

Dynamics of high-speed Railways

ME5670: Vehicle Dynamics Presentation
Group B

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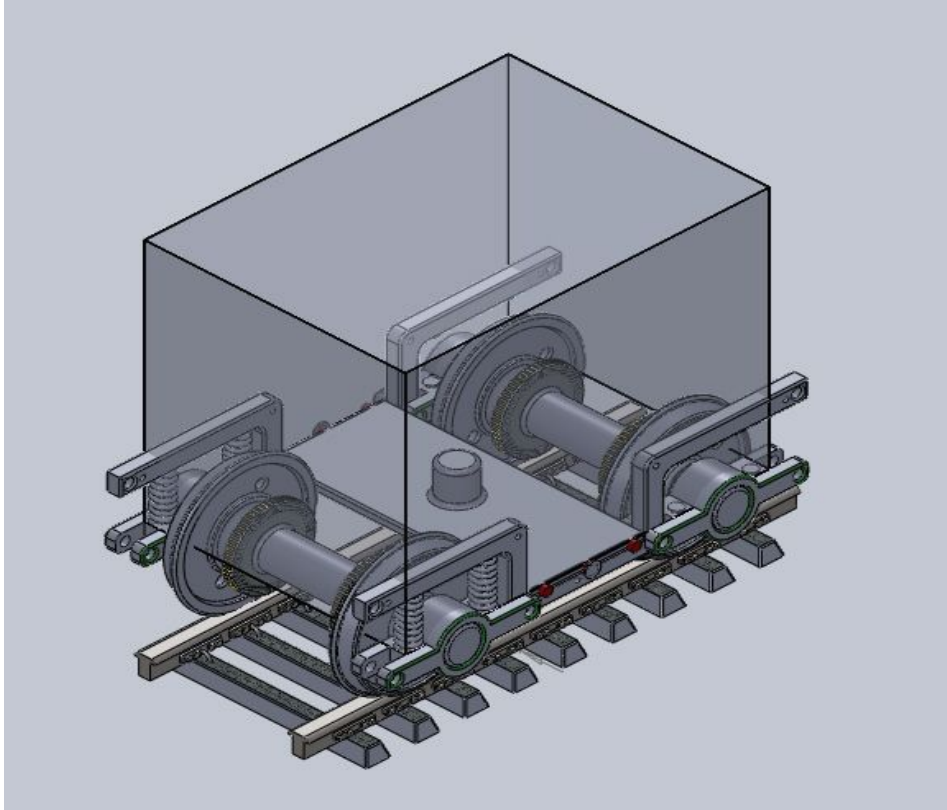
Introduction

- With immense development in the field of science and technology, high-speed railways have become a go-to in every country.
- More and more people consider high-speed trains as a comfortable, safe, low emission, and clean energy consumption transportation tool with a high on-schedule rate.
- Increasing the speed to meet the needs requires high service performance, running safety and vibration control in environments which are all closely related to the dynamic performance of the train/track coupling system.

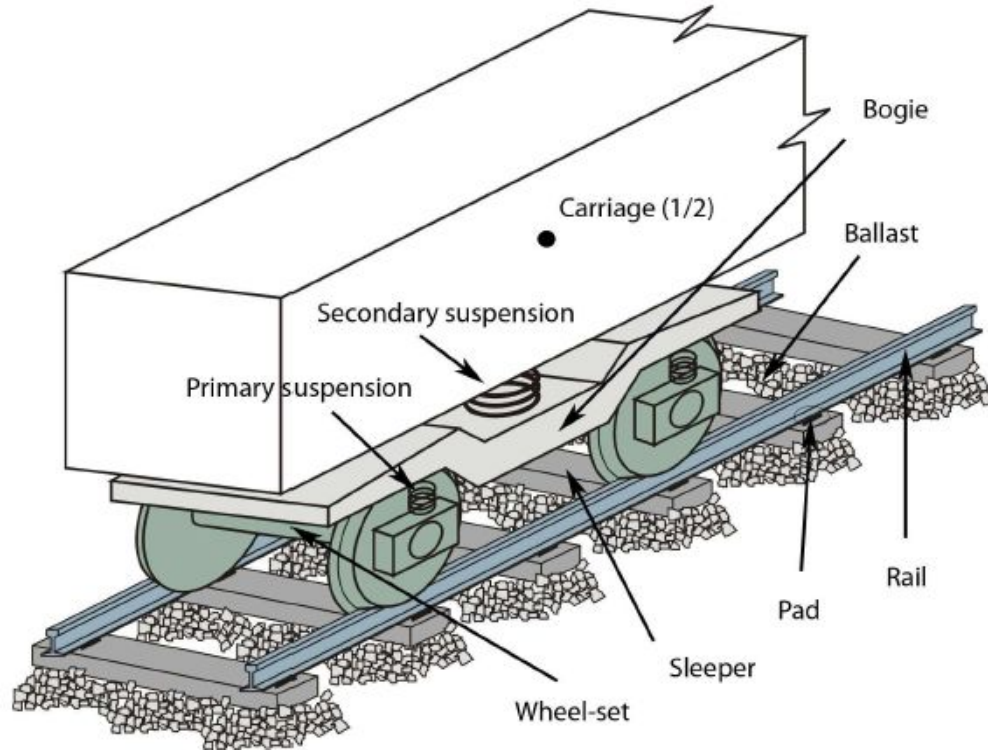
Introduction

- There are mainly two types of simulations: single vehicle/track dynamics and models for multi vehicle (train)/track coupled systems.
- Track flexibility has a significant impact on the wheel/ rail interaction and vehicle-track dynamics.
- We need to characterize the dynamical behavior of track components or the ground vibration induced by high-speed trains in operation.
- To understand the effects of inter-vehicle dynamic behaviour, there is a necessity to develop a 3D model of high-speed train coupled with a flexible track.

The Overall Model (CAD - Solidworks)



Understanding the model

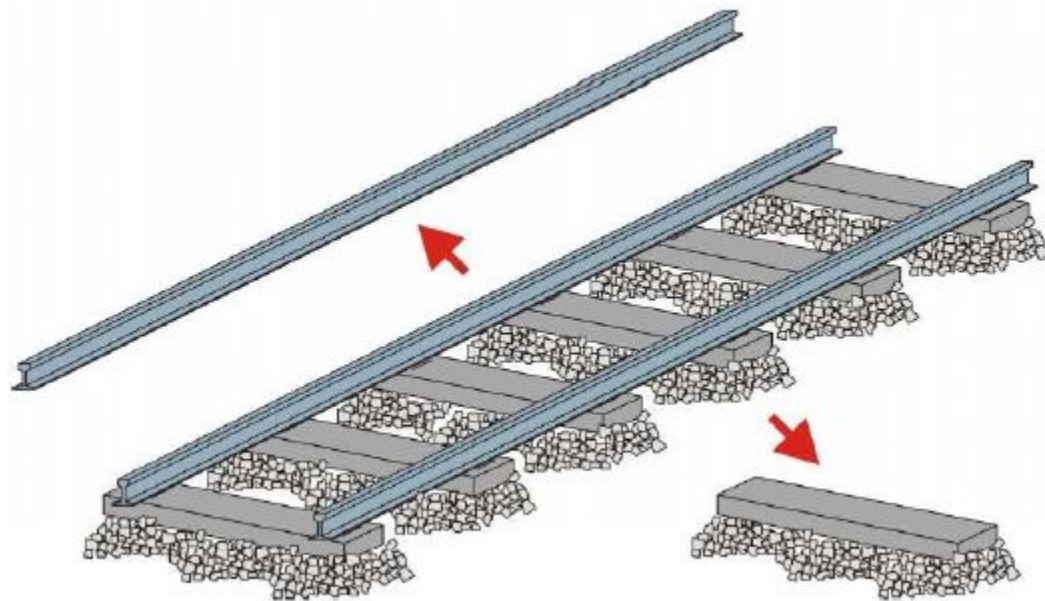


Understanding the model

- A 3D dynamic model of a high speed train coupled with a ballast track is developed, which extends the single-vehicle/track vertical-lateral coupling model to a multi-vehicle/track vertical lateral- longitudinal coupling model.
- To simulate the interaction between adjacent vehicles, a detailed connection model is developed, which includes non-linear couplers, non-linear inter-vehicle dampers and linear tight-lock diaphragm.

Track components

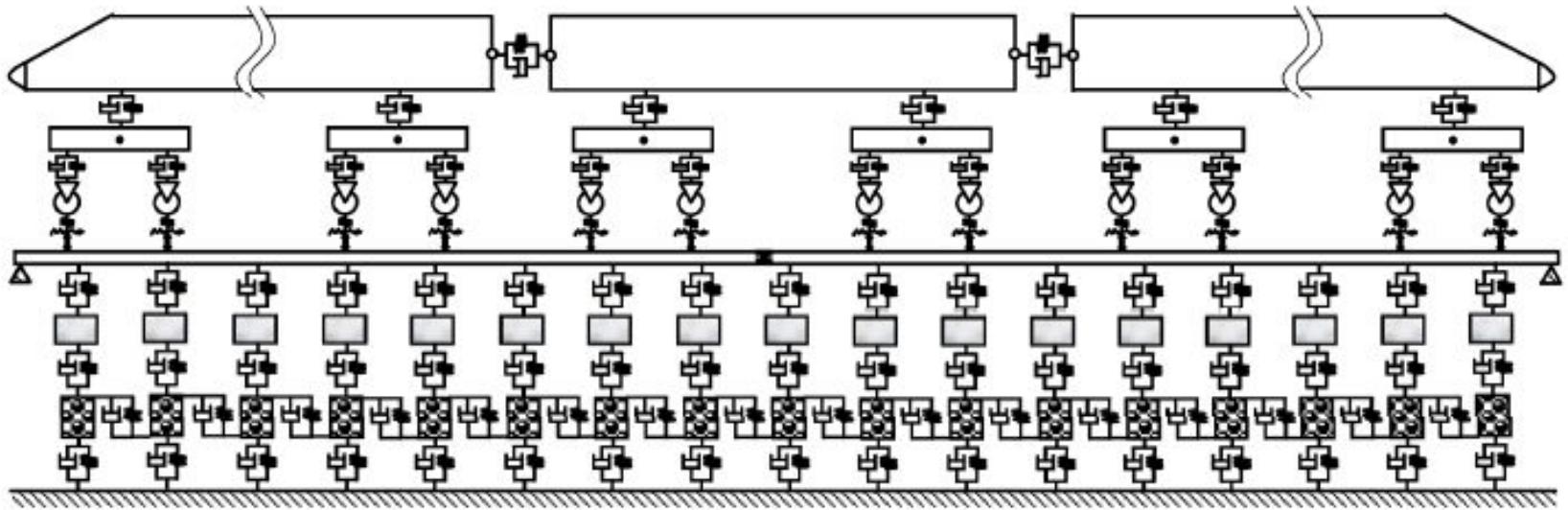
- The track is a flexible 3 layer model consisting of rails, sleepers and ballast.



The Mathematical Model

Vehicle Subsystem

- A dynamic coupled model that involves the non linear multi-body system is displayed below: (Two bogies for one car body)



Vehicle Subsystem

- The coordinate system is Cartesian x-y-z, with x moving in the direction of the train, z is the vertical direction and y is the lateral direction of the track.
- Each component of the vehicle has 6 DOFs: longitudinal displacement X , lateral displacement Y , vertical displacement Z , pitch angle β , the roll angle ϕ , the yaw angle Ψ .
- C and K represent the coefficients of equivalent dampers and spring stiffnesses respectively.

Car body equations

The equations of motion of the car body in longitudinal, lateral, vertical, rolling, pitching, and yawing directions are:

$$M_c \ddot{X}_c = -F_{xs1} - F_{xs2} - F_{xcf} - F_{xcb},$$

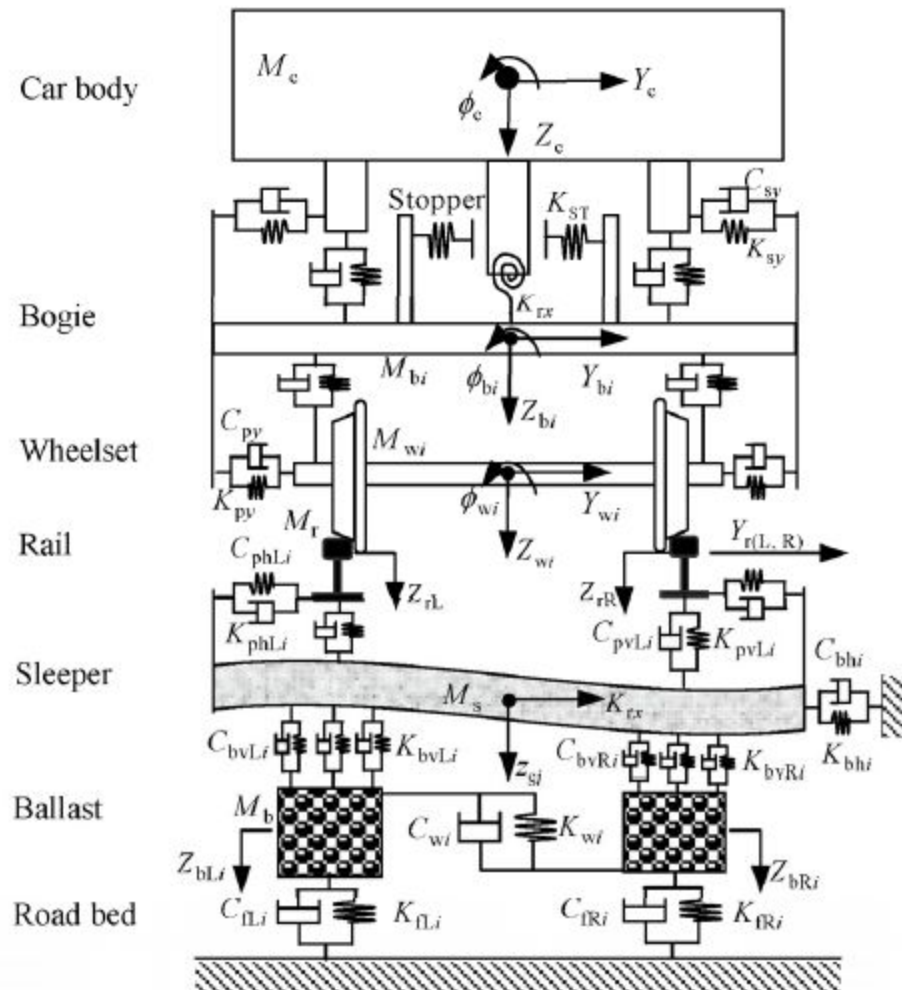
$$M_c \ddot{Y}_c = F_{ys1} + F_{ys2} - F_{ycf} - F_{ycb} + M_c g \phi_{sec} + F_{ycc},$$

$$M_c \ddot{Z}_c = -F_{zs1} - F_{zs2} - F_{zcf} - F_{zcb} + M_c g + F_{zcc},$$

$$I_{\text{cx}} \ddot{\phi}_{\text{c}} = -M_{\text{xs1}} - M_{\text{xs2}} + M_{\text{xcf}} + M_{\text{xcb}} + M_{\text{xcc}},$$

$$I_{cy}\ddot{\beta}_c = -M_{ys1} - M_{ys2} + M_{ycf} + M_{ycb},$$

$$I_{cz}\ddot{\psi}_c = -M_{zs1} - M_{zs2} + M_{zcf} + M_{zcb} + M_{zcc},$$



Car body parameters

- M_c is the mass of the car body.
- I_{cx} , I_{cy} , I_{cz} are the rolling, pitching, and yawing moments of inertia, respectively.
- F_{xsi} , F_{ysi} , F_{zsi} , M_{xsi} , M_{ysi} , and M_{zsi} ($i=1, 2$) denote the mutual forces and moments between car body and bogie frames in the x, y, and z directions.
- Subscripts 1 and 2 indicate the front and rear bogies.

Car body parameters

- F_{xci} , F_{yci} , F_{zci} , M_{xci} , M_{yci} , and M_{zci} ($i=f$ or b) denote the inter-vehicle forces and moments caused by inter-vehicle connections between the adjacent car bodies in the x , y , and z directions.
- Subscripts f and b indicate the front and end of each car body.
- F_{ycc} , F_{zcc} , M_{xcc} , and M_{zcc} denote the external forces on the car bodies resulting from the centripetal acceleration when a train is negotiating a curved track.

Bogie equations

The equations of motion of the bogie i ($i=1, 2$), in the longitudinal, lateral, vertical, rolling, pitching, and yawing directions are

$$M_b \ddot{X}_{bi} = F_{xsi} - F_{xf(2i-1)} - F_{xf(2i)},$$

$$M_b \ddot{Y}_{bi} = F_{yf(2i-1)} + F_{yf(2i)} - F_{ysi} + M_b g \phi_{sebi} + F_{ycbi},$$

$$M_b \ddot{Z}_{bi} = F_{zsi} - F_{zf(2i-1)} - F_{zf(2i)} + M_b g + F_{zcbi},$$

$$I_{bx} \ddot{\phi}_{bi} = -M_{xf(2i-1)} - M_{xf(2i)} + M_{xsi} + M_{xcbi},$$

$$I_{by} \ddot{\beta}_{bi} = -M_{yf(2i-1)} - M_{yf(2i)} + M_{ysi},$$

$$I_{bz} \ddot{\psi}_{bi} = -M_{zf(2i-1)} - M_{zf(2i)} + M_{zsi} + M_{zcbi},$$

Bogie EoM contd.

- M_b is the mass of the bogie
- I_{bx} , I_{by} , and I_{bz} are the moments of inertia of the bogie in rolling, pitching and yawing motions
- X_b , Y_b , Z_b are the accelerations of the bogie center in the longitudinal, lateral, vertical, rolling, pitching, and yawing directions, respectively
- F_{xfi} , F_{yfi} , F_{zfi} , M_{xfi} , M_{yfi} , and M_{zfi} ($i=1, 2, 3, 4$) denote the mutual forces and moments between bogie frames and wheelsets in the x, y, and z directions
- Subscripts 1, 2, 3, 4 indicate the four wheelsets of the vehicle, respectively; and F_{ycbi} , F_{zcbi} , M_{xcbi} , and M_{zcbi} ($i=1, 2$) denote the external forces on bogies resulting from the centripetal acceleration when the vehicle is negotiating curved track.

Wheel set equations

The equations of motion of the wheelset i ($i=1, 2, 3, 4$) in the longitudinal, lateral, vertical, rolling, pitching, and yawing directions are

$$M_w \ddot{X}_{wi} = F_{xfi} + F_{wrx i},$$

$$M_w \ddot{Y}_{wi} = -F_{yfi} + F_{wry i} + M_w g \phi_{sewi} + F_{ycwi},$$

$$M_w \ddot{Z}_{wi} = F_{zfi} - F_{wrz i} + M_w g + F_{zcow i},$$

$$I_{wx} \ddot{\phi}_{wi} = M_{xfi} - M_{wrz i} + M_{xcwi},$$

$$I_{wy} \ddot{\beta}_{wi} = M_{wry i} + M_{TBi},$$

$$I_{wz} \ddot{\psi}_{wi} = M_{zfi} + M_{wrz i} + M_{zcow i},$$

Wheel set equations' parameters

- M_w is the mass of the wheelset; I_{wx} , I_{wy} , and I_{wz} are the moments of inertia of the wheelset in rolling, pitching, and yawing motions, respectively.
- F_{wrx_i} , F_{wry_i} , F_{wrz_i} , M_{wrx_i} , M_{wry_i} , and M_{wrz_i} ($i=1, 2, 3, 4$) denote the contact forces and moments between the wheels and the rails in the x , y , and z directions, respectively.
- F_{ycw_i} , $F_{z cw_i}$, M_{xcw_i} , and M_{zcw_i} ($i=1, 2, 3, 4$) denote the external forces on the wheelsets resulting from the centripetal acceleration when the train is negotiating curved track.
- M_{TB_i} is the traction or braking moment acting on the wheelsets when the train is accelerating or decelerating.

Car body - Bogie interaction

According to the bilinear postulation, the forces between the bogies and the car body or the wheelsets are:

$$F_{xYD} = \begin{cases} C_{YD1} \Delta \dot{x}_{YD}, & |\Delta \dot{x}_{YD}| < V_{0YD}, \\ \text{sign}(\Delta \dot{x}_{YD}) [C_{YD1} V_{0YD} + C_{YD2} (|\Delta \dot{x}_{YD}| - V_{0YD})], & |\Delta \dot{x}_{YD}| \geq V_{0YD}, \end{cases}$$

$$F_{yST} = \begin{cases} 0, & |\Delta y_{ST}| < \delta, \\ K_{ST} (|\Delta y_{ST}| - \delta), & |\Delta y_{ST}| \geq \delta, \end{cases}$$

$$F_{yLD} = \begin{cases} C_{LD1} \Delta \dot{y}_{LD}, & |\Delta \dot{y}_{LD}| < V_{0LD}, \\ \text{sign}(\Delta \dot{y}_{LD}) [C_{LD1} V_{0LD} + C_{LD2} (|\Delta \dot{y}_{LD}| - V_{0LD})], & |\Delta \dot{y}_{LD}| \geq V_{0LD}, \end{cases}$$

$$F_{zVD} = \begin{cases} C_{VD1} \Delta \dot{z}_{VD}, & |\Delta \dot{z}_{VD}| < V_{0VD}, \\ \text{sign}(\Delta \dot{z}_{VD}) [C_{VD1} V_{0VD} + C_{VD2} (|\Delta \dot{z}_{VD}| - V_{0VD})], & |\Delta \dot{z}_{VD}| \geq V_{0VD}, \end{cases}$$

Inter-vehicle subsystem

According to the bilinear assumption, the coupler forces are

$$F_{cg} = \begin{cases} 0, & |\Delta x| < \Delta x_0, \\ K_{CB1}(\Delta x - \Delta x_0), & \Delta x_0 \leq |\Delta x| \leq X_{0CB}, \\ K_{CB1}(X_{0CB} - \Delta x_0) + K_{CB2}(\Delta x - X_{0CB}), & |\Delta x| > X_{0CB}, \end{cases}$$

where Δx is the relative displacement between the two ends of the couplers connecting the adjacent vehicles in the axial direction, Δx_0 is the slackless of the coupler, X_{0CB} is the initial length of the coupler, and K_{CB} is its equivalent stiffness coefficient.

Inter-vehicle dampers

- The forces on the inter-vehicle dampers are

$$F_{\text{CDL,R}} = \begin{cases} C_{\text{CD1}} \Delta V_{\text{CDL,R}}, & |\Delta V_{\text{CDL,R}}| < V_{0\text{CD}}, |\Delta X_{\text{CDL,R}}| \leq X_{0\text{CD}}, \\ \text{sign}(\Delta V_{\text{CDL,R}}) [C_{\text{CD1}} V_{0\text{CD}} + C_{\text{CD2}} (|\Delta V_{\text{CDL,R}}| - V_{0\text{CD}})], & |\Delta V_{\text{CDL,R}}| \geq V_{0\text{CD}}, |\Delta X_{\text{CDL,R}}| \leq X_{0\text{CD}}, \\ \text{sign}(\Delta X_{\text{CDL,R}}) K_{\text{CD}} (|\Delta X_{\text{CDL,R}}| - X_{0\text{CD}}), & |\Delta V_{\text{CDL,R}}| \geq V_{0\text{CD}}, |\Delta X_{\text{CDL,R}}| > X_{0\text{CD}}, \end{cases}$$

Track subsystem

- The bending deformations of the rails are described by the Timoshenko beam theory.
- Using the modal synthesis method and normalized shape functions of a Timoshenko beam, the fourth-order partial differential equations of the rails are converted into second-order ordinary differential equations.

Lateral Bending Motion

$$\left\{ \begin{aligned} & \ddot{q}_{\gamma k}(t) + \frac{\kappa_{\gamma} G_{\Gamma} A_{\Gamma}}{\rho_{\Gamma} A_{\Gamma}} \left(\frac{k\pi}{l_{\Gamma}} \right)^2 q_{\gamma k}(t) - \kappa_{\gamma} G_{\Gamma} A_{\Gamma} \frac{k\pi}{l_{\Gamma}} \sqrt{\frac{1}{m_{\Gamma} \rho_{\Gamma} I_{\Gamma z}}} w_{\gamma k}(t) \\ & = - \sum_{i=1}^{N_s} R_{\gamma i}(t) Y_{\Gamma k}(x_{si}) + \sum_{j=1}^{N_w} F_{w\gamma j}(t) Y_{\Gamma k}(x_{wj}), \\ & \ddot{w}_{\gamma k}(t) + \left[\frac{\kappa_{\gamma} G_{\Gamma} A_{\Gamma}}{\rho_{\Gamma} I_{\Gamma z}} + \frac{E_{\Gamma} I_{\Gamma z}}{\rho_{\Gamma} I_{\Gamma z}} \left(\frac{k\pi}{l_{\Gamma}} \right)^2 \right] w_{\gamma k}(t) \\ & - \kappa_{\gamma} G_{\Gamma} A_{\Gamma} \frac{k\pi}{l_{\Gamma}} \sqrt{\frac{1}{m_{\Gamma} \rho_{\Gamma} I_{\Gamma z}}} q_{\gamma k}(t) = 0, \quad k = 1, 2, \dots, NMY; \end{aligned} \right.$$

Vertical Bending Motion

$$\left\{ \begin{aligned} & \ddot{q}_{\text{rzk}}(t) + \frac{\kappa_{\text{rz}} G_{\text{r}} A_{\text{r}}}{\rho_{\text{r}} A_{\text{r}}} \left(\frac{k\pi}{l_{\text{r}}} \right)^2 q_{\text{rzk}}(t) - \kappa_{\text{rz}} G_{\text{r}} A_{\text{r}} \frac{k\pi}{l_{\text{r}}} \sqrt{\frac{1}{m_{\text{r}} \rho_{\text{r}} I_{\text{ry}}}} w_{\text{rzk}}(t) \\ &= - \sum_{i=1}^{N_{\text{s}}} R_{\text{zi}}(t) Z_{\text{rk}}(x_{\text{si}}) + \sum_{j=1}^{N_{\text{w}}} F_{\text{wrzj}}(t) Z_{\text{rk}}(x_{\text{wj}}), \\ & \ddot{w}_{\text{rzk}}(t) + \left[\frac{\kappa_{\text{rz}} G_{\text{r}} A_{\text{r}}}{\rho_{\text{r}} I_{\text{ry}}} + \frac{E_{\text{r}} I_{\text{ry}}}{\rho_{\text{r}} I_{\text{ry}}} \left(\frac{k\pi}{l_{\text{r}}} \right)^2 \right] w_{\text{rzk}}(t) \\ & - \kappa_{\text{rz}} G_{\text{r}} A_{\text{r}} \frac{k\pi}{l_{\text{r}}} \sqrt{\frac{1}{m_{\text{r}} \rho_{\text{r}} I_{\text{ry}}}} q_{\text{rzk}}(t) = 0, \quad k = 1, 2, \dots, \text{NMZ}; \end{aligned} \right.$$

Torsional Motion

$$\ddot{q}_{rTk}(t) + \frac{G_r K_r}{\rho_r I_{r0}} \left(\frac{k\pi}{l_r} \right)^2 q_{rTk}(t) = - \sum_{i=1}^{N_s} M_{si}(t) \Phi_{rk}(x_{si}) + \sum_{j=1}^{N_w} M_{Gj}(t) \Phi_{rk}(x_{wj}), \quad k = 1, 2, \dots, NMT.$$

The material properties of the rail are indicated by the density ρ , the shear modulus G_r , and Young's modulus E_r . m_r is the mass per unit longitudinal length. The geometry of the cross section of the rail is represented by the area A_r , the second moments of area I_{ry} and I_{rz} around the y-axis and the z-axis, respectively, and the polar moment of inertia I_{r0} .

Ballast Body equations

- The ballast bed is replaced by equivalent rigid ballast blocks in this calculation model, while only the vertical motion of each ballast body is taken into account.
- The vertical equations of motion of the ballast body i are

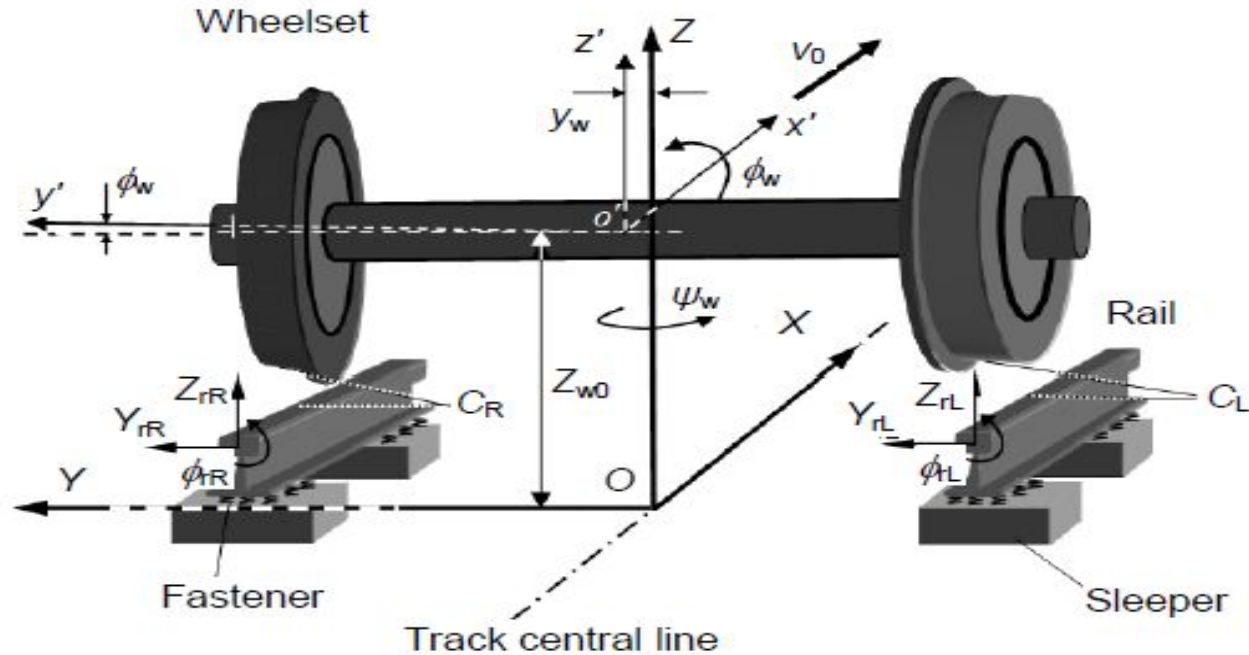
$$M_{bs} \ddot{Z}_{bLi} = F_{bzLi} + F_{zrLi} + F_{zLRi} - F_{zgLi} - F_{zfLi},$$

$$M_{bs} \ddot{Z}_{bRi} = F_{bzRi} + F_{zrRi} - F_{zLRi} - F_{zgRi} - F_{zfRi},$$

Ballast Body parameters

- F_{zfLi} , F_{zrLi} , F_{zfRi} , F_{zrRi} , and F_{zLRi} are the vertical shear forces between neighboring ballast bodies.
- F_{zgLi} and F_{zgRi} are the vertical forces between ballast bodies and the roadbed, and M_{bs} is the mass of each ballast body.
- Such a ballast model can represent the in-phase and out-of-phase motions of two vertical rigid modes in the vertical-lateral plane of the track.

Wheel/rail contact geometry calculation model



Wheel/rail contact subsystem

The calculation model of the wheel/rail normal force, which characterizes the relationship law of the normal load and deformation between the wheel and rail, is described by a Hertzian nonlinear contact spring with a unilateral restraint.

$$F_n(t) = \begin{cases} \left[\frac{1}{G} Z_{\text{wmc}}(t) \right]^{3/2}, & Z_{\text{wmc}}(t) > 0, \\ 0, & Z_{\text{wmc}}(t) \leq 0, \end{cases}$$

G is the contact constant. Z is the normal compressing amount.

Initial and Boundary conditions

- Both ends of the Timoshenko beam modeling the rails are hinged, and the deflections and the bending moments at the hinged beam ends are assumed to be zero.
- The static state of the systems is regarded as the original point of reference.
- The initial displacements and velocities of all components of the track are set to zero.
- The initial displacements and the initial vertical and lateral velocities of all components of the high-speed train are also set to zero, and the initial longitudinal velocity is the running speed of the train, which is a constant.

Simulation Results



Contact force at Rear Right Wheel

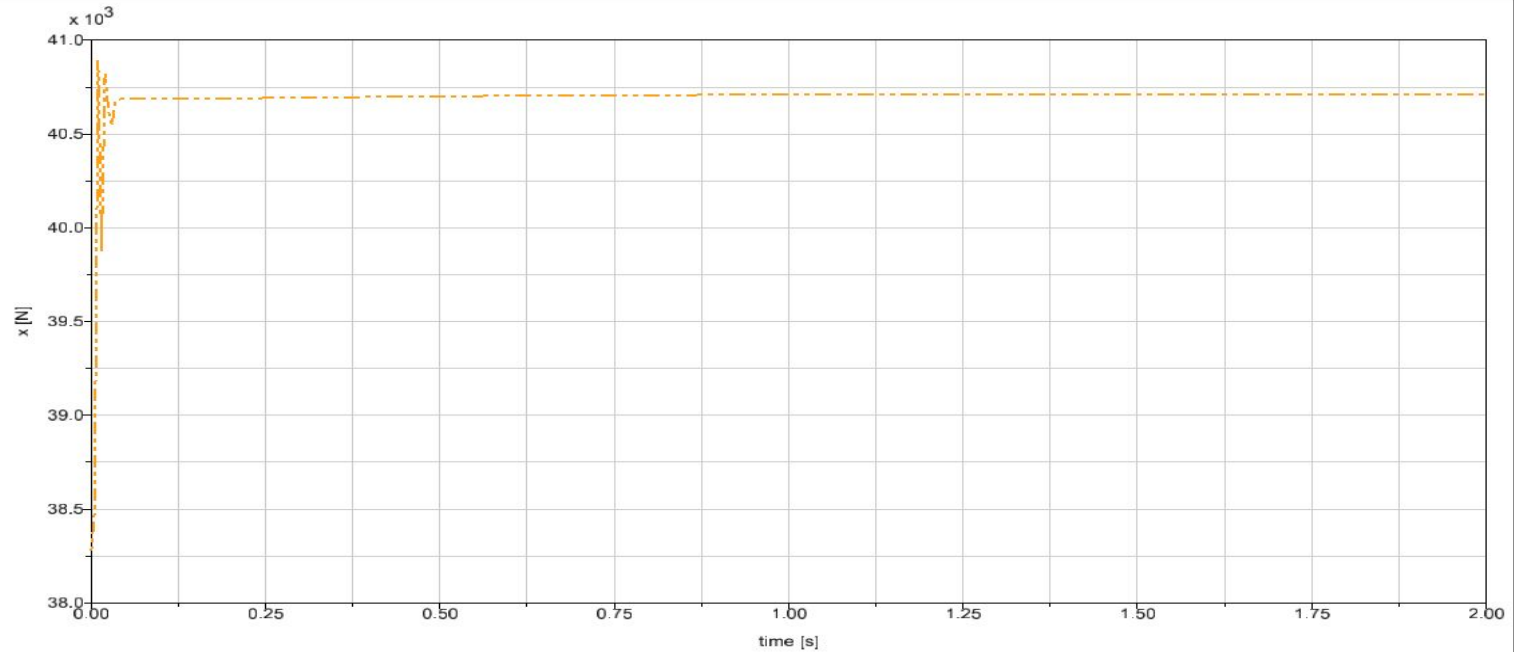


Contact force at Rear Left Wheel

Page 1

Diagram

force force.\$F_RWContact_RWP_Left.z

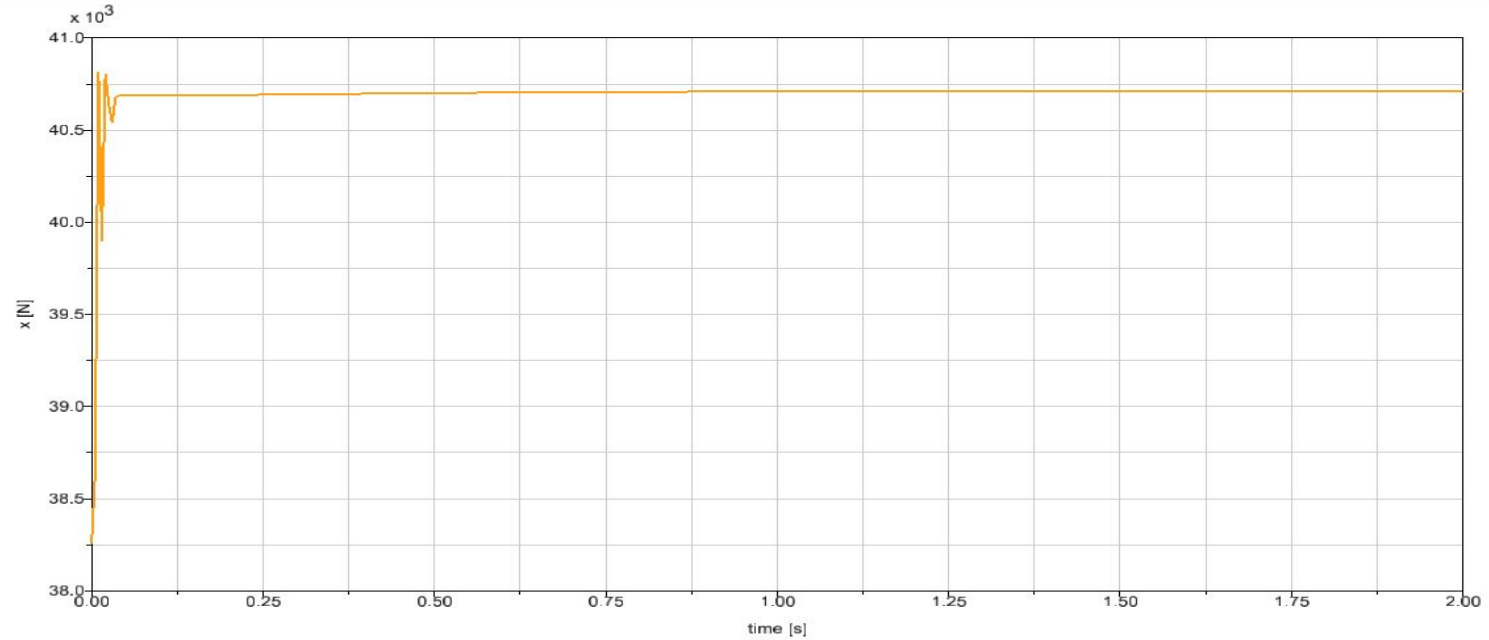


Contact force at Front Right Wheel

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Diagram

force force.\$F_RWContact_RWP_Wheelact_2_Right.z

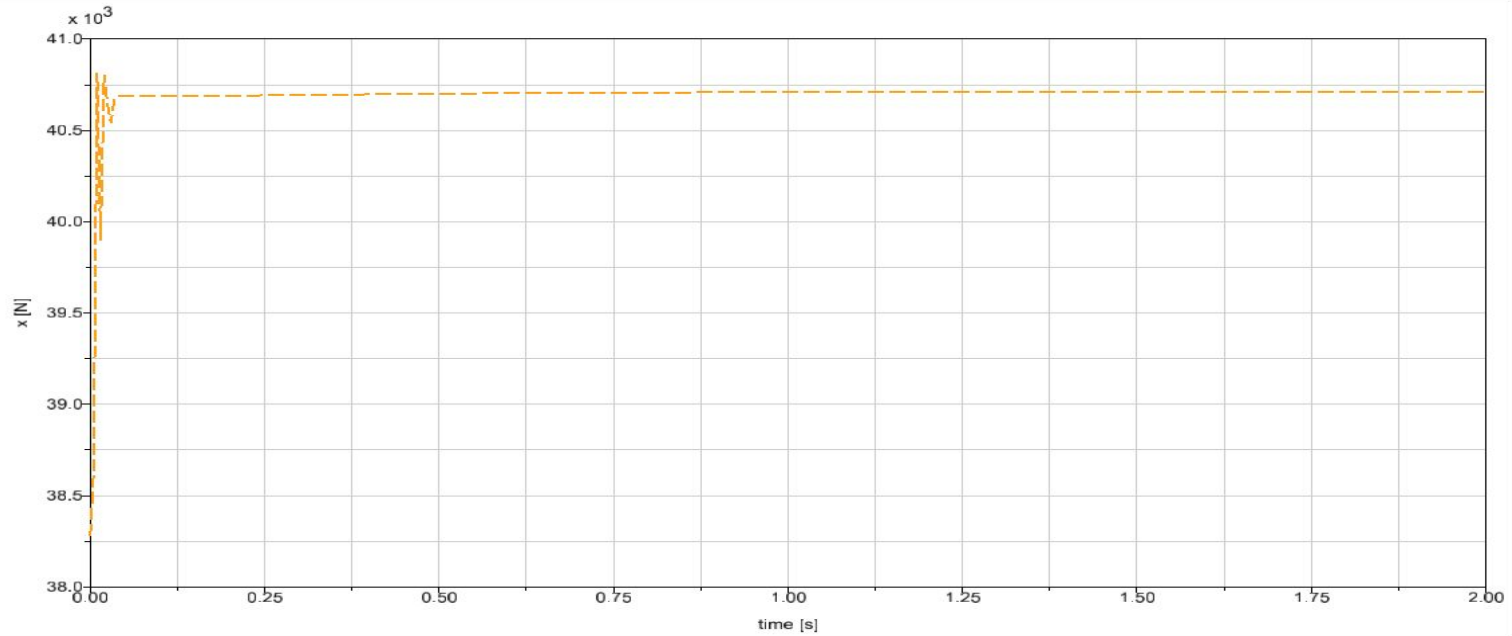


Contact force at Front Left Wheel

Page 1

Diagram


force force.\$F_RWContact_RWP_Wheelset_2_Left.z



References

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- A numerical program for railway vehicle-track-structure dynamic interaction using a modal substructuring approach, Gabriel Savini, 2009-2010

Work Division

- Aparna: Designed the model and created the structure - SolidWorks
- Faizaan: Derived the equations of motion and prepared the mathematical model
- Dheeraj: Performed the required simulations and obtained the contact forces required to make the model work 
- Venkat, Pavan: Prepared the slides and report of the project