GROUP 6

EFFECTS OF MULTIPLE AXLES ON THE LATERAL DYNAMICS OF MULTI-ARTICULATED VEHICLES

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INTRODUCTION

- An **articulated vehicle** is a **vehicle** which has a permanent or semi-permanent pivoting joint in its construction, allowing the **vehicle** to turn more sharply. There are many kinds of **articulated vehicles**, from heavy equipment to buses, trams and trains.
- In **multi-articulated vehicles**, there are more than one pivoting joints involved. So, this essentially consists of a tractor (contains the engine) and two or more trailers/semi-trailers pivoted to it.
- In our discussion here, we will be considering a combination of a tractor and two semi-trailers.

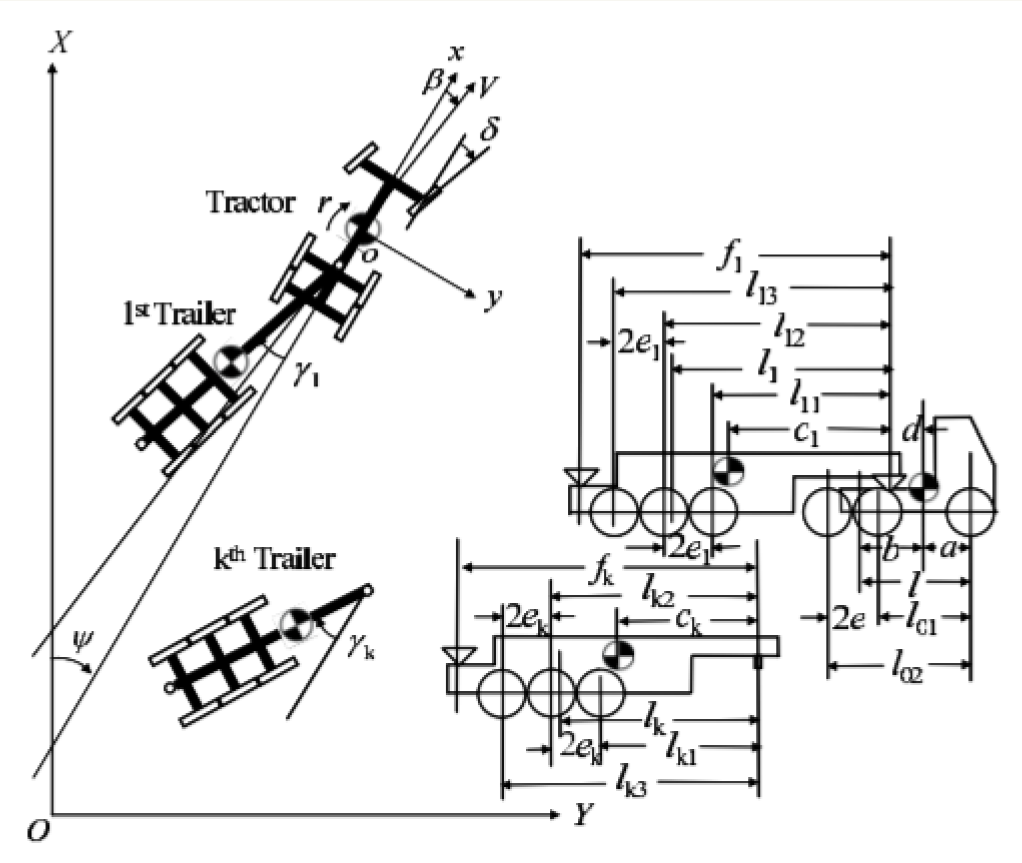
https://images-na.ssl-images-amazon.com/images/I/71SLuhiluvL._SL1500_.jpg

- The advantage of a multi-articulated vehicle is that it can carry higher loads than a normal vehicle.
- Also, it is able to make turns easily about the pivoted joints even if the total length of the vehicle is very large.

http://i.imgur.com/rX06b2i.jpg

MODEL

- The model considered here consists of a tractor and two semi-trailers joined using pivoted joints.



For our case, ‘k’ varies from 1-2

Total no. of semi-trailers, n = 2 (k = 1 or 2)

The masses and different dimensions are mentioned in the next slide.

PARAMETERS

The useful parameters are:

a = 1.167 m, b = 2.333 m, c1 = 2.853 m, c2 = 3.669 m, d = 1.983 m, f1 = 6.000 m, l = 3.500 m, l1 = 5.300 m, l2 = 5.300 m, M = 6000 kg, M1 = 13, 000 kg, M2 = 13, 000 kg, I = 19, 600 kgm2, I1 = 68, 600 kg m2, I2 = 58, 800 kg m2, Cf = 186 kN/rad,Cr = 332 kN/rad, C1 = 405 kN/rad,C2 = 370 kN/rad

e = e1 = e2 = 0 (single axle) and e = e1 = e2 = 0.65 m (multi-axle)

No. of axles on tractor, p = 1 (or 2)

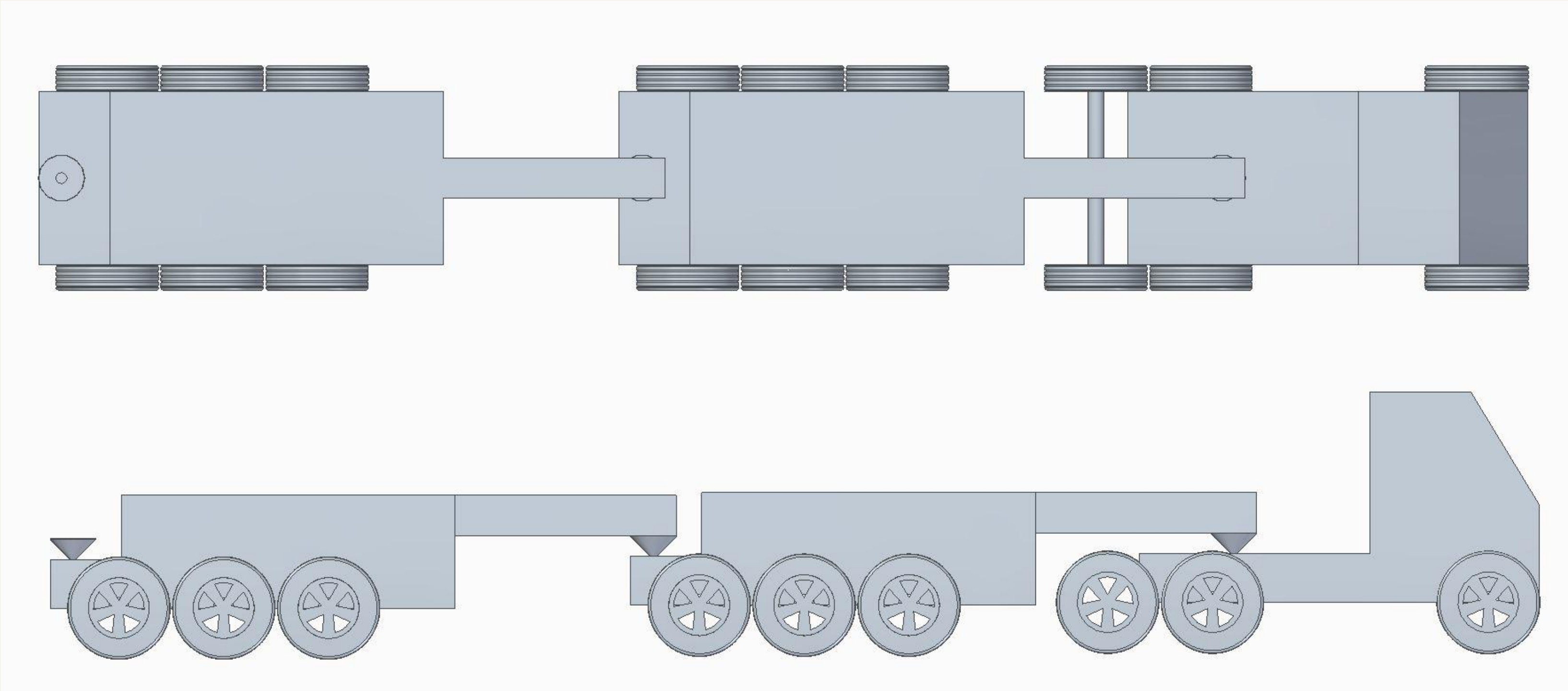
No. of axles on 1^{st} semi-trailer, q1 = 1 (or 2)

No. of axles on 2^{nd} semi-trailer, q2 = 1 (or 2)

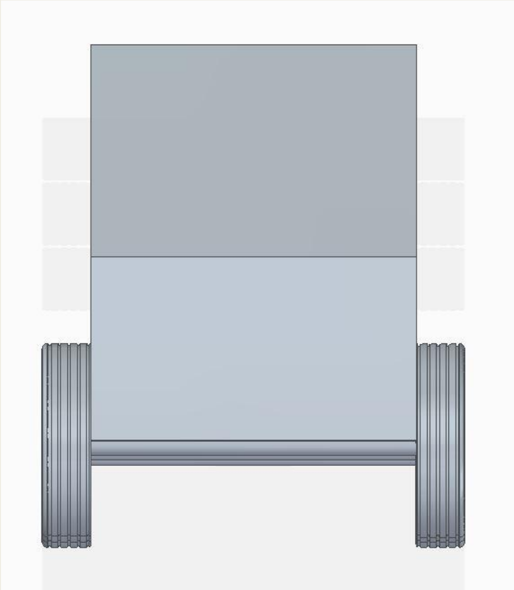
CAD MODELS

1. WHOLE TRUCK

Top view

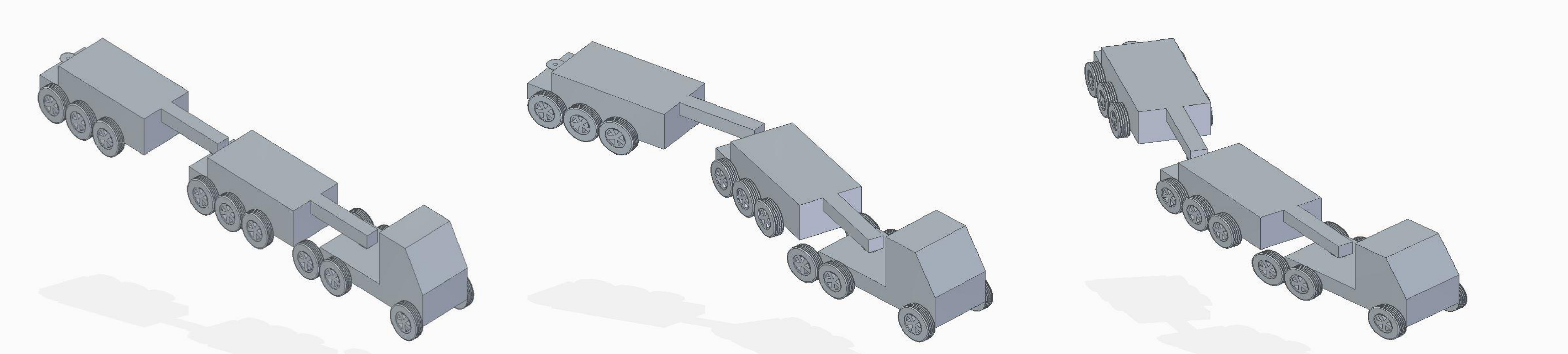


Side view

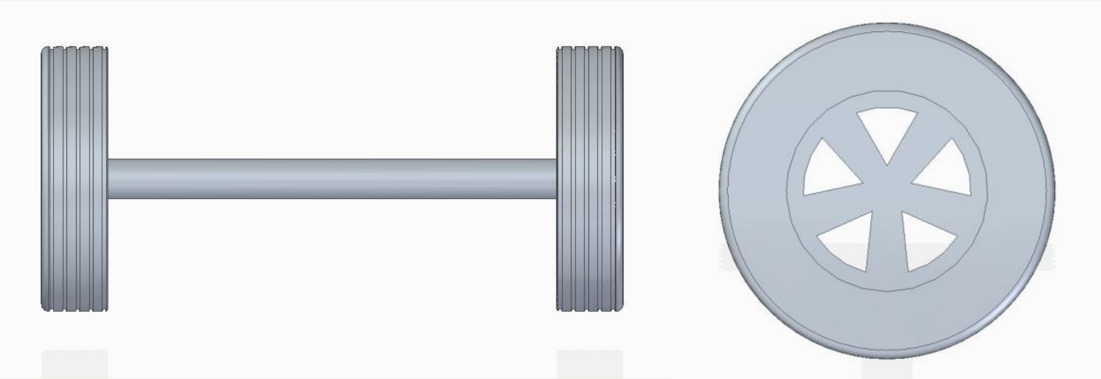


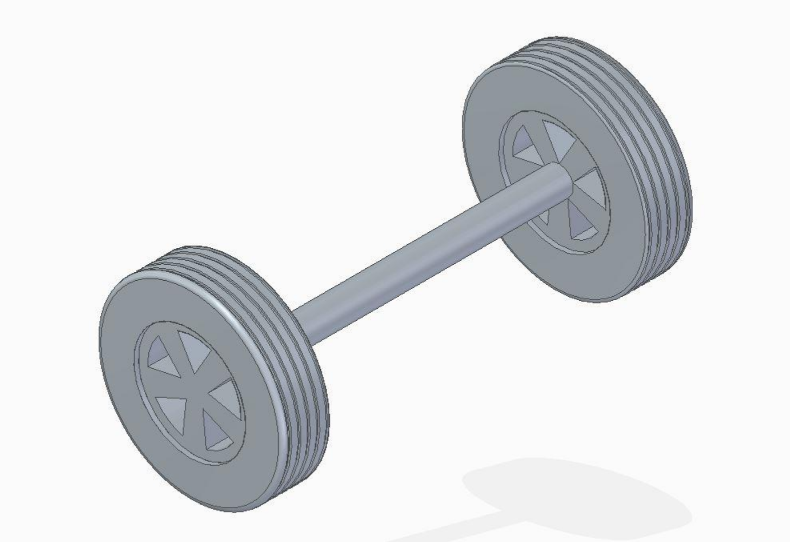
Front view

The trailers can move around the pivoting joints as shown below:

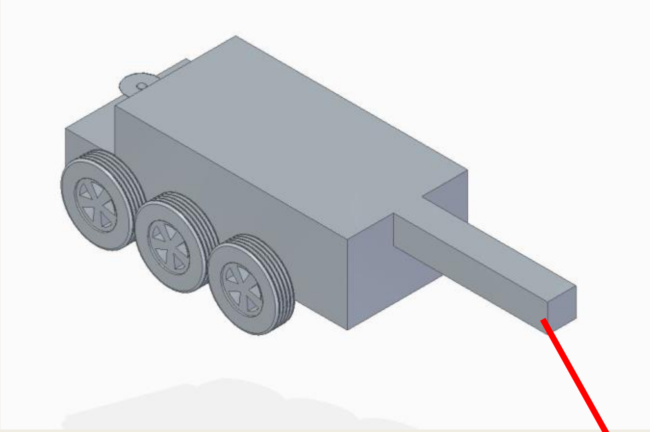


2. Tire and Axle



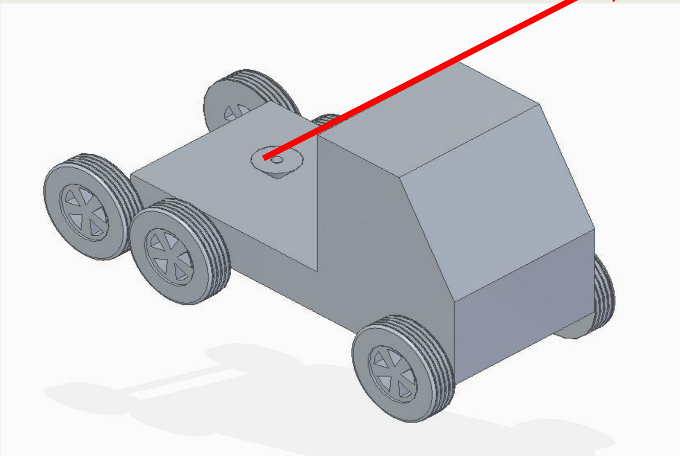


3. Semi-trailer



4. Tractor

Pivoting joint



EQUATIONS OF MOTION

The equations of motion for moving coordinate system are:

$$
\begin{aligned}
\beta &= \frac{\dot{Y}}{V} - \psi, \\
\beta_{\mathrm{f}} &= \delta - \beta - \frac{ar}{V}, \\
\beta_{\mathrm{r}m} &= -\beta + \frac{(l_{0m} - a)r}{V}, \\
\beta_{km} &= -\beta + \frac{\left(d + \sum_{i=1}^{k-1} f + l_{km}\right) r}{V} + \frac{\sum_{i=1}^{k-1} f_i \dot{\gamma}_i}{V} + \frac{l_{km}\dot{\gamma}_k}{V} + \gamma_k.
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
&\left(M + \sum_{i=1}^{n} M_i\right) V\dot{\beta} + \left(C_{\mathrm{f}} + \sum_{m=1}^{p} C_{\mathrm{r}m} + \sum_{i=1}^{n}\sum_{m=1}^{q_i} C_{im}\right)\beta - \sum_{j=1}^{n} M_j \left(d + \sum_{i=1}^{j-1} f_i + c_j\right)\dot{r} \\
&+ \left[aC_{\mathrm{f}} - \sum_{m=1}^{p} (l_{0m} - a)\, C_{\mathrm{r}m} - \sum_{j=1}^{n}\sum_{m=1}^{q_j}\left(d + \sum_{i=1}^{j-1} f_i + l_{jm}\right) C_{jm} + \left(M + \sum_{i=1}^{n} M_i\right) V^2\right] r/V \\
&- \sum_{j=1}^{n}\left(M_j c_j + \sum_{i=j+1}^{n} M_i f_j\right)\ddot{\gamma}_j - \sum_{j=1}^{n}\left(\sum_{m=1}^{q_j} l_{jm} C_{jm} + f_j \sum_{i=j+1}^{n}\sum_{m=1}^{q_i} C_{im}\right)\dot{\gamma}_j/V - \sum_{i=1}^{n}\sum_{m=1}^{q_i} C_{im}\gamma_i = C_{\mathrm{f}}\delta.
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
&MdV\dot{\beta} + \left[(a+d)C_{\mathrm{f}} + \sum_{m=1}^{p}(a+d-l_{0m})C_{\mathrm{r}m}\right]\beta + I\dot{r} \\
&\quad + \left[a(a+d)C_{\mathrm{f}} - \sum_{m=1}^{p}(l_{0m}-a)(a+d-l_{0m})C_{\mathrm{r}m} + MdV^2\right]r/V = (a+d)C_{\mathrm{f}}\delta.
\end{aligned}
\tag{A3}
$$

$$
\begin{aligned}
&-\left(M_k c_k + \sum_{i=k+1}^{n} M_i f_k\right)V\dot{\beta} - \left(\sum_{m=1}^{q_k} l_{km}C_{km} + f_k \sum_{i=k+1}^{n}\sum_{m=1}^{q_i} C_{im}\right)\beta \\
&\quad + \left[I_k + M_k c_k\left(d + \sum_{i=1}^{k-1} f_i + c_k\right) + \sum_{j=k+1}^{n} M_j\left(d + \sum_{i=1}^{j-1} f_i + c_j\right) f_k\right]\dot{r} \\
&\quad + \left[\sum_{m=1}^{q_k} l_{km}\left(d + \sum_{i=1}^{k-1} f_i + l_{km}\right)C_{km} + f_k \sum_{j=k+1}^{n}\sum_{m=1}^{q_j}\left(d + \sum_{i=1}^{j-1} f_i + l_{jm}\right)C_{jm} - \left(M_k c_k + \sum_{i=k+1}^{n} M_i f_k\right)V^2\right]r/V \\
&\quad + \sum_{j=1}^{k-1}\left(M_k c_k + \sum_{i=k+1}^{n} M_i f_k\right)f_j\ddot{\gamma}_j + \left(I_k + M_k c_k^2 + \sum_{i=k+1}^{n} M_i f_k^2\right)\ddot{\gamma}_k + \sum_{j=k+1}^{n}\left(M_j c_j + \sum_{i=j+1}^{n} M_i f_j\right)f_k\ddot{\gamma}_j \\
&\quad + \sum_{j=1}^{k-1}\left(\sum_{m=1}^{q_k} l_{km}C_{km} + f_k \sum_{i=k+1}^{n}\sum_{m=1}^{q_i} C_{im}\right)f_j\dot{\gamma}_j/V + \left(\sum_{m=1}^{q_k} l_{km}^2 C_{km} + f_k^2 \sum_{i=k+1}^{n}\sum_{m=1}^{q_i} C_{im}\right)\dot{\gamma}_k/V \\
&\quad + f_k \sum_{j=k+1}^{n}\left(\sum_{m=1}^{q_j} l_{jm}C_{jm} + f_j \sum_{i=j+1}^{n}\sum_{m=1}^{q_i} C_{im}\right)\dot{\gamma}_j/V + \sum_{m=1}^{q_k} l_{km}C_{km}\gamma_k + f_k \sum_{i=k+1}^{n}\sum_{m=1}^{q_i} C_{im}\gamma_i = 0.
\end{aligned}
\tag{A4}
$$

- Equations A2 to A4 are derived after the articulation point force is cancelled, and the cornering stiffness multiplied by the wheel sideslip angle is substituted into the wheel lateral force in the equilibrium equations of the lateral forces and yawing moments of each vehicle at constant speed.
- Secondly, the equations of motion for the moving coordinate system and those for the fixed coordinate system in a horizontal plane can be mutually transformed using the first equation A1.

STEERING SENSITIVITY IN STEADY-STATE TURNING

$$\left[\frac{\delta_0}{\delta}\right]_{\text{steady-state}} = \frac{1}{1+K^*V^2}$$ where $K^* = K/F$

$$F = 1 + \left(\frac{A}{l^2}\right)\left(1+\frac{C_r}{C_f}\right) + \sum_{i=1}^{n}\left(\frac{B_i}{ll_i}\right)\left[D_i\left(\frac{C_i}{C_f}\right) - E_i\left(\frac{C_i}{C_r}\right)\right]$$ where

$$A = \left[\frac{2\sum_{m=1}^{p}(m-1)^2}{p}\right]e^2,$$
$$B_k = \left[\frac{2\sum_{m=1}^{q_k}(m-1)^2}{q_k}\right]e_k^2,$$

when m = 1,3,5,...,p(qk)

and

$$A = \left[\frac{2\sum_{m=2}^{P}(m-1)^2}{p}\right]e^2$$
$$B_k = \left[\frac{2\sum_{m=2}^{q_k}(m-1)^2}{q_k}\right]e_k^2,$$

when m = 1,3,5,...,p(qk)

$$D_i = \left[\frac{b-d}{l}\right]\prod_{j=1}^{i-1}\left(1-\frac{f_j}{l_j}\right)$$
$$E_i = \left[\frac{a+d}{l}\right]\prod_{j=1}^{i-1}\left(1-\frac{f_j}{l_j}\right)$$

$$K = \frac{P_f/C_f - P_r/C_r}{gl}$$ where

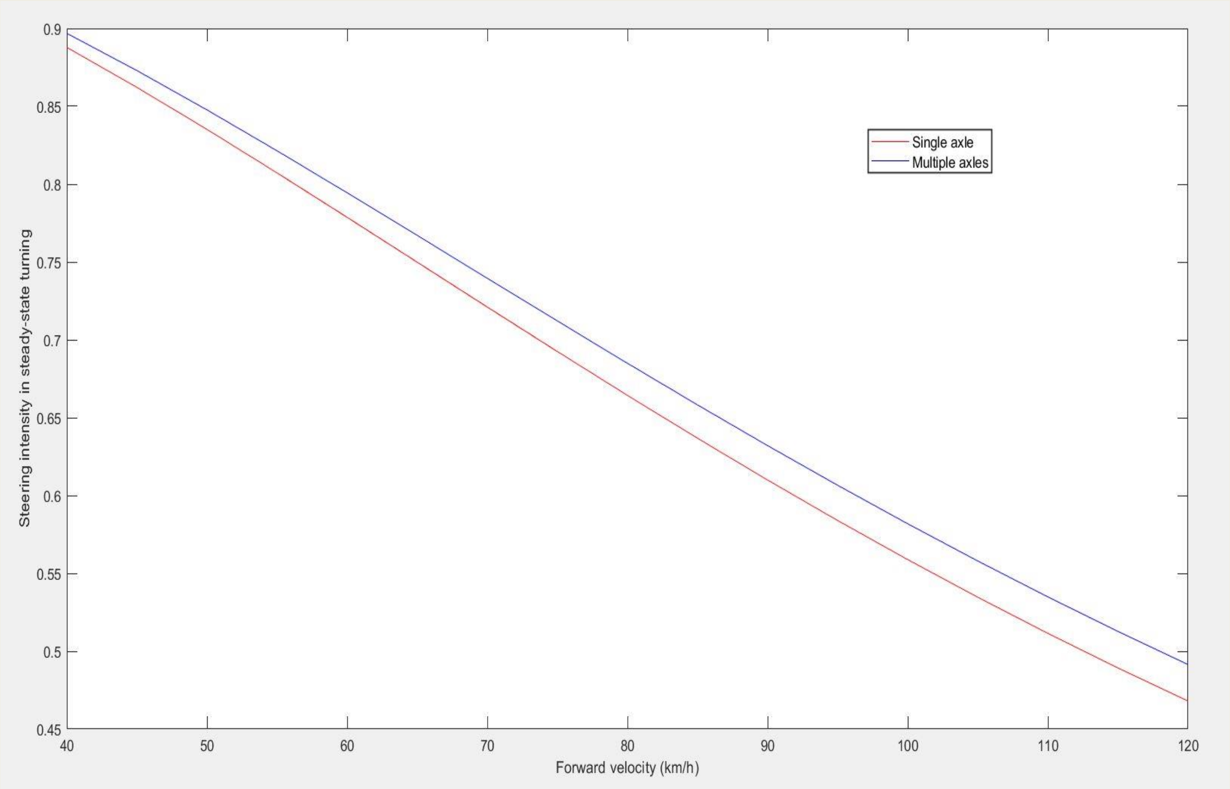
$$P_f = \left(\frac{b}{l}\right)Mg + \left[\frac{b-d}{l}\right]\sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\left(1-\frac{f_j}{l_j}\right)\right]\left(1-\frac{c_i}{l_i}\right)M_i g,$$
$$P_r = \left(\frac{a}{l}\right)Mg + \left[\frac{a+d}{l}\right]\sum_{i=1}^{n}\left[\prod_{j=1}^{i-1}\left(1-\frac{f_j}{l_j}\right)\right]\left(1-\frac{c_i}{l_i}\right)M_i g.$$

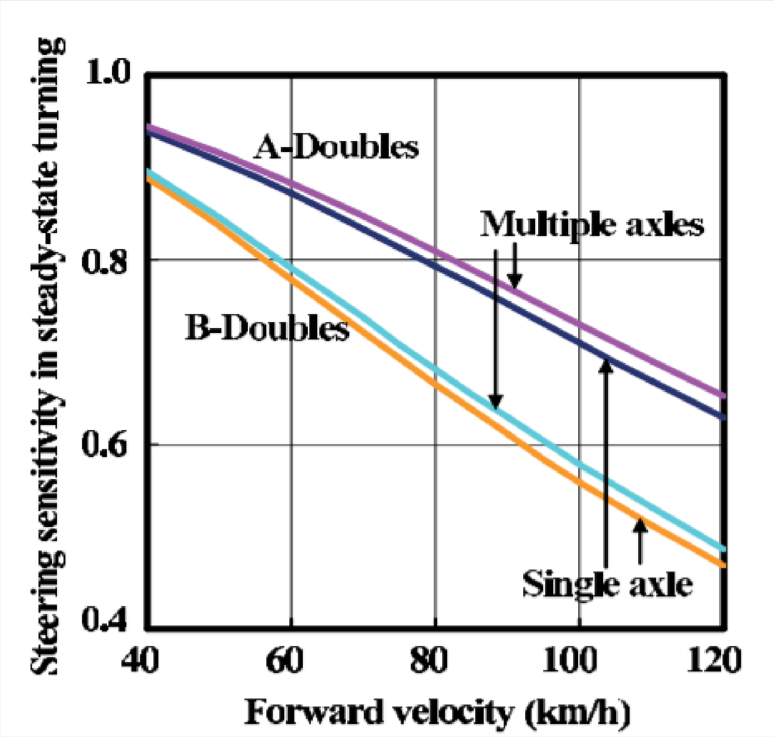
The above formulae are implemented using MATLAB to give out the steering sensitivity in steady-state turning vs forward velocity graph:

```
clc
clear all
close all

a = 1.167; b = 2.333; c1 = 2.853; c2 = 3.669; d = 1.983; f1 = 6.000;
l = 3.500; l1 = 5.3; l2 = 5.3; M = 6000; M1 = 13000; M2 = 13000;
I = 19600; I1 = 68600; I2 = 58800; Cf = 186000; Cr = 332000;
C1 = 405000; C2 = 370000; g = 9.81;

w1 = pi/2; Q = 3.6;

p = 1; q1 = 1; q2 = 1; n = 2; e = 0; e1 = 0; e2 = 0;
%p = 2; q1 = 2; q2 = 2; n = 2; e = 0.65; e1 = 0.65; e2 = 0.65;

m = 1:1:p;
m1 = 1:1:q1;
m2 = 1:1:q2;

sum = 0;
if mod(p,2)~=0
    for i = 1:1:(length(m)+1)/2
        sum = sum + (m(2*i-1)-1)^2;
    end
elseif mod(p,2)==0
    for i = 1:1:(length(m))/2
        sum = sum + (m(2*i)-1)^2;
    end
end
A = (2*sum*e^2)/p;

sum = 0;
if mod(q1,2)~=0
    for i = 1:1:(length(m1)+1)/2
        sum = sum + (m1(2*i-1)-1)^2;
    end
elseif mod(q1,2)==0
    for i = 1:1:(length(m1))/2
        sum = sum + (m1(2*i)-1)^2;
    end
end
B1 = (2*sum*e1^2)/q1;
```

```
sum = 0;
if mod(q2,2)~=0
    for i = 1:1:(length(m2)+1)/2
        sum = sum + (m2(2*i-1)-1)^2;
    end
elseif mod(p,2)==0
    for i = 1:1:(length(m2))/2
        sum = sum + (m2(2*i)-1)^2;
    end
end

B2 = (2*sum*e2^2)/q2;

D1 = 0;
D2 = ((b-d)/l)*(1-(f1/l1));
E1 = 0;
E2 = ((a+d)/l)*(1-(f1/l1));

Pf = ((b/l)*M*g) + ((b-d)/l)*(1-(c1/l1))*M1*g + ((b-d)/l)*(1-(f1/l1))*(1-(c2/l2))*M2*g;
Pr = ((a/l)*M*g) + ((a+d)/l)*(1-(c1/l1))*M1*g + ((a+d)/l)*(1-(f1/l1))*(1-(c2/l2))*M2*g;

K = ((Pf/Cf) - (Pr/Cr))/(g*l);
F = 1 + (A/l^2)*(1+(Cr/Cf)) + (B2/(l*l2))*((D2*(C2/Cf))-(E2*(C2/Cr)));

K_star = K/F;

V = 40:5:120;
Vbar = (5/18)*V;

for i = 1:length(V)
    ss(i) = 1/(1+K_star*(Vbar(i)^2));
    lc(i) = ((2*pi*Vbar(i)^2)/(w1^2*Q*l*F))*ss(i);
end
plot(V,ss,'r')
plot(V,lc,'r')
```

The graph obtained is as follows:





We can observe that the results obtained using MATLAB and the ones presented in the paper are close to each other for B-doubles case. The steering sensitivity in steady-state turning is increased by increasing the number of axles; however, it is decreased by increasing forward velocities.

STEERING SENSITIVITY IN LANE CHANGING

$$\frac{1}{\delta_1} = \left(\frac{2\pi V^2}{\omega_1^2 QlF}\right) \times \left[\frac{\delta_0}{\delta}\right]_{\text{steady-state}}$$

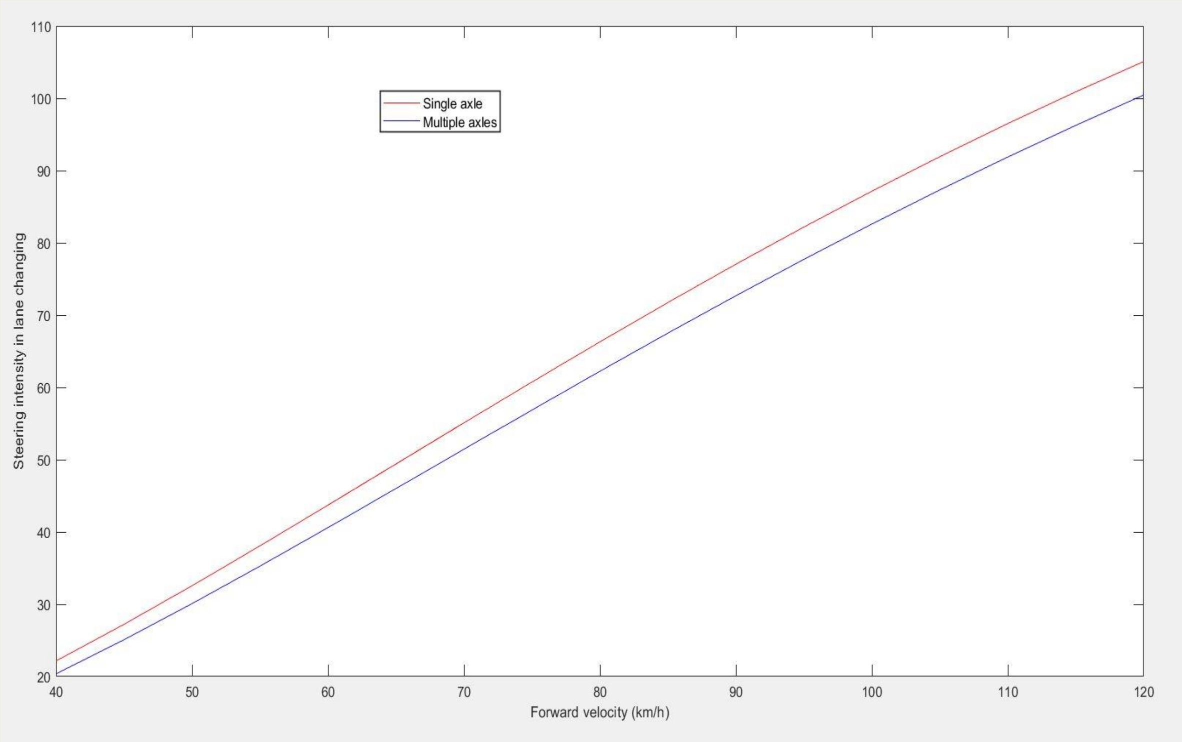
where '*F*' has the same formulae as before

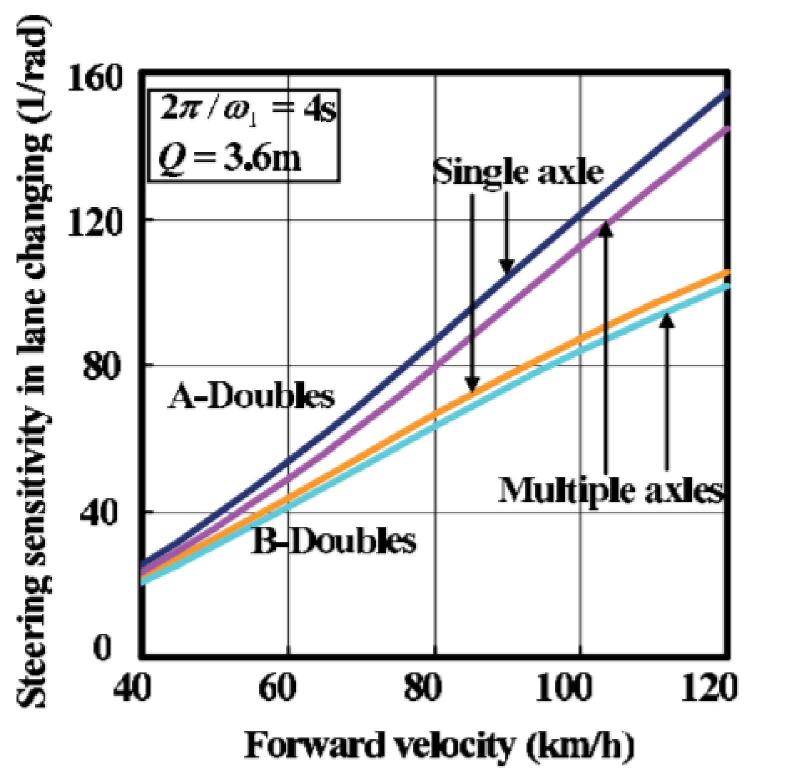
And $2\pi/\omega_1 = 4\ s$, *and* $Q = 3.6\ m$

The graph of steering sensitivity in lane changing vs the forward velocity (V) is obtained using the same code as before.

The graphs are shown below.

The graph obtained is as follows:





We can observe that the results obtained using MATLAB and the ones presented in the paper are close to each other for B-doubles case. The steering sensitivity in lane changing is decreased by increasing the number of axles; however, it is increased by increasing forward velocities. In addition, it is increased by increasing the time duration of steering input and decreasing the lane-changing width and the tractor wheelbase.

Also, the parameters p, q_k, e, and e_k are varied and the steering sensitivity, stability factor (K*) and multiple-axle factor (F) values are tabulated as follows:

PARAMETERS				STEERING SENSITIVITY		K* (s^2/m^2)	F
p	q_k	e (m)	e_k (m)	Steady-state turning	Lane changing		
1	1	0	0	0.468	105	0.0010	1.00
2	1	0.65	0	0.491	101	0.0009	1.10
3	1	0.65	0	0.525	94	0.0008	1.26
1	1	0	0	0.468	105	0.0010	1.00
1	2	0	0.65	0.464	106	0.0010	0.98
1	3	0	0.65	0.456	108	0.0011	0.95
2	2	0.65	0.65	0.487	102	0.0009	1.08
2	2	0.75	0.75	0.493	100	0.0009	1.10
2	2	0.85	0.85	0.500	99	0.0009	1.13

Parameters: *V = 120 km/h, $2\pi/\omega_1$ = 4 s, and Q = 3.6 m*

OFF-TRACKING IN STEADY-STATE TURNING

$$F_{\mathrm{b}} = b + \left(\frac{A}{l}\right) - \sum_{i=1}^{n} \left(\frac{B_i}{l_i}\right) E_i \left(\frac{C_i}{C_{\mathrm{r}}}\right)$$

and

$$F_k = \left(d + \sum_{i=1}^{k-1} f_i + l_k\right) + \left(\frac{B_k}{l_k}\right) - \sum_{i=k+1}^{n} \left[\left(\frac{B_i}{l_i}\right)\left(\frac{f_k}{l_k}\right)\left(\frac{C_i}{C_k}\right) \prod_{j=k+1}^{i-1} \left(1 - \frac{f_j}{l_j}\right)\right]$$

where ***F, A, Bk*** have the same formulae as before

$$\Delta R_{\mathrm{f}} = \frac{-a(P_{\mathrm{r}}/C_{\mathrm{r}})V^2}{gR} \qquad \Delta R_{\mathrm{r}} = \frac{b(P_{\mathrm{r}}/C_{\mathrm{r}})V^2}{gR} \qquad \Delta R_k = \frac{\left[d(P_{\mathrm{r}}/C_{\mathrm{r}}) + \sum_{i=1}^{k-1} f_i (P_i/C_i) + l_k (P_k/C_k)\right] V^2}{gR}$$

$$R_{\mathrm{f}} = \sqrt{R^2 - b^2 + l^2} + \frac{a(F_{\mathrm{b}} - b)}{R} \qquad R_{\mathrm{r}} = \sqrt{R^2 - b^2} - \frac{b[F_{\mathrm{b}} - b]}{R}$$

$$R_k = \sqrt{R^2 - b^2 + (b-d)^2 + \sum_{i=1}^{k-1} (f_i - l_i)^2 - \sum_{i=1}^{k} l_i^2} - \frac{d(F_{\mathrm{b}} - b)}{R} - \frac{\sum_{i=1}^{k-1} f_i \left[F_i - \left(d + \sum_{j=1}^{i-1} f_i + l_i\right)\right]}{R} - \frac{l_k \left[F_k - \left(d + \sum_{i=1}^{k-1} f_i + l_k\right)\right]}{R}$$

$$\xi_h = R_h + \Delta R_h - R$$

where h = f, r, 1, 2

$$e_s = |\xi_i - \xi_j|_{\max}$$

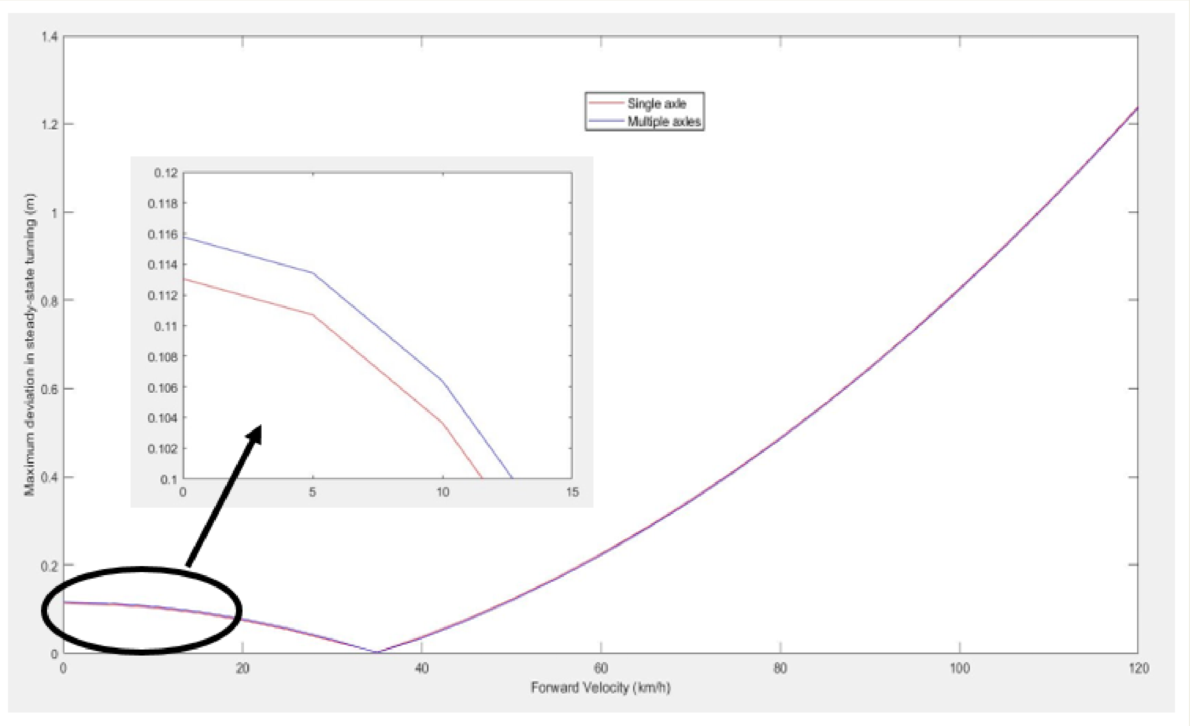
where i, j = f, r, 1, 2

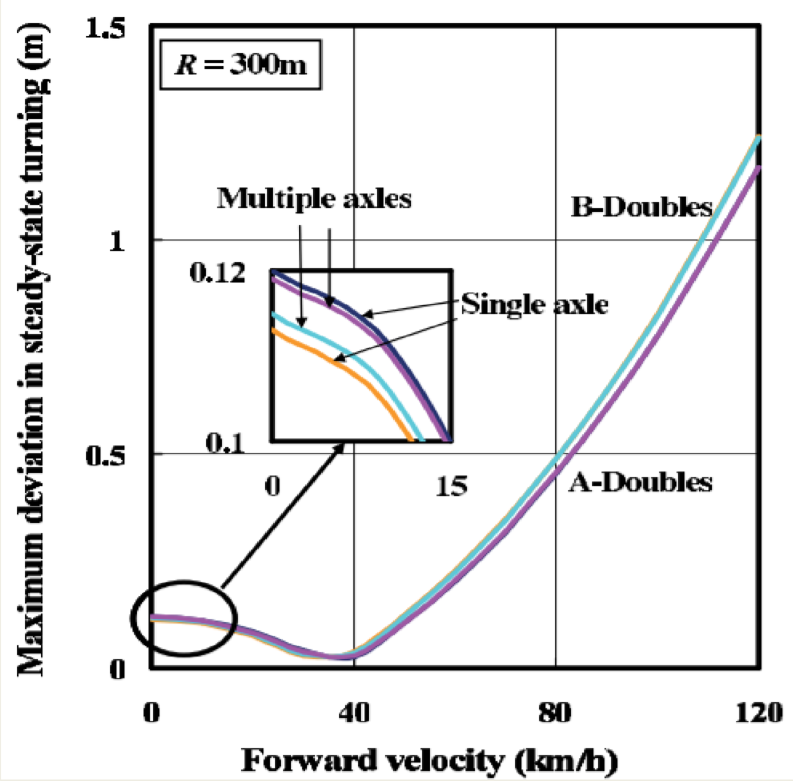
We plot e_s vs V graph using MATLAB code

```
clc
clear all
close all

a = 1.167; b = 2.333; c1 = 2.853; c2 = 3.669; d = 1.983; f1 = 6.000;
l = 3.500; l1 = 5.3; l2 = 5.3; M = 6000; M1 = 13000; M2 = 13000;
I = 19600; I1 = 68600; I2 = 58800; Cf = 186000; Cr = 332000;
C1 = 405000; C2 = 370000; g = 9.81;

R = 300; Q = 3.6; w1 = (2*pi)/2.5;

p = 1; q1 = 1; q2 = 1; n = 2; e = 0; e1 = 0; e2 = 0;
%p = 2; q1 = 2; q2 = 2; n = 2; e = 0.65; e1 = 0.65; e2 = 0.65;

m = 1:1:p;
m1 = 1:1:q1;
m2 = 1:1:q2;

sum = 0;
if mod(p,2)~=0
    for i = 1:1:(length(m)+1)/2
        sum = sum + (m(2*i-1)-1)^2;
    end
elseif mod(p,2)==0
    for i = 1:1:(length(m))/2
        sum = sum + (m(2*i)-1)^2;
    end
end
A = (2*sum*e^2)/p;

sum = 0;
if mod(q1,2)~=0
    for i = 1:1:(length(m1)+1)/2
        sum = sum + (m1(2*i-1)-1)^2;
    end
elseif mod(q1,2)==0
    for i = 1:1:(length(m1))/2
        sum = sum + (m1(2*i)-1)^2;
    end
end
B1 = (2*sum*e1^2)/q1;

sum = 0;
if mod(q2,2)~=0
    for i = 1:1:(length(m2)+1)/2
        sum = sum + (m2(2*i-1)-1)^2;
    end
elseif mod(p,2)==0
    for i = 1:1:(length(m2))/2
        sum = sum + (m2(2*i)-1)^2;
    end
end
```

```
B2 = (2*sum*e2^2)/q2;

D1 = 0;
D2 = ((b-d)/l)*(1-(f1/l1));
E1 = 0;
E2 = ((a+d)/l)*(1-(f1/l1));

Pf = ((b/l)*M*g) + ((b-d)/l)*(1-(c1/l1))*M1*g + ((b-d)/l)*(1-(f1/l1))*(1-(c2/l2))*M2*g;
Pr = ((a/l)*M*g) + ((a+d)/l)*(1-(c1/l1))*M1*g + ((a+d)/l)*(1-(f1/l1))*(1-(c2/l2))*M2*g;

K = ((Pf/Cf) - (Pr/Cr))/(g*l);
F = 1 + (A/l^2)*(1+(Cr/Cf)) + (B2/(l*l2))*((D2*(C2/Cf))-(E2*(C2/Cr)));

K_star = K/F;

Fb = b + (A/l) - ((B1*E1*C1)/(l1*Cr)) - ((B2*E2*C2)/(l2*Cr));
F1 = (d+l1) + (B1/l1) - ((B2*f1*C2)/(l2*l1*C1));
F2 = (d+l1+f1) + (B2/l2);

P1 = (c1/l1)*M1*g + (f1/l1)*(1-(c2/l2))*M2*g;
P2 = (c2/l2)*M2*g;

Rf = sqrt(R^2-b^2+l^2) + ((a*(Fb-b))/R);
Rr = sqrt(R^2-b^2) - ((b*(Fb-b))/R);
R1 = sqrt(R^2-b^2+(b-d)^2-l1^2) - ((d*(Fb-b))/R) - (l1*(F1-(d+l1)))/R;
R2 = sqrt(R^2-b^2+(b-d)^2+(f1-l1)^2-l1^2-l2^2) - ((d*(Fb-b))/R) - (f1*(F1-(d+l1))/R) -
(l2*(F2-(d+f1+l2)))/R;

for i = 1:length(V)
    del_Rf(i) = (-a/(g*R))*(Pr/Cr)*Vbar(i)^2;
    del_Rr(i) = (b/(g*R))*(Pr/Cr)*Vbar(i)^2;
    del_R1(i) = Vbar(i)^2*(1/(g*R))*((d*Pr)/Cr + (l1*P1)/C1);
    del_R2(i) = Vbar(i)^2*(1/(g*R))*((d*Pr)/Cr + (l2*P2)/C2 + (f1*P1)/C1);
end

eps = zeros(4,length(V));
for j = 1:length(V)
    eps(1,j) = R1 + del_R1(j) - R;
    eps(2,j) = R2 + del_R2(j) - R;
    eps(3,j) = Rf + del_Rf(j) - R;
    eps(4,j) = Rr + del_Rr(j) - R;
end

for j = 1:length(V)
    for i = 1:4
        for k = 1:4
            ess(i,k,j) = (abs(eps(i,j)-eps(k,j)));
        end
    end
    es(j) = max(max(ess(:,:,j)));
end


V = 0:5:120;
Vbar = (5/18)*V;
plot(V,es,'r')
```

The graph obtained is as follows:





We can observe that the results obtained using MATLAB and the ones presented in the paper are close to each other for B-doubles case. We can see that there are no differences between multiple-axle and single-axle vehicle combinations for the maximum deviation in steady-state turning. Also, with increasing forward velocity, there is an initial dip in the maximum deviation, but it again increases after some velocity for both single and multiple axle models.

CONCLUSION

This study focuses on the fundamentals of lateral dynamics of multiple-axle vehicle combinations. First, the equations of motion are reduced to those of multi-articulated vehicles with hypothetical axles and additional terms, for equal vertical loads on all axles as well as equal cornering coefficients of all wheels on multiple axles. The terms on multiple axles are added in the tractor yawing moment and part of the trailer yawing moment. Second, non-oscillatory stability, steering sensitivities in steady-state turning and lane changing, and off-tracking in steady-state turning are analyzed for B- Double combination with multiple axles. The analyses indicate that multiple-axle vehicle combinations differ from single-axle vehicle combinations in non-oscillatory stability and steering sensitivity, where steering sensitivity in steady-state turning is included in that in lane changing. In addition, the following conclusions regarding off-tracking are drawn. Off-tracking in steady-state turning for multiple-axle vehicle combinations is reduced to a performance issue at very low forward velocities, and the kinematic radii, except for tight turning, cannot be distinguished from those of single-axle vehicle combinations and off-tracking in lane changing for multiple-axle vehicle combinations with low steering sensitivity exceeds that for combinations with high steering sensitivity.

FUTURE WORK

- Similar procedure can be implemented for higher number of semi-trailers.
- Also, more combinations of axles can be added to study the variations in the properties.

REFERENCES

- Akira Aoki , Yoshitaka Marumo & Ichiro Kageyama (2013) Effects of multiple axles on the lateral dynamics of multi-articulated vehicles, Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 51:3, 338-359, DOI: 10.1080/00423114.2012.743667

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APPENDIX – NOMENCLATURE

a	distance between the centre of front axle and centre of gravity of tractor
b	distance between the centre of gravity and centre of hypothetical rear axle of the tractor
c_k	distance between the front articulation point and centre of gravity of the kth trailer
d	distance between the centre of gravity and articulation point of tractor
$2e$	distance between adjoining axles in rear axles of the tractor
$2e_k$	distance between adjoining axles of the kth trailer
f_k	distance between front and rear articulation points of the kth trailer
l	distance between the centre of front axle and centre of hypothetical rear axle of the tractor (wheelbase of tractor)
l_{0m}	distance between the centre of front axle and centre of the mth rear axle of the tractor
l_k	distance between the front articulation point and centre of hypothetical axle of the kth trailer (wheelbase of the kth trailer)
l_{km}	distance between the front articulation point and centre of the mth axle of the kth trailer
M	mass of the tractor
M_k	mass of the kth trailer
I	yawing moment of inertia of the tractor
I_k	yawing moment of inertia of the kth trailer
C_{f}	cornering stiffness of front-axle wheels of the tractor
C_{r}	cornering stiffness of hypothetical rear-axle wheels of the tractor
$C_{\mathrm{r}m}$	cornering stiffness of the mth rear-axle wheels of the tractor
C_k	cornering stiffness of hypothetical axle wheels of the kth trailer
C_{km}	cornering stiffness of the mth axle wheels of the kth trailer
F_{f}	lateral force of front-axle wheels of the tractor
F_{r}	lateral force of hypothetical rear-axle wheels of the tractor
$F_{\mathrm{r}m}$	lateral force of the mth rear-axle wheels of the tractor
F_k	lateral force of hypothetical axle wheels of the kth trailer
F_{km}	lateral force of the mth axle wheels of the kth trailer
$F_{\mathrm{c}k}$	lateral force at the front articulation point of the kth trailer
P_{f}	vertical load of the front axle of the tractor
P_{r}	vertical load of the hypothetical rear axle of the tractor
$P_{\mathrm{r}m}$	vertical load of the mth rear axle of the tractor
$P_{\mathrm{c}k}$	vertical load of the front articulation point of the kth trailer
P_k	vertical load of the hypothetical axle of the kth trailer
P_{km}	vertical load of the mth axle of the kth trailer
β	sideslip angle of the centre of gravity of the tractor

β_f	sideslip angle of front-axle wheels of the tractor
β_r	sideslip angle of hypothetical rear-axle wheels of the tractor
β_{rm}	sideslip angle of the mth rear-axle wheels of the tractor
β_k	sideslip angle of the hypothetical axle wheels of the kth trailer
β_{km}	sideslip angle of the mth axle wheels of the kth trailer
γ_k	relative yawing angle between the tractor and the kth trailer
δ	front wheel angle of the tractor
ψ	yawing angle of the tractor
r	yawing angular velocity of the tractor
Y	lateral displacement of the centre of gravity of the tractor
V	forward velocity of vehicle combinations
n	number of trailers ($n = 3$ for A-Doubles; $n = 2$ for B-Doubles or truck and full-trailer combinations; $n = 1$ for tractor and semitrailer combinations; $n = 0$ for tractors or trucks)
p	number of rear axles of the tractor
q_k	number of axles of the kth trailer
oxy	fixed coordinate system in the moving tractor
OXY	fixed coordinate system in the horizontal plane
f	abbreviation of the front axle of the tractor
r	abbreviation of the rear axle of the tractor
k	abbreviation of the kth trailer ($k = 1, 2, \ldots, n$)
R	steady-state turning radius of the centre of gravity of the tractor
g	gravitational acceleration
δ_1	amplitude of the sinusoidal front wheel angle of the tractor in lane changing
ω_1	frequency of the sinusoidal front wheel angle of the tractor in lane changing
Q	lateral distance between initial and final lanes of the centre of gravity of the tractor
s	Laplace operator
$U(t)$	unit step function
$Y(s)$	Laplace transformation of lateral displacement of the centre of gravity of the tractor
$\Psi(s)$	Laplace transformation of the yawing angle of the tractor
$\Gamma_k(s)$	Laplace transformation of the relative yawing angle between the tractor and the kth trailer
$\Delta(s)$	Laplace transformation of the front wheel angle of the tractor
F	multiple-axle factor
K	stability factor of single-axle vehicle combinations
K^*	stability factor of multiple-axle vehicle combinations; by definition $= K/F$
δ_0	kinematic steering angle; by definition $= lF/R$
k_0	non-oscillatory coefficient in characteristic equation used in stability analysis
V_{cr}	critical speed
$[\delta_0/\delta]_{\text{steady-state}}$	steering sensitivity in steady-state turning
$1/\delta_1$	steering sensitivity in lane changing

ΔR_f	fluctuation of radius of the front-axle centre of tractor in steady-state turning; by definition $= -a(P_r/C_r)V^2/gR$
ΔR_r	fluctuations of radius of the hypothetical rear-axle centre of the tractor in steady-state turning; by definition $= b(P_r/C_r)V^2/gR$
ΔR_k	fluctuations of radius of the hypothetical axle centre of the kth trailer in steady-state turning; by definition $= [d(P_r/C_r) + \sum_{i=1}^{k-1} f_i(P_i/C_i) + l_k(P_k/C_k)]V^2/gR$
R_f	kinematic radius of the front-axle centre of the tractor
R_r	kinematic radius of the hypothetical rear-axle centre of the tractor
R_k	kinematic radius of the hypothetical axle centre of the kth trailer
ξ_h	deviation of each axle centre from radius of the centre of gravity of the tractor in steady-state turning
e_s	maximum deviation among tracks of each axle centre in steady-state turning
β_{f0}	kinematic sideslip angle of front-axle wheels of the tractor
β_{r0}	kinematic sideslip angle of hypothetical rear-axle wheels of the tractor
β_{k0}	kinematic sideslip angle of hypothetical axle wheels of the kth trailer
P_i	pole of vehicle combinations (i: ordinary number of poles)
Z_{kj}	zero of relative yawing angle of the kth trailer (j: ordinary number of zeroes)
ζ	damping ratio of pole and zero