भारतीय प्रौद्योगिकी संस्थान हैदराबाद
Indian Institute of Technology Hyderabad

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VEHICLE DYNAMICS(ME5670)

LATERAL DYNAMICS OF MULTI-ARTICULATED VEHICLES WITH MULTIPLE AXLES

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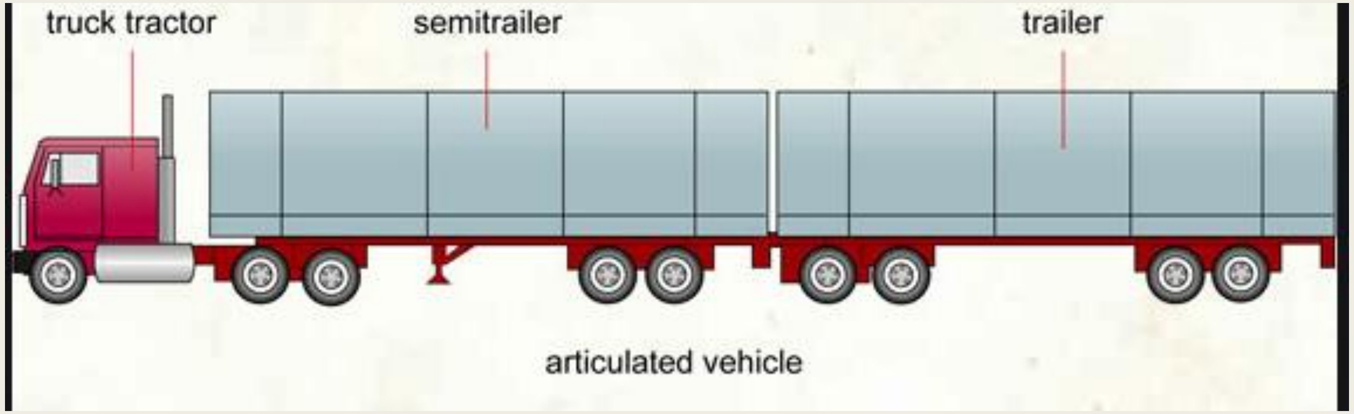
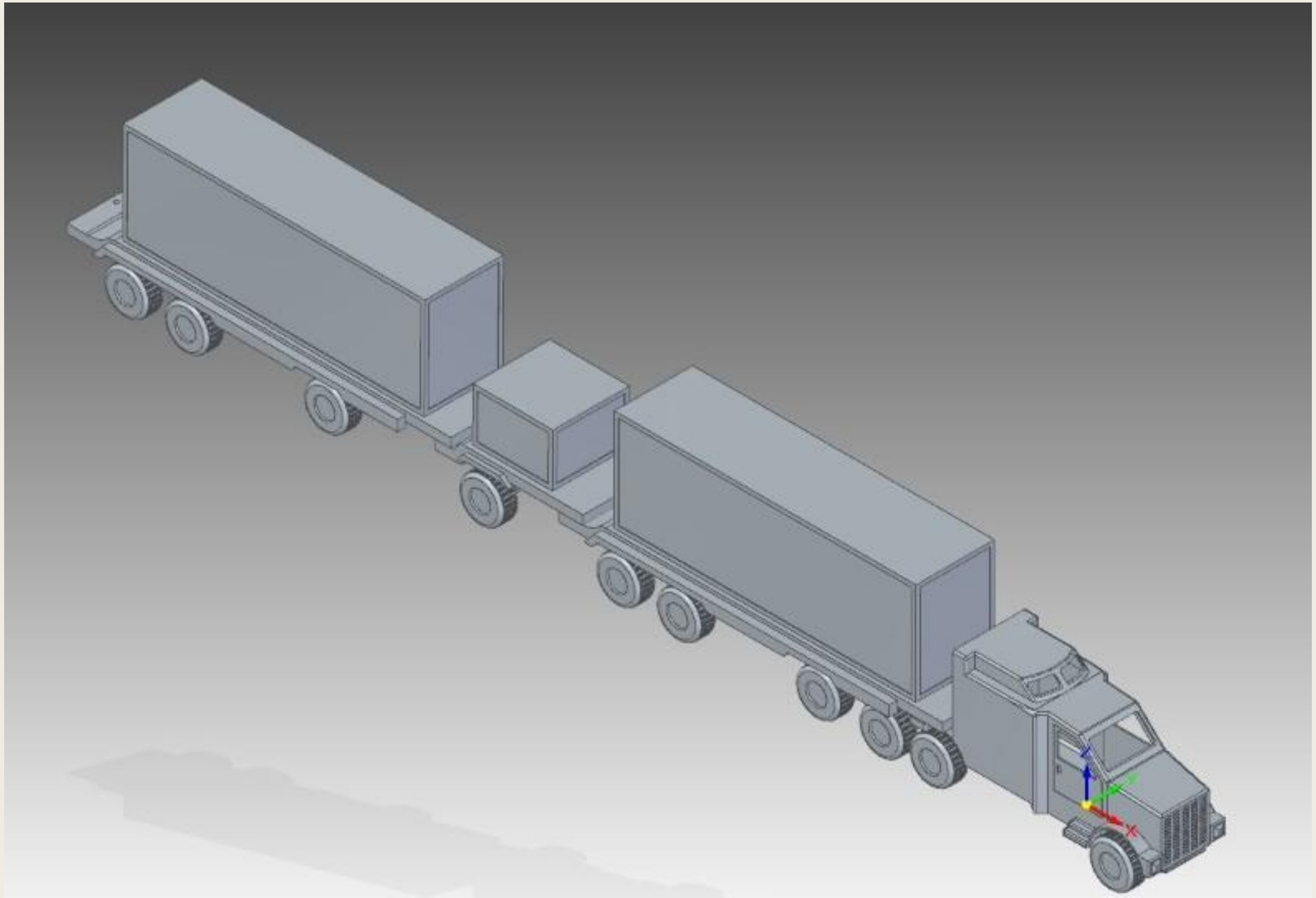
Articulated vehicles

- An **articulated vehicle** is a vehicle which has a permanent or semi-permanent pivot joint in its construction, allowing the vehicle to turn more sharply.
- There are many kinds of articulated vehicles, from heavy equipment to buses, trams and trains.
- Steam locomotives were sometimes articulated in that the driving wheels could pivot around.

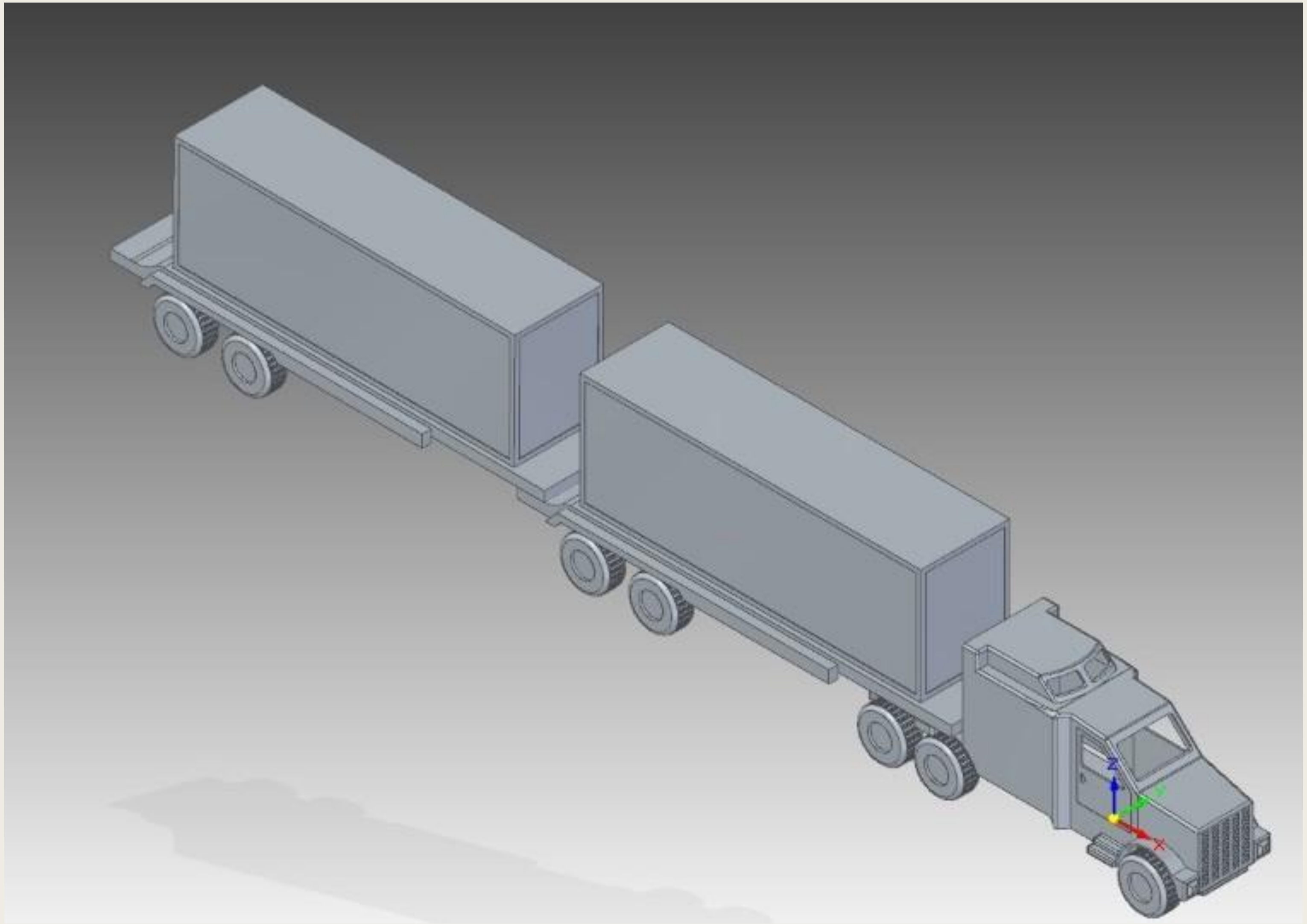


A-double Truck



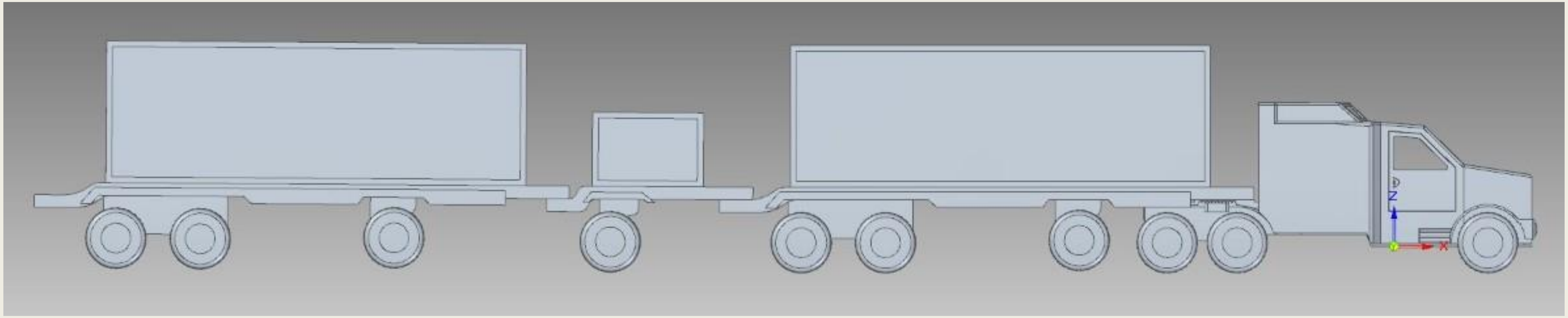
The CAD Models are designed in Solid Edge ST8

B-double Truck

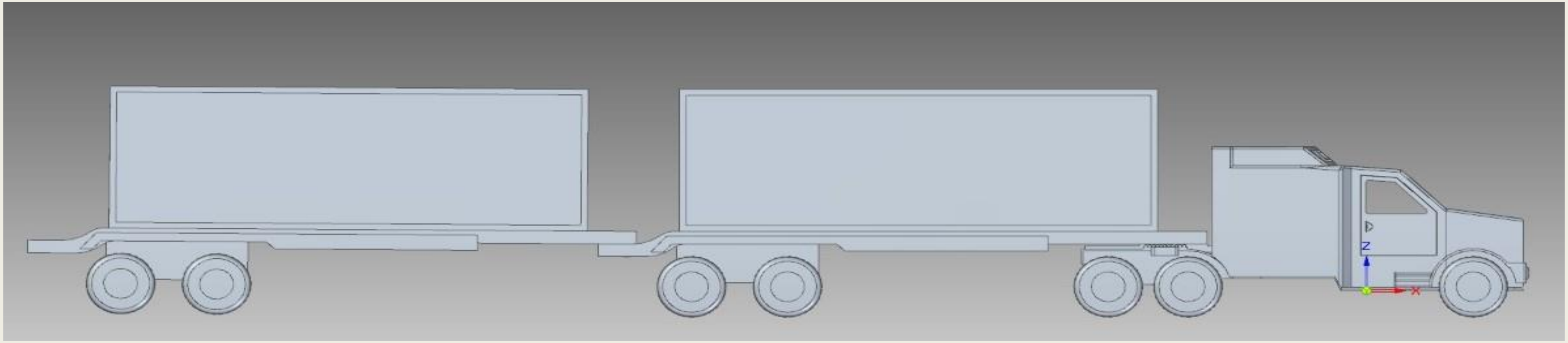


The CAD Models are designed in Solid Edge ST8

A-double Truck



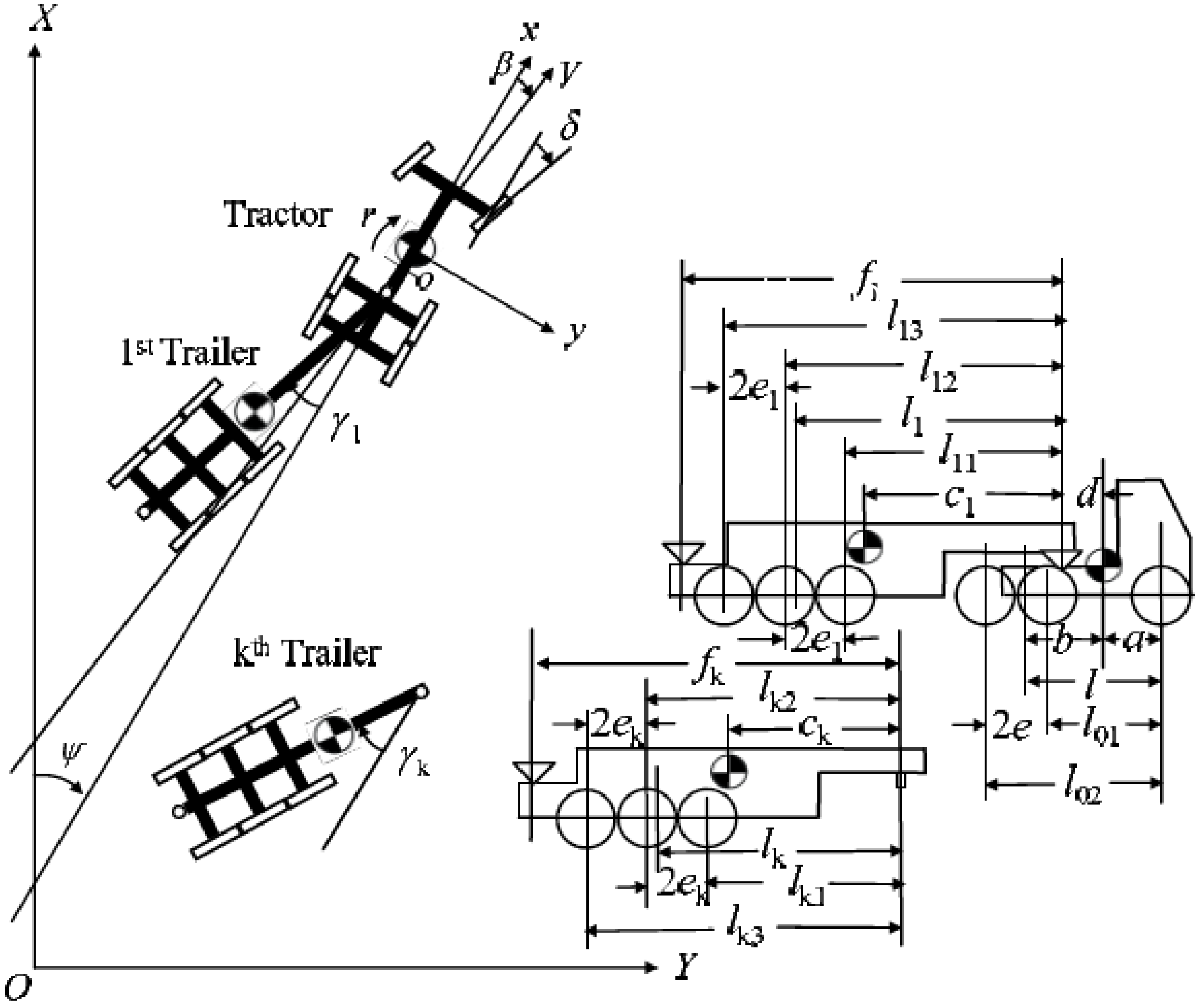
B-double Truck



Model description

- A simplified mathematical planar model is adopted for the fixed right-hand rectangular coordinate system in a moving tractor.
- Roll dynamics are neglected.
- The degrees of freedom are the side-slipping and yawing of the tractor, and the yawing of trailers relative to the tractor.

Co-ordinate Reference Model



- Multiple axles in each vehicle are transformed into a hypothetical axle in the equations of motion.
- For an equal vertical load of each axle of multiple axles, the center of each hypothetical axle is located at the kinematic center of multiple axles by the equilibrium of pitching moments.
- The vertical load and cornering stiffness in each axle are expressed as those in each hypothetical axle divided by the number of axles;

Equation of motion

$$
\begin{aligned}
&\left(M+\sum_{i=1}^{n} M_i\right) V\dot{\beta}+\left(C_f+C_r+\sum_{i=1}^{n} C_i\right)\beta-\sum_{j=1}^{n} M_j\left(d+\sum_{i=1}^{j-1} f_i+c_j\right)\dot{r}\\
&\quad+\left[aC_f-bC_r-\sum_{j=1}^{n}\left(d+\sum_{i=1}^{j-1} f_i+l_j\right)C_j+\left(M+\sum_{i=1}^{n} M_i\right)V^2\right] r/V\\
&\quad-\sum_{j=1}^{n}\left(M_j c_j+\sum_{i=j+1}^{n} M_i f_j\right)\ddot{\gamma}_j-\sum_{j=1}^{n}\left(l_j C_j+f_j\sum_{i=j+1}^{n} C_i\right)\dot{\gamma}_j/V-\sum_{i=1}^{n} C_i\gamma_i=C_f\delta, \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
&MdV\dot{\beta}+[(a+d)C_f+(d-b)C_r]\beta+I\dot{r}\\
&\quad+[a(a+d)C_f-b(d-b)C_r+AC_r+MdV^2]r/V=(a+d)C_f\delta, \qquad (2)
\end{aligned}
$$

$$
\begin{aligned}
&-\left(M_k c_k+\sum_{i=k+1}^{n} M_i f_k\right)V\dot{\beta}-\left(l_k C_k+f_k\sum_{i=k+1}^{n} C_i\right)\beta\\
&\quad+\left[I_k+M_k c_k\left(d+\sum_{i=1}^{k-1} f_i+c_k\right)+\sum_{j=k+1}^{n} M_j\left(d+\sum_{i=1}^{j-1} f_i+c_j\right)f_k\right]\dot{r}\\
&\quad+\left[l_k\left(d+\sum_{i=1}^{k-1} f_i+l_k\right)C_k+B_k C_k+f_k\sum_{j=k+1}^{n}\left(d+\sum_{i=1}^{j-1} f_i+l_j\right)C_j\right.\\
&\quad\left.-\left(M_k c_k+\sum_{i=k+1}^{n} M_i f_k\right)V^2\right]r/V\\
&\quad+\sum_{j=1}^{k-1}\left(M_k c_k+\sum_{i=k+1}^{n} M_i f_k\right)f_j\ddot{\gamma}_j+\left(I_k+M_k c_k^2+\sum_{i=k+1}^{n} M_i f_k^2\right)\ddot{\gamma}_k\\
&\quad+\sum_{j=k+1}^{n}\left(M_j c_j+\sum_{i=j+1}^{n} M_i f_j\right)f_k\ddot{\gamma}_j\\
&\quad+\sum_{j=1}^{k-1}\left(l_k C_k+f_k\sum_{i=k+1}^{n} C_i\right)f_j\dot{\gamma}_j/V+\left(l_k^2 C_k+B_k C_k+f_k^2\sum_{i=k+1}^{n} C_i\right)\dot{\gamma}_k/V\\
&\quad+f_k\sum_{j=k+1}^{n}\left(l_j C_j+f_j\sum_{i=j+1}^{n} C_i\right)\dot{\gamma}_j/V+l_k C_k\gamma_k+f_k\sum_{i=k+1}^{n} C_i\gamma_i=0, \qquad (3)
\end{aligned}
$$

where $k=1,2,3,\ldots,n$ and

Analysis of lateral dynamics

- Stability criterion:
- The characteristic equations are obtained by transforming Equations (1)–(3) using the Laplace operator.
- Routh Hurwitz method is used to determine the non-oscillatory stability of the system.
- The sign of the highest-order coefficients of A- and B-Doubles (n = 3 and 2) is positive and independent of forward velocity as are those of single-axle vehicle combinations.

The coefficient k_o of A – and B-Doubles is given as:

$$k_0 = \frac{l^2 C_f C_r \prod_{i=1}^{n} l_i C_i (F + KV^2)}{V}, \tag{11}$$

where

$$\begin{aligned}
F &= 1 + \left(\frac{A}{l^2}\right)\left(1 + \frac{C_r}{C_f}\right) + \sum_{i=1}^{n}\left(\frac{B_i}{l l_i}\right)\left[D_i\left(\frac{C_i}{C_f}\right) - E_i\left(\frac{C_i}{C_r}\right)\right],\\
D_i &= \left[\frac{b-d}{l}\right]\prod_{j=1}^{i-1}\left(1 - \frac{f_j}{l_j}\right),\\
E_i &= \left[\frac{a+d}{l}\right]\prod_{j=1}^{i-1}\left(1 - \frac{f_j}{l_j}\right),\\
K &= \frac{P_f/C_f - P_r/C_r}{gl},\\
P_f &= \left(\frac{b}{l}\right)Mg + \left[\frac{b-d}{l}\right]\sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\left(1 - \frac{f_j}{l_j}\right)\right]\left(1 - \frac{c_i}{l_i}\right)M_i g,\\
P_r &= \left(\frac{a}{l}\right)Mg + \left[\frac{a+d}{l}\right]\sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\left(1 - \frac{f_j}{l_j}\right)\right]\left(1 - \frac{c_i}{l_i}\right)M_i g.
\end{aligned} \tag{12}$$

Steering sensitivity in steady state running:

- The steering of the vehicle combinations are determined using

$$\delta = \frac{\left(F + KV^2\right)l}{R}$$

where R - steady-state turning radius of the center of gravity of a tractor$(r = V/R)$

K- stability factor

- Steering sensitivity

$$\left[\frac{\delta_0}{\delta}\right]_{steady-state} = \frac{1}{1 + K^* V^2}$$

Where,

$$K^* = \frac{K}{F}$$

- Substituting in the non oscillatory stability criteria

$$k_0 = \frac{l^2 C_f C_r \prod_{i=1}^{n} l_i\, C_i F (1 + K^* V^2)}{V}$$

$$= \frac{l^2 C_f C_r \prod_{i=1}^{n} l_i\, C_i F \Big/ V}{\left[\frac{\delta_0}{\delta}\right]_{steady-state}}$$

- k_0 is +ve for $K* \geq 0$ independent of forward velocity V.

- k_0 is +ve for $K* < 0$ when $V < V_{cr} = \sqrt{(-\frac{1}{K^*})}$

Steering sensitivity in lane changing:

- The front wheel angles of a tractor in lane changing can be treated as a one-cycle sinusoidal wave by observing running tests

$$\delta = \delta_1 \sin \omega_1 t \left(U(t) - U\left(t - \frac{2}{\omega_1}\right)\right)$$

$$\Delta s = \left[\frac{\delta_1 \omega_1}{s^2 + \omega_1^2}\right]\left(1 - e^{-2\pi s/\omega_1}\right)$$

- Lateral Displacement

$$\frac{Y(s)}{\Delta(s)} = \frac{\sum_{i=0}^{2n+2} \bar{k}_{iY} s^i}{s^2 \sum_{i=0}^{2n+2} \bar{k}_i s^i}$$

After inverse laplace transform,

$$Q = \lim_{t \to \infty} Y = \left(\frac{2\pi\delta_1}{\omega_1^2}\right)\left(\frac{\bar{k}_{0Y}}{\bar{k}_0}\right)$$

- The steering sensitivity in lane changing is given by

$$\frac{1}{\delta_1} = \left(\frac{2\pi}{\omega_1^2 Q}\right)\left(\frac{\bar{k}_{0Y}}{\bar{k}_0}\right)$$

- Substituting the values of $\bar{k}_0$ and $\bar{k}_{0Y}$ we get

$$\frac{1}{\delta_1} = \frac{\left({}^{2\pi}/{}_{\omega_1^2 Q}\right)V^2}{lF(1 + K^* V^2)}$$

$$\frac{1}{\delta_1} = \left(\frac{2\pi V^2}{\omega_1^2 Q l F}\right)\left[\frac{\delta_0}{\delta}\right]_{steady-state}$$

Simulation:

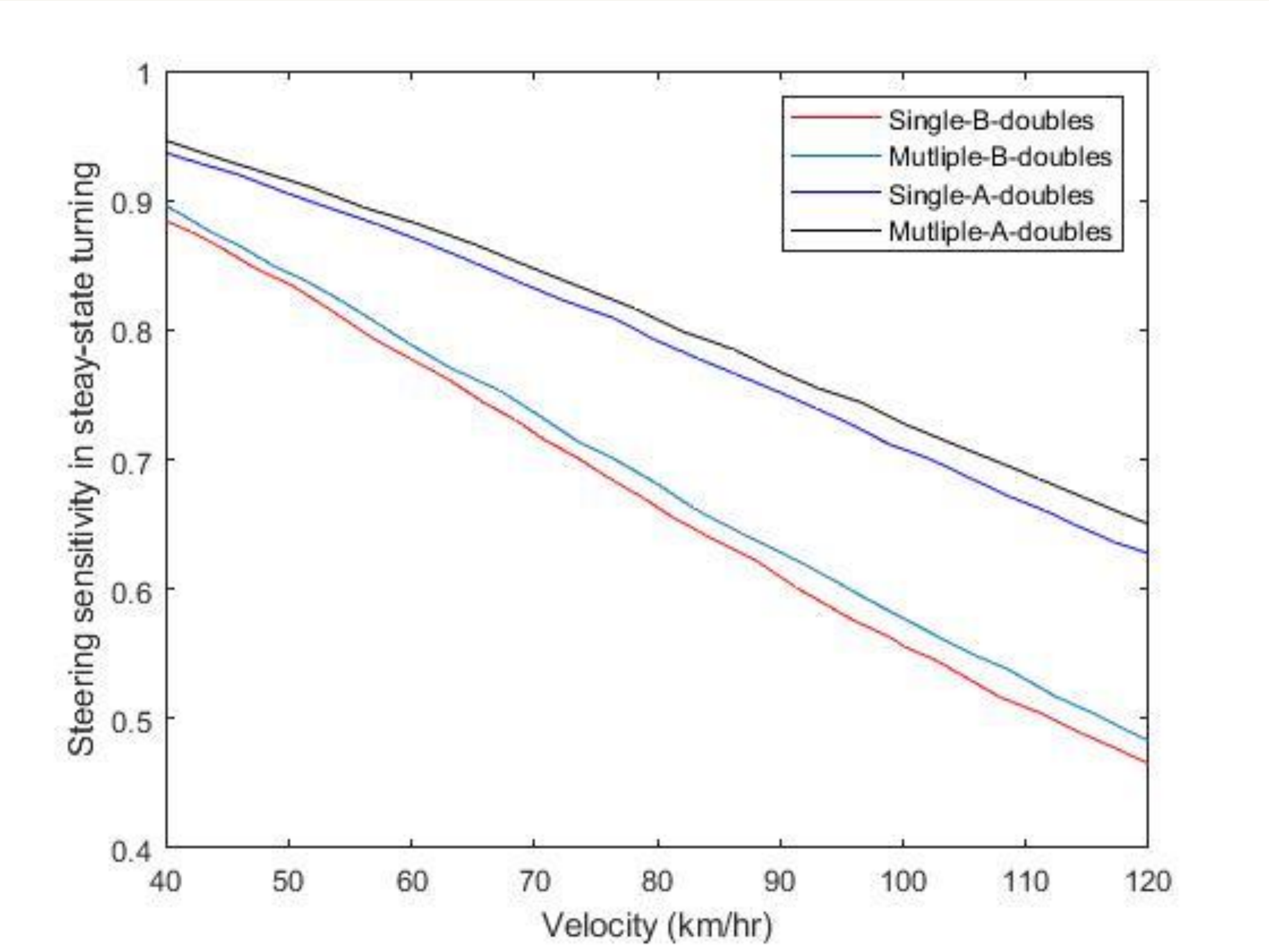
MATLAB is utilized as a tool for simulation of the mathematical model.

Data was created for different parameter like stability factor ‘K’ and multiple axle factor ‘F’.

Graphs were created using the data for steering sensitivity in steady-state steering and for Steering sensitivity in lane changing.

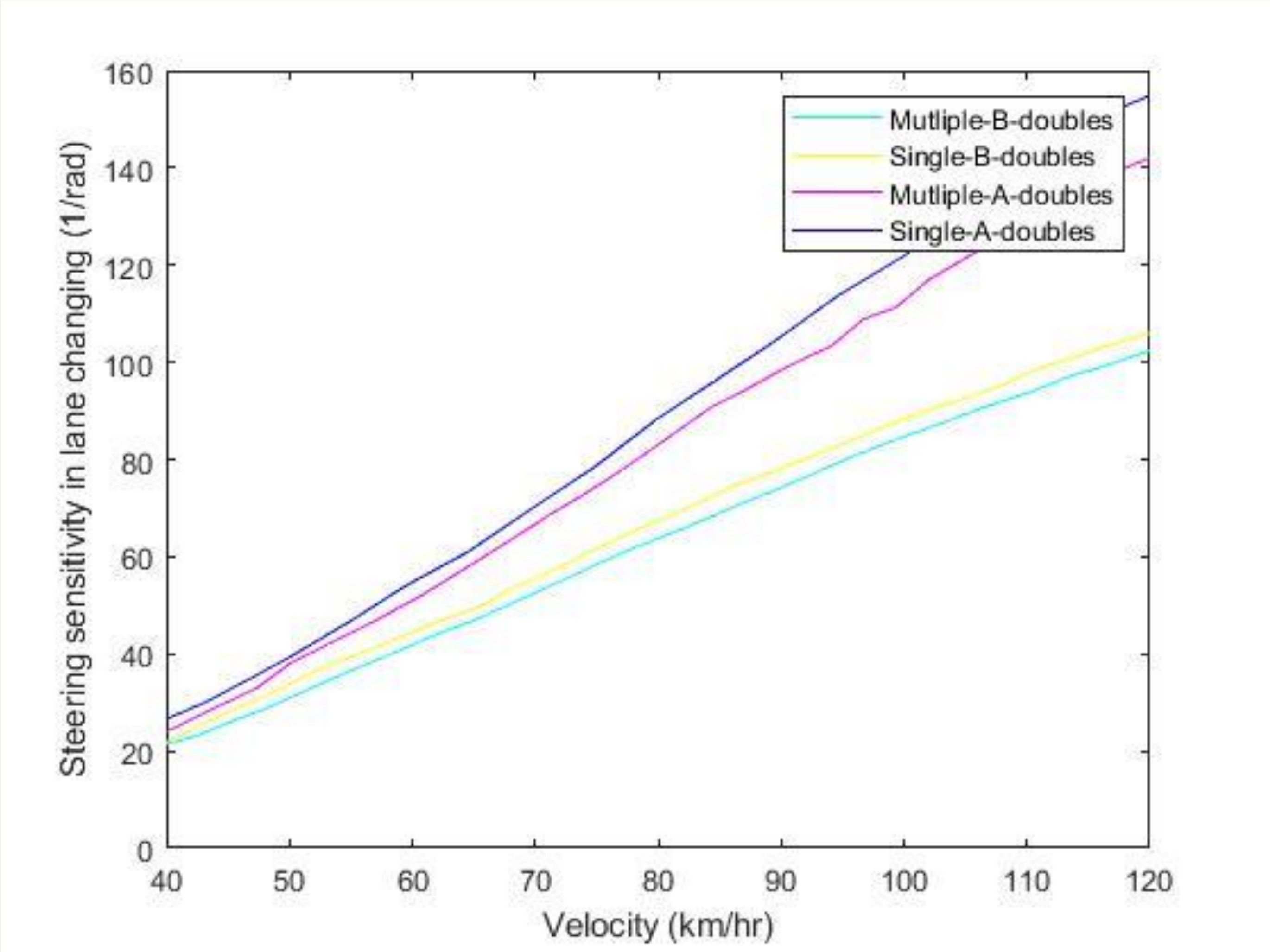
RESULTS:

- Steering sensitivity in steady-state steering



RESULTS:

- Steering sensitivity in Lane changing steering



Conclusions:

Equation of motion reduced to those of multi-articulated vehicles with hypothetical axles and additional terms, for equal vertical loads on all axles as well as equal cornering coefficients of all wheels on multiple axles.

Steering sensitivity decreased for Lane changing where as increased for Steady-state steering with Multiple axle than single axle.

Future Work:

The paper also deals with prediction of zeros and poles for further analysis of the multiple axle and their location for oscillatory system.

Off-tracking analysis is done for the above mentioned system.

We would like to understand the modelling and work on them.

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- And we are also thankful for the accepting our request for extension of project present.

References:

- Akira Aoki, Yoshitaka Marumo, and Ichiro Kageyama – '**Effects of multiple axles on the lateral dynamics of multi-articulated vehicles**'

Contribution:

- ME16BTECH11008 – Mathematical modelling and presentation
- ME16BTECH11027 – CAD modelling
- ME16BTECH11033 – Simulation and presentation
- ME16BTECH11034 – CAD modelling
- ME16BTECH11038 – Report work