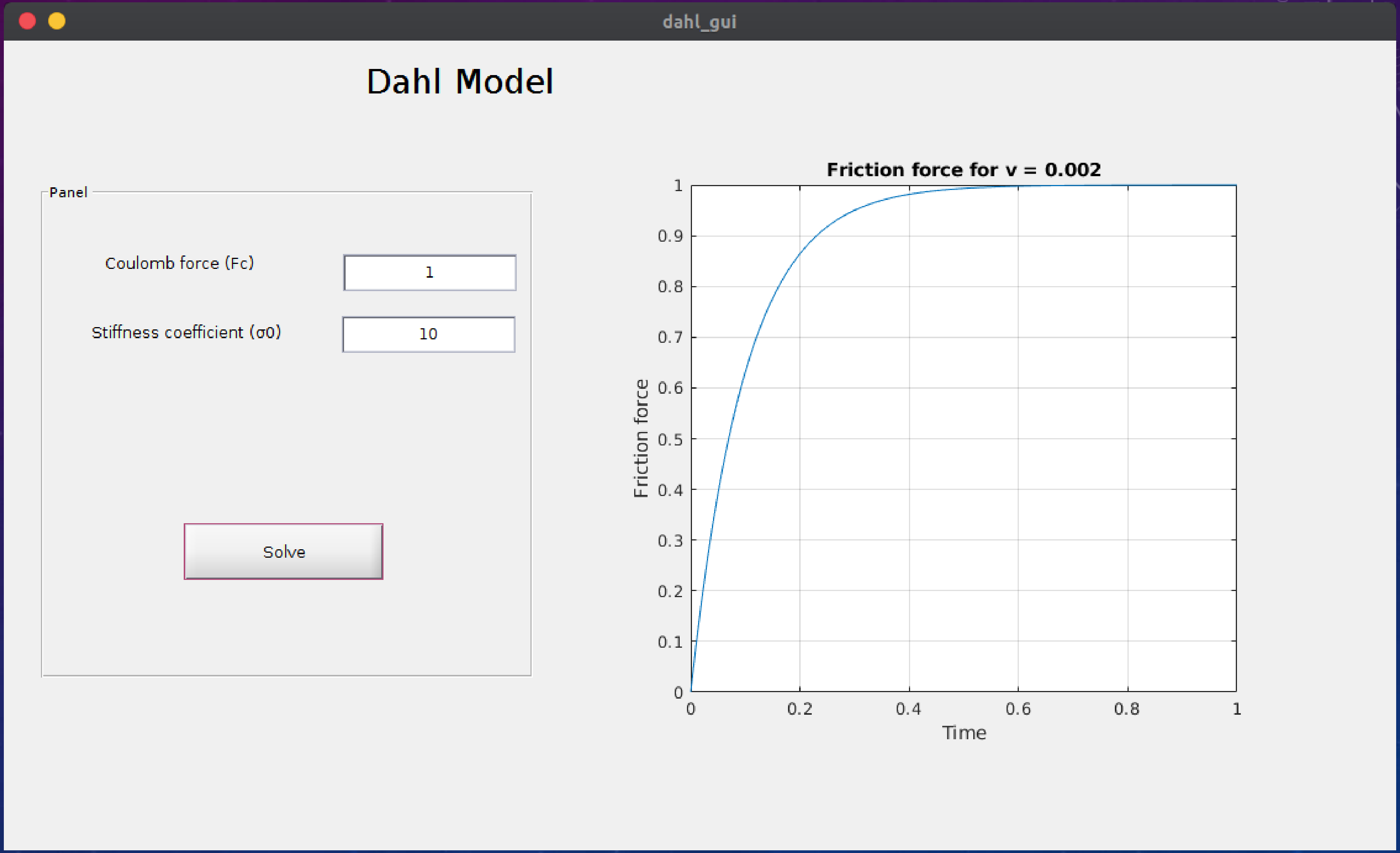
# MODEL DESCRIPTION :

$$\frac{dz}{dt} = v_r - \frac{\sigma_0 z |v_r|}{F_c}$$

$$F = \sigma_0 z$$

Where $z$ is the relative displacement of the bristles.

# NORMALIZED TOTAL FRICTION FORCE VS TIME



# SOME COMMON TERMS

The longitudinal force $F$ is usually described as a static function of the longitudinal slip rate $s$ which is defined as,

$$s = \begin{cases} \dfrac{v - r\omega}{v} & , for \qquad r\omega \leq v \neq 0 \; (braking) \\ \dfrac{r\omega - v}{r\omega} & , for \qquad r\omega < v \neq 0 \; (driving) \end{cases}$$

Where $v$ = vehicle speed, $\omega$ = wheel angular velocity , $r$ = effective tire radius.

It is assumed that the tire/road contact is realized through a lot of tiny, massless and elastic elements called bristles. The contact patch has a rectangular form with the length L. The uniform normal pressure distribution is assumed in the paper. The relative speed between the bristle base point attached to the belt, and the tip which adheres to the ground is (considering traction)

$$v_r = r\omega - v$$

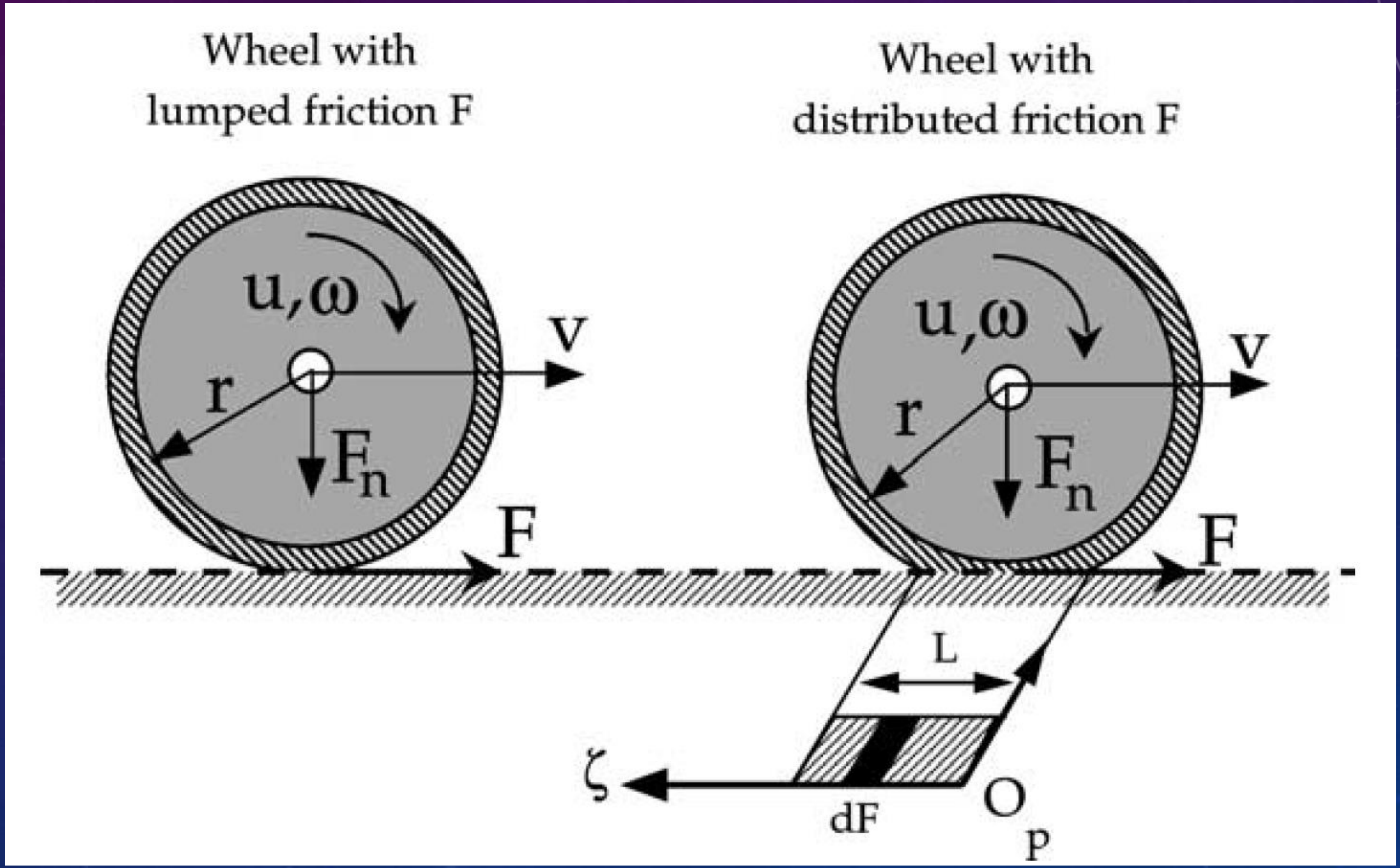
The bristles deform producing the tire longitudinal force $F$.

# LUMPED FRICTION VS DISTRIBUTED FRICTION

A lumped friction model assumes a point tire-road friction contact. As a result, the mathematical model describing such a model is an ordinary differential equations that can be easily solved by time integration.

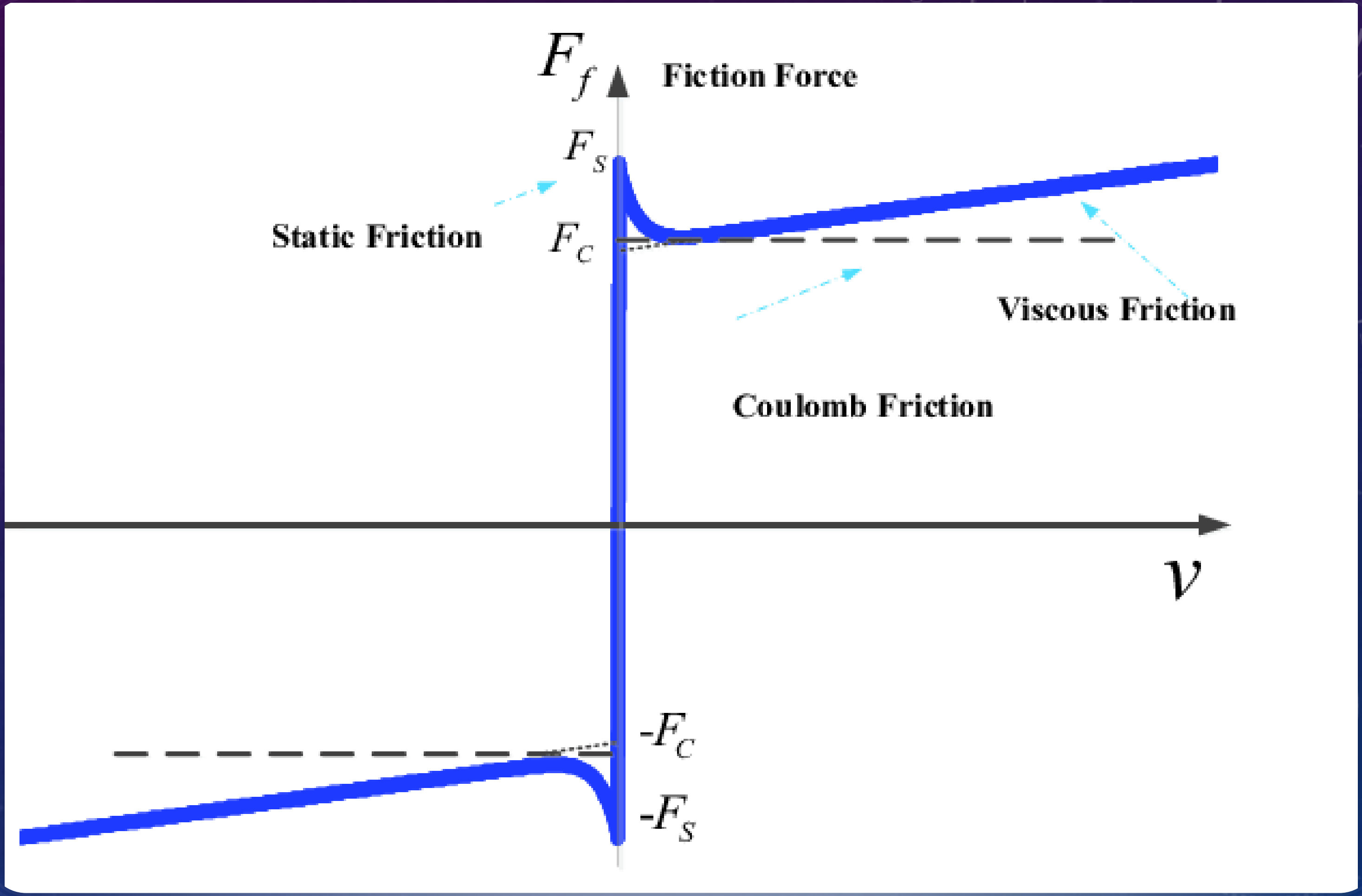
Distributed friction models, on the other hand, assume the existence of a contact patch between the tire and the ground with an associated normal pressure distribution. This formulation results in a partial differential equation, that needs to be solved both in time and space.

# ONE-WHEEL SYSTEM WITH LUMPED FRICTION (LEFT), AND DISTRIBUTED FRICTION (RIGHT)



# STRIBECK EFFECT

When the thickness of the film is large enough to completely separate the bodies in contact, the friction coefficient may increase with velocity as hydrodynamic effects become significant. This is called the **Stribeck effect**.



# LUMPED LUGRE MODEL

The LuGre model is an extension of the Dahl model that includes the Stribeck effect.

SIMULATION OF MODEL :

$$\frac{dz}{dt} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z$$

$$F = \left( \sigma_0 z + \sigma_1 \frac{\partial z}{\partial t} + \sigma_2 v_r \right) F_n$$

$$g(v_r) = \theta \left[ \mu_c + (\mu_s - \mu_c) * \exp\left( - \left( {}^{v_r}/_{v_s} \right)^{\alpha} \right) \right]$$

Where $\sigma_0$ = rubber longitudinal lumped stiffness

$\sigma_1$ = rubber longitudinal lumped damping (bristle damping coefficient),

$\sigma_2$ = viscous relative damping (friction viscous coefficient),

$F_n$ = normal force.

$\mu_c$ = normalized Coulomb friction

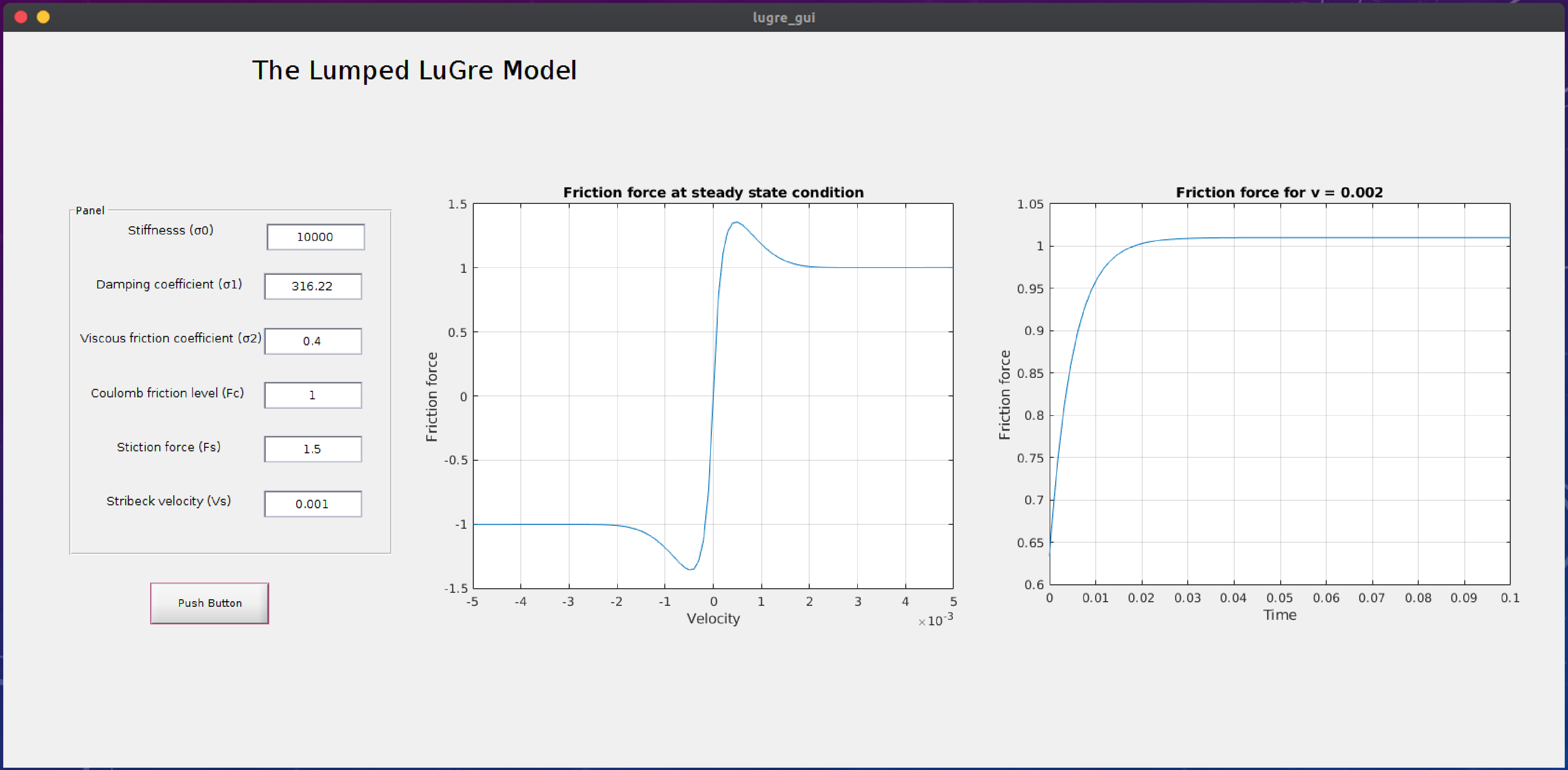
$\mu_s$ = normalized Static friction

$v_s$ = Stribeck relative velocity

$\alpha$ is a constant. Here we use $\alpha = {}^{1}/_{2}$.

$g(v_r)$ is the Stribeck-type tire/road sliding friction function.

# LUMPED LUGRE MODEL RESULTS



# DISTRIBUTED LUGRE MODEL

In this model, we extend the point friction model to a distributed friction model along the patch by letting $z(\zeta, t)$ denote the friction state of the bristle element at a certain time $t$.

At every time instant $z(\zeta, t)$ provides the deflection distribution along the contact patch.

SIMULATION MODEL :

The equation for this model can be written as (using first two equations of Lumped LuGre model)

$$\frac{dz(\zeta,t)}{dt} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z$$

$$F(t) = \int_0^L dF(\zeta,t)$$

Where $g(v_r)$ is same as defined in lumped model.

$$dF(\zeta,t)=\left(\sigma_0 z(\zeta,t)+\sigma_1\frac{\partial z(\zeta,t)}{\partial t}+\sigma_2 v_r\right)dF_n(\zeta,t)$$

Where $dF(\zeta,t)$ is the differential friction force developed in the element $d\zeta$and $dF_n(\zeta,t)$ is the differential normal force applied in the element $d\zeta$ at time $t$.

This model assumes that the contact velocity of each differential state element is equal to $v_r$ .

# RELATION WITH MAGIC FORMULA :

Steady state expressions depend on v and w. Therefore two cases are formed accordingly.

**Driving case :**

In this case $v < r\omega$, therefore the force at steady state is given by

$$F_d(s) = sgn(v_r)F_n g(s)\left[1 + \frac{g(s)\left(e^{-\sigma_0 L|s|/g(s)} - 1\right)}{\sigma_0 L|s|}\right] + F_n \sigma_2 r\omega s$$

$$g(s) = \mu_c + (\mu_s - \mu_c) * \exp\left(-\left|{}^{r\omega s}/_{v_s}\right|^{\alpha}\right)$$

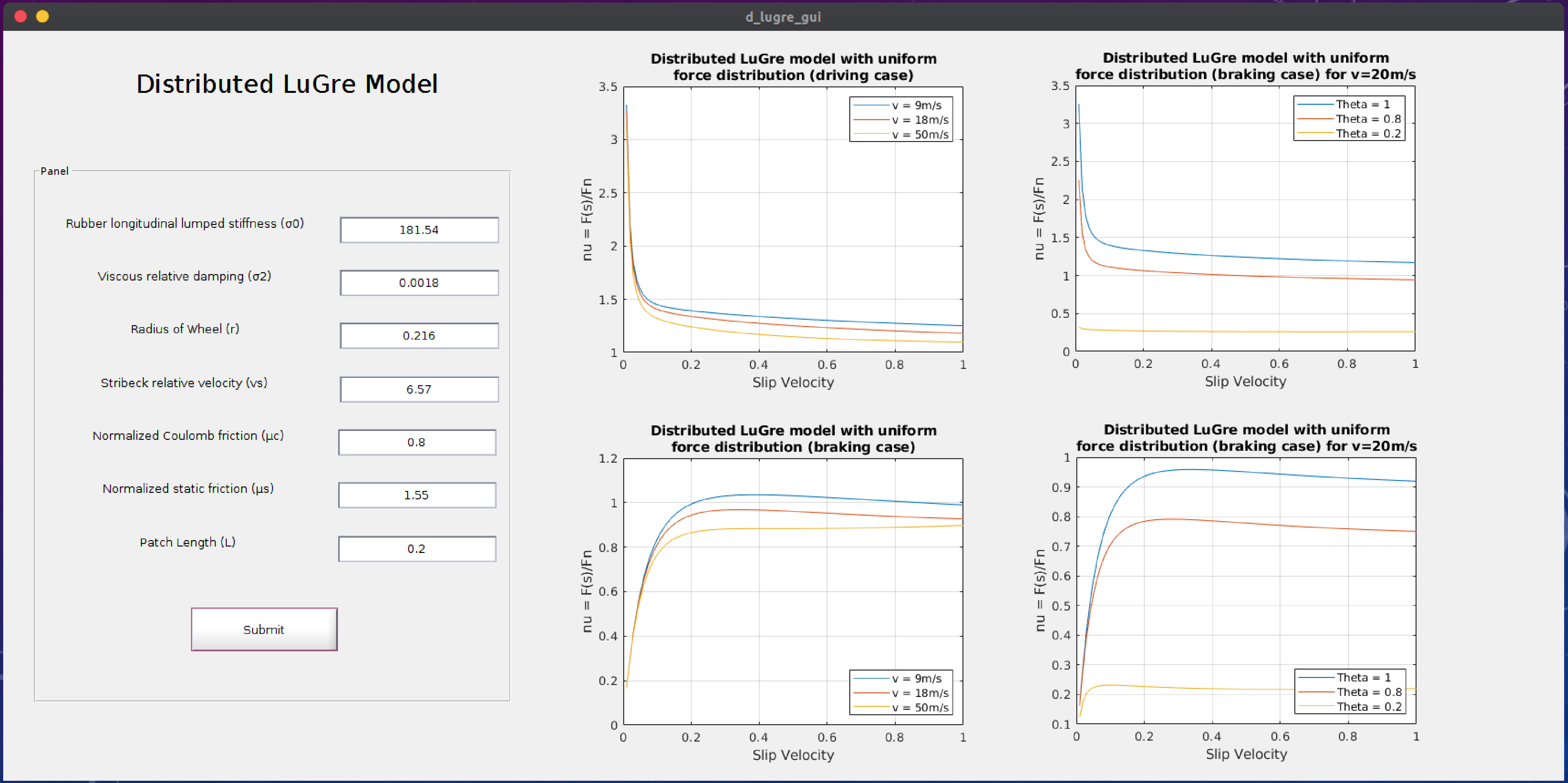
**Braking case :**

In this case $v > r\omega$, therefore the force at steady state is given by

$$F_b(s) = sgn(v_r)F_n g(s)\left[1 + \frac{g(s)|1+s|}{\sigma_0 L|s|}\left(e^{-\sigma_0 L|s| / g(s)|1+s|} - 1\right)\right] + F_n \sigma_2 v s$$

$$g(s) = \mu_c + (\mu_s - \mu_c) * \exp\left(-\left|{}^{vs}/_{v_s}\right|^{\alpha}\right)$$

# DISTRIBUTED LUGRE MODEL RESULTS



# CAD MODEL OF THE VEHICLE

# THANK YOU