

Modelling and Analysis of a Vehicle Crash using Kelvin Model

Group 4

Vidyasagar(Me15btech11031)

Sairaj(Me15btech11006)

Revanth(Me15btech11027)

Sitansh(Me15btech11031)

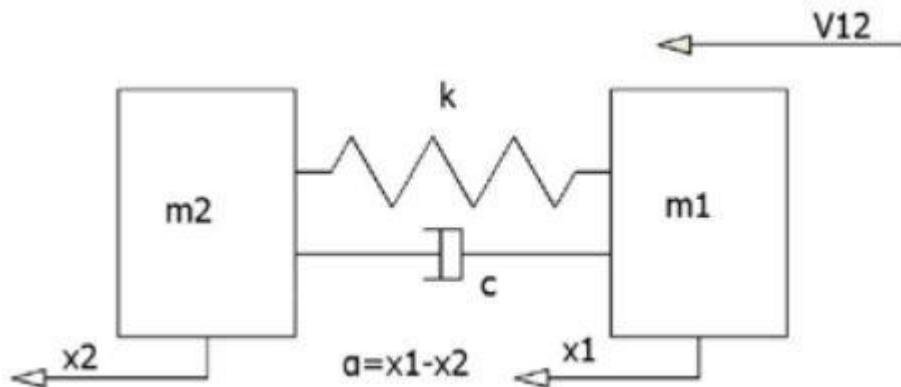
Sandeep(Me15bech11017)

Why Mathematical modelling:

- Real crash tests are complex
- Real crash test are time consuming and costly
- Simple Mathematical modelling gives you a first level analysis at a fraction of time and cost

Description of Kelvin Model

In Kelvin Model, we use spring mass and damper models to analyse collisions. Vehicle into barrier, Vehicle into vehicle collisions can be analysed using the model. We consider the system to be underdamped.



k	spring constant
m_1	Mass1
m_2	Mass 2
C	Damping coefficient
Alpha	Displacement

Final equations:

Transient responses of the underdamped system are:

$$\alpha(t) = \frac{v_0 e^{-\zeta \omega_e t}}{\sqrt{1 - \zeta^2} \omega_e} \sin(\sqrt{1 - \zeta^2} \omega_e t) \quad (2.2)$$

displacement (dynamic crush)

$$\dot{\alpha}(t) = v_0 e^{-\zeta \omega_e t} [\cos(\sqrt{1 - \zeta^2} \omega_e t) - \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin(\sqrt{1 - \zeta^2} \omega_e t)] \quad (2.3)$$

velocity

$$\ddot{\alpha}(t) = v_0 \omega_e e^{-\zeta \omega_e t} [-2\zeta \cos(\sqrt{1 - \zeta^2} \omega_e t) + \frac{2\zeta^2 - 1}{\sqrt{1 - \zeta^2}} \sin(\sqrt{1 - \zeta^2} \omega_e t)] \quad (2.4)$$

deceleration

Analytical analysis of vehicle into barrier model:

Fig. 4.1 presents a Kelvin model of a vehicle-to-barrier impact.

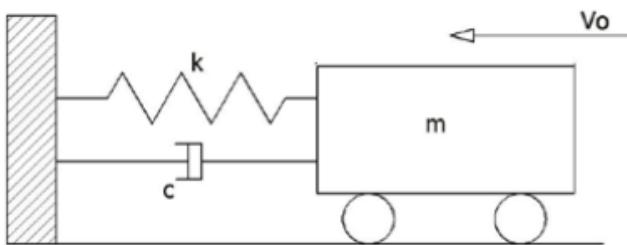


Fig. 4.1: VTB collision – Kelvin model

k – spring stiffness

c – damping coefficient

m – mass of the vehicle

v_0 – barrier initial impact velocity

We estimate the parameters of ζ and f by using the data of t_c and t_m . now from values of ζ and f we determine structural parameters of the model – k and c :

$$\text{Since } \omega_e = 2\pi f = \sqrt{\frac{k}{m}} \text{ and } \zeta = \frac{c}{2m\omega_e}$$

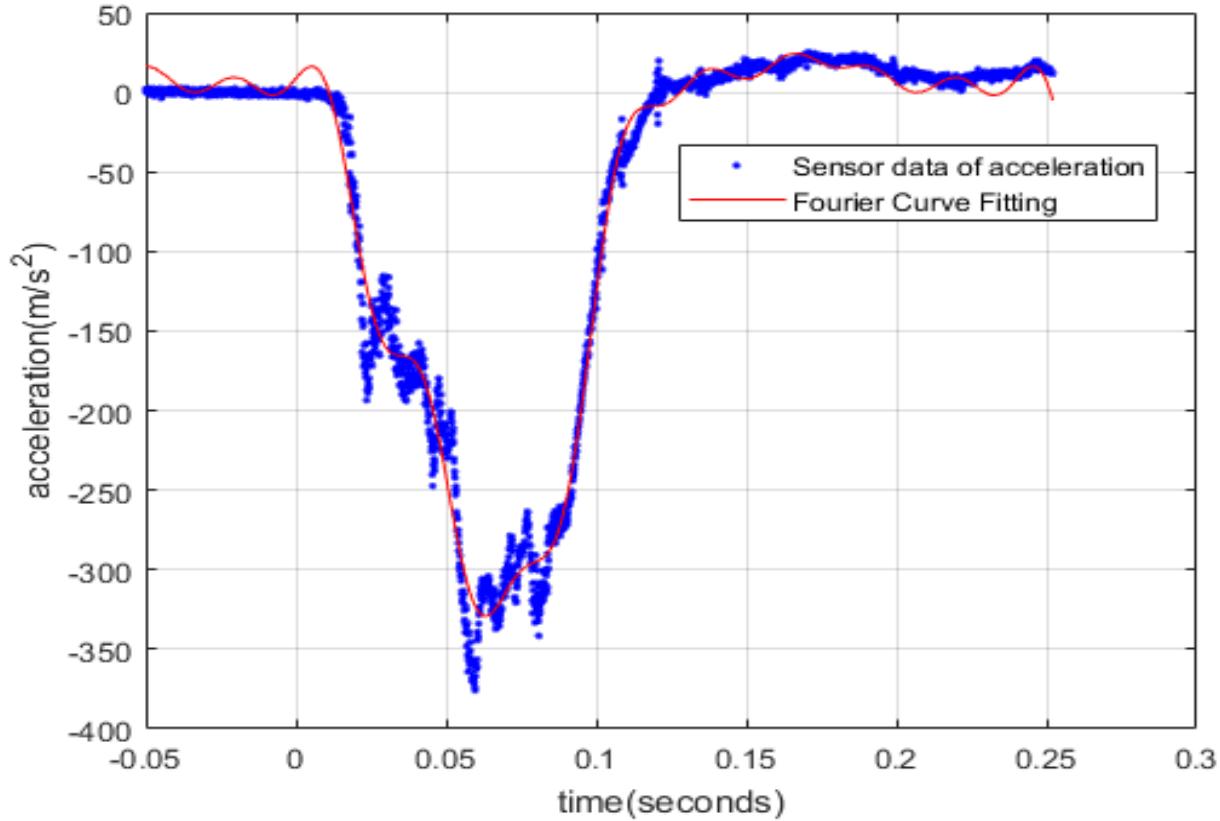
$$k = 4\pi^2 f^2 m$$

$$c = 4\pi f \zeta m$$

Data

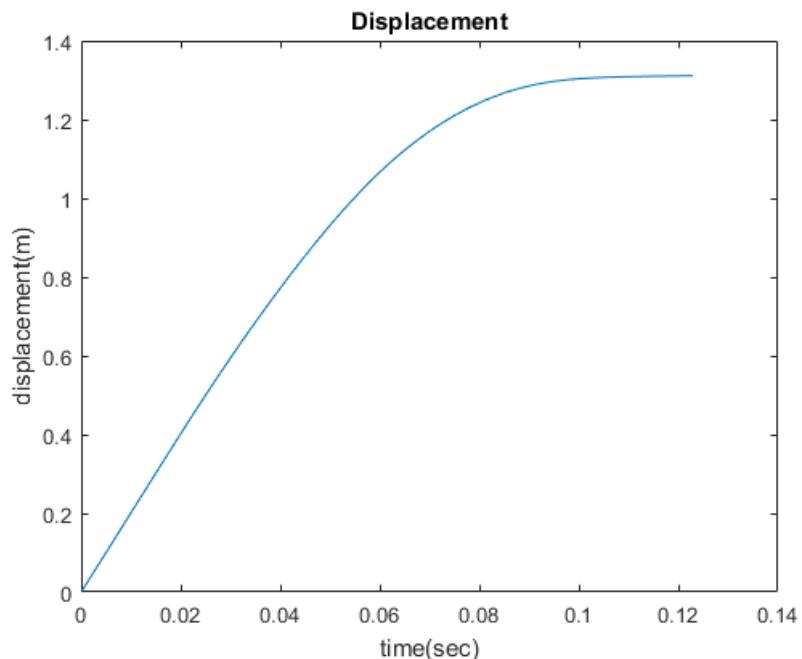
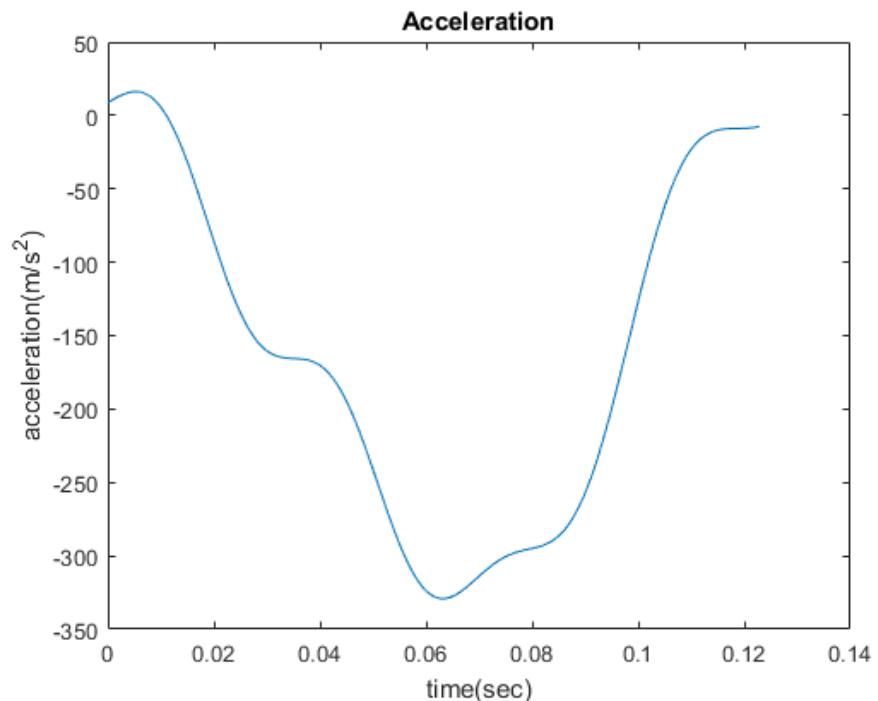
Acceleration in x direction measured. [Source](#)

Mass of car: 2382 Kg
Initial velocity: 56 Kmph
Restrain Slack: 0.267 m



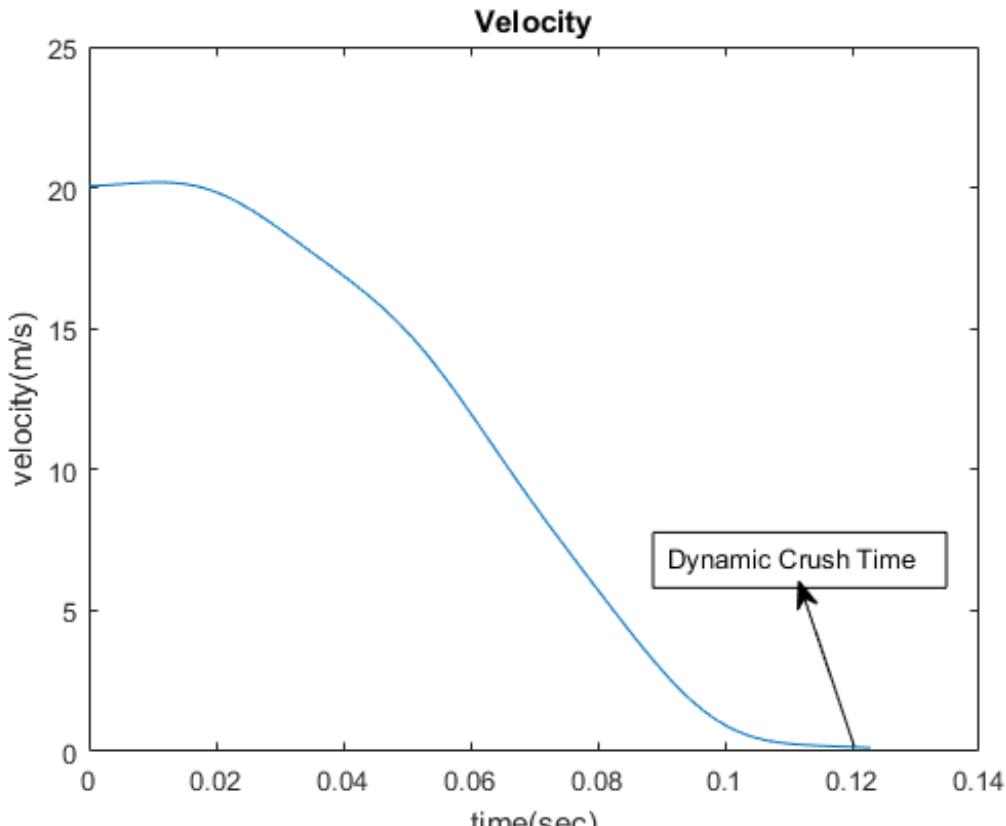
Acceleration and Displacement

Curve Fitting in Matlab



Velocity

Dynamic Crush time of 0.12 seconds



- Initial velocity is greater than the actual velocity (15 m/s).
- This is because of using the raw data from IMUs

Estimating Kelvin Parameters from raw data

From the graphs, dynamic crush time is 0.12 seconds

$$t_c = \frac{C}{v_0} = \frac{1.313}{20.1894} = 0.0650$$

$$\frac{t_c}{t_m} = \frac{0.0650}{0.12} = 0.5420$$

From equation 4.2 in [ref](#), we estimate $\zeta = 0.189$

From equation 4.3, $f = 1.86 \text{ Hz}$

Estimating Kelvin Parameters from raw data

- $k = 4\pi^2 f^2 m = 3223926 \frac{N}{m}$
- $c = 4\pi f \zeta m = 10598 \frac{Ns}{m}$

Estimation of maximum chest deceleration of the occupant

- $ESW = 0.5 \frac{v^2}{c} = 153.35 \frac{m}{s^2}$
- $DAF = 1 + \sqrt{1 + (2\pi f t^*)^2} = 2$
- $Maximum\ occupant\ chest\ deceleration = DAF * ESW = 306.72\ m.s^{-2}$
- $a_0 = 31.26g$

Estimating model parameters using matlab identification toolbox

EOM of the kelvin model: $mu(t) = m \frac{d^2y}{dt^2} + c \frac{dy}{dt} + ky$

Applying Laplace transformation to the above equation gives

$$G(s) = \frac{Y(s)}{U(s)} = \frac{m}{ms^2 + cs + k}$$

- With $U(s)$ or acceleration as the input and $Y(s)$ or relative displacement as the output

Estimating model parameters using matlab identification toolbox

- Using the raw input data(acceleration) and estimated displacement, we use system identification tool box to find the transfer function.
- By comparing the coefficients, we get the kelvin parameter models(k and c)
- From the [reference](#), the following relations have been presented.

$$T = \sqrt{\frac{m}{k}} \quad \text{and} \quad K = \frac{m}{k} \quad \text{and} \quad \zeta = \frac{c}{2m\omega_e}$$

Estimating model parameters using matlab identification toolbox

K = -2.7

T = 0.28287

Damping coefficient = 61.7734

The screenshot shows the 'Process Models' window in MATLAB. The left panel displays the transfer function $\frac{K}{(1 + (2 \text{ Zeta } T_w) s + (T_w s)^2)}$, poles (2, Underdamped), and zero/delay/integrator options. The right panel shows estimated parameters: K = -2.7812, T_w = 0.28287, Zeta = 61.7734, Tp3 = 0, Tz = 0, Td = 0. The 'User-defined' radio button is selected for initial guess. The bottom panel includes settings for disturbance model (None), focus (Simulation), covariance (Estimate), and regularization.

Par	Known	Value	Initial Guess	Bounds
K	<input type="checkbox"/>	-2.7812	0	[-Inf Inf]
Tw	<input type="checkbox"/>	0.28287	Auto	[0 10000]
Zeta	<input type="checkbox"/>	61.7734	Auto	[0 Inf]
Tp3	<input type="checkbox"/>	0	0	[0 Inf]
Tz	<input type="checkbox"/>	0	0	[-Inf Inf]
Td	<input type="checkbox"/>	0	0	[0 Inf]

Initial Guess

Auto-selected

From existing model:

User-defined

Disturbance Model: Initial condition:

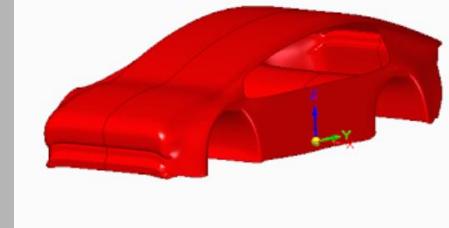
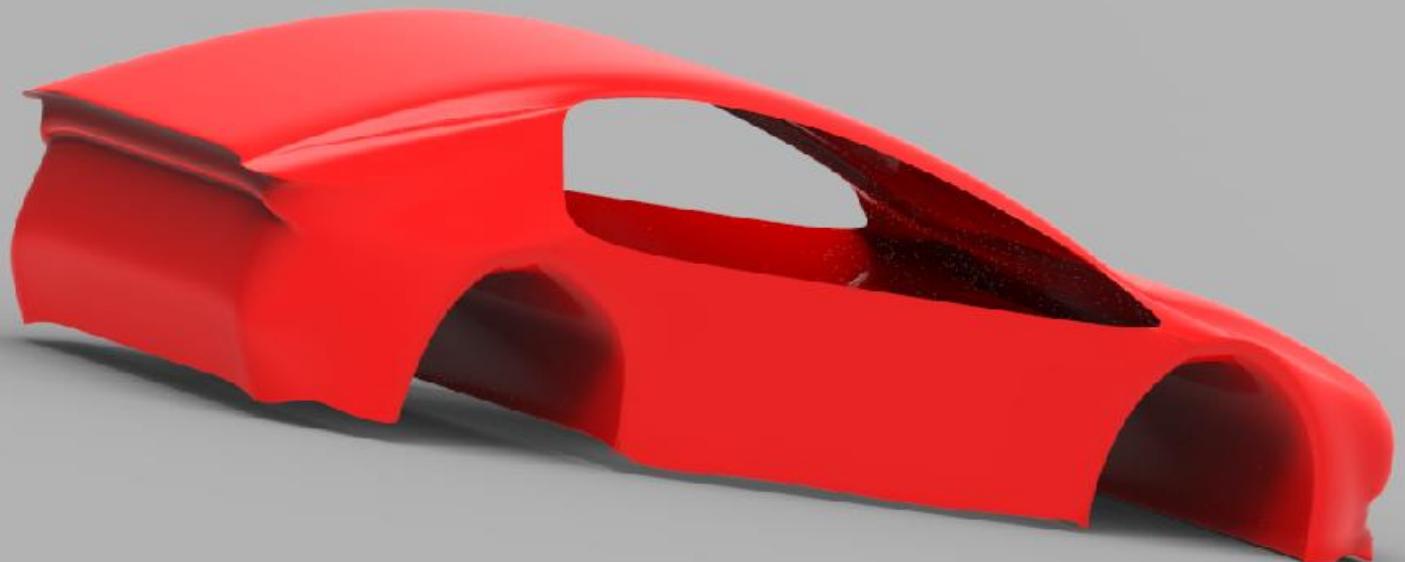
Focus: Covariance:

Display progress

Name:

References:

1. <https://www-nrd.nhtsa.dot.gov/database/VSR/veh/TestDetail.aspx?LJC=11821>
2. <https://www-nrd.nhtsa.dot.gov/database/VSR/veh/instrumentationinfo.aspx?LJC=11821>
3. [Mathematical Modeling and Analysis of a Vehicle Crash](#)



Thank you