DYNAMIC FRICTION MODELS

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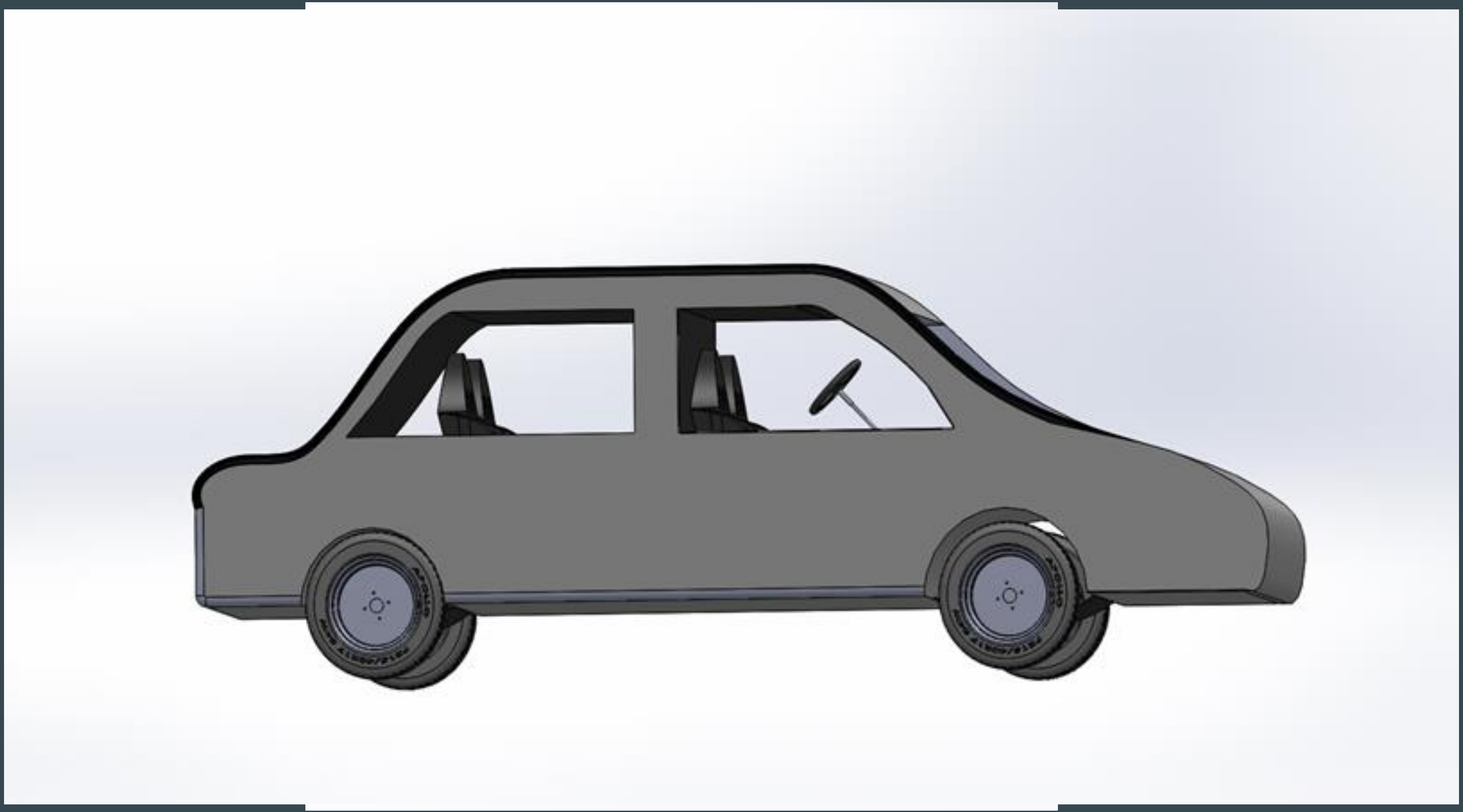
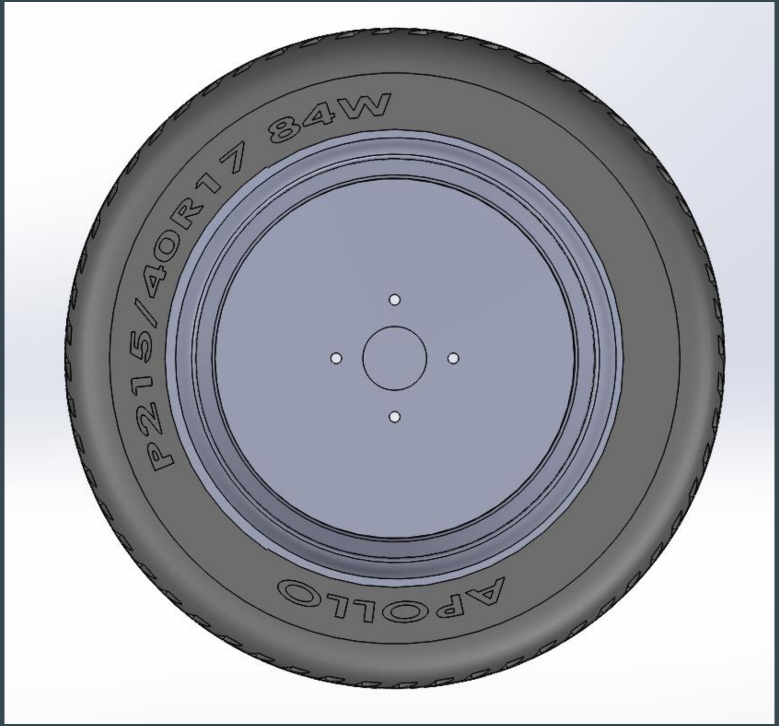
Introduction

- Vehicle Motion primarily determined by friction forces transferred to the road by the tires.
- Tire friction model forms the basis for the development of tire road friction estimators and various vehicle dynamic control systems.
- Traction control aims to achieve maximum torque transfer from the wheel axle to forward acceleration.
- Anti-lock braking systems (ABS) prohibit wheel lock and skidding during braking by regulating the pressure applied on the brakes, thus increasing lateral stability and steerability, especially during wet and icy road conditions

CAD Model



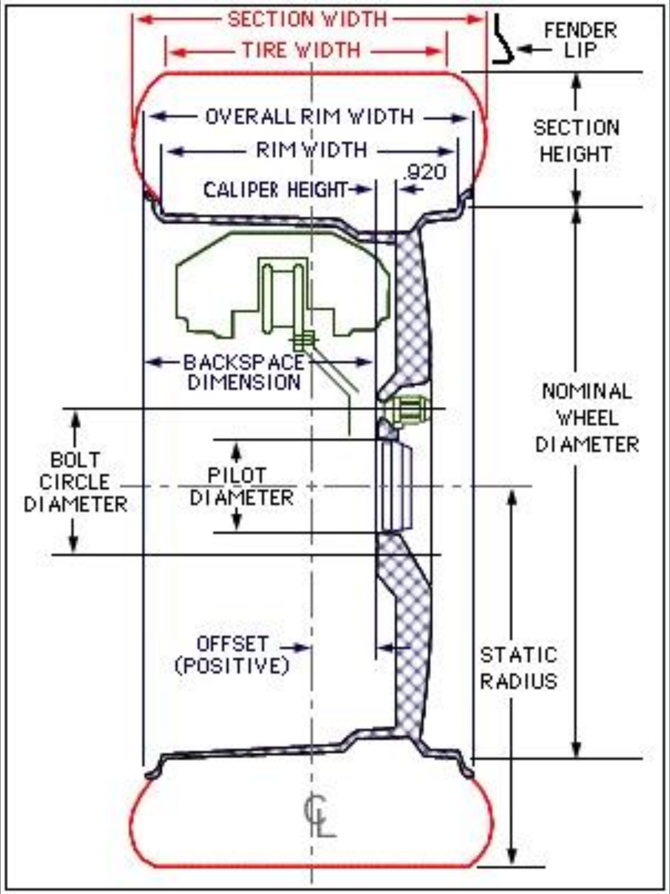
Tire



Tire

Tire : P215 / 40R17 84W

P: Passenger Car
215: Tire Width
40: Aspect Ratio
R: Rim Radius = 17
84: Load Index
W: Speed symbol

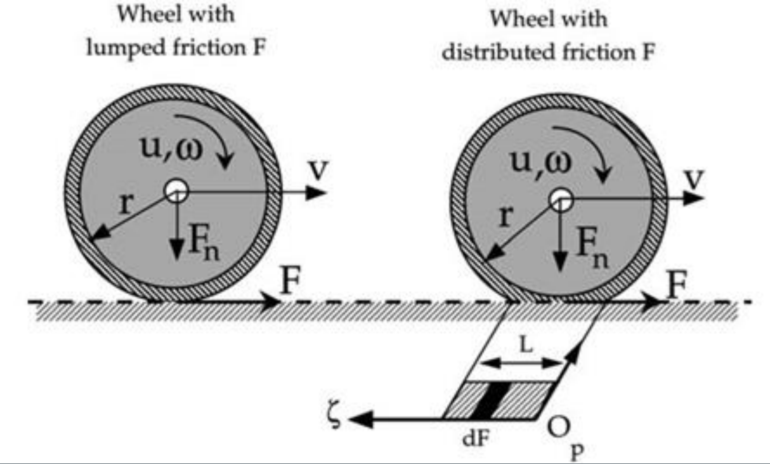


Static Model vs Dynamic Model

- Static friction models are appropriate when we have steady-state conditions for the linear and angular velocities.
- Static model can be described by brush model expressions or empirical formulae (e.g., ‘‘magic’’ formula)
- Magic Formula : $R(k) = d \cdot \sin\{c \cdot \arctan[b\,(1-e)\,k + e \cdot \arctan(bk)]\}$
- Dynamic friction models attempt to capture the transient behavior of the tire-road contact forces under time-varying velocity conditions.
- Dynamic models can be formulated either as lumped or as distributed models

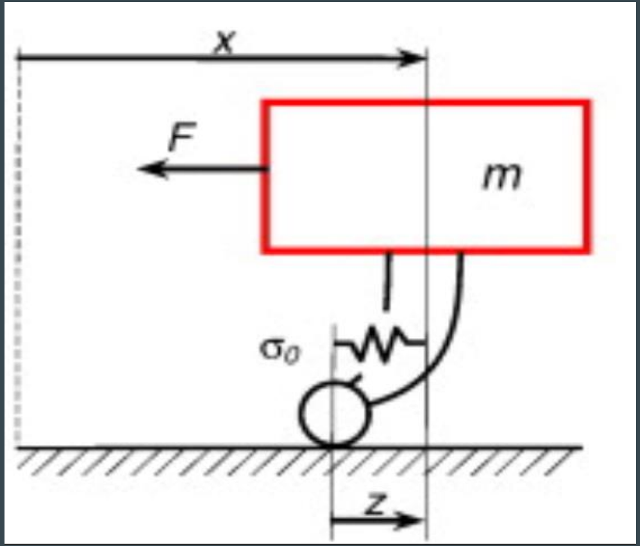
Lumped vs Distributed Model

- A lumped friction model assumes a point tire-road friction contact. As a result, the mathematical model describing such a model is an ordinary differential equations that can be easily solved by time integration.
- Distributed friction models, assume the existence of a contact patch between the tire and the ground with an associated normal pressure distribution. This formulation results in a partial differential equation, that needs to be solved both in time and space.



Dahl Model

- Developed for simulating control systems with friction.
- The starting point of Dahl's model is the stress-strain curve in classical solid mechanics.
- When subject to stress, the friction force increases gradually until rupture occurs.
- Designed to simulate a symmetrical hysteresis loops



Equation used

- $\frac{dF}{dx} = \sigma_0(1 - F * sgn(v_r))$

Where F : friction force

x : Relative displacement

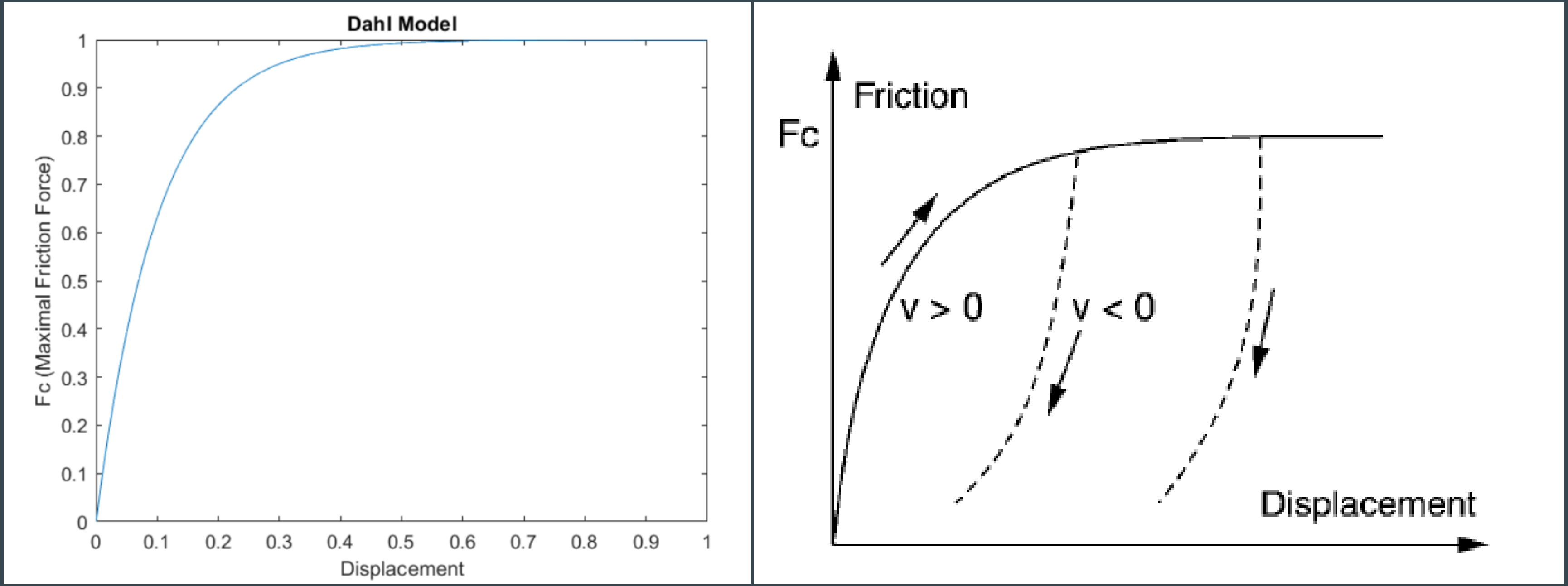
σ_0: stiffness coefficient

v_r: Relative velocity

BC's

F = 0 at x = 0

ODE45 was used for solving the model.



Friction force as a function of displacement for the Dahl's Model

Lumped LuGre Model

An extension of the Dahl model that includes the Stribeck effect.

A lumped form makes the model more suitable for the development and implementation of on-line estimation and control algorithms. The main objective of the lumped model is to be able to capture the steady-state behavior of the distributed model exactly

Equation Used:

$$\frac{dz}{dt} = v_r - \left(\sigma_0 \frac{v_r}{g(v_r)}\right) z \quad ;$$

$$g(v_r) = \mu_c + (\mu_s - \mu_c) e^{-abs\left(\frac{v_r}{v_s}\right)^{\alpha}}$$

$$F = (\sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 v_r) F_n$$

BC's

F = 0 at $\frac{dz}{dt}$ = 0 at t = 0

$\sigma_o =$ *The rubber longitudinal lumped stiffness.*

σ_1 = *The rubber longitudinal lumped damping*

σ_2 = *The viscous relative damping*

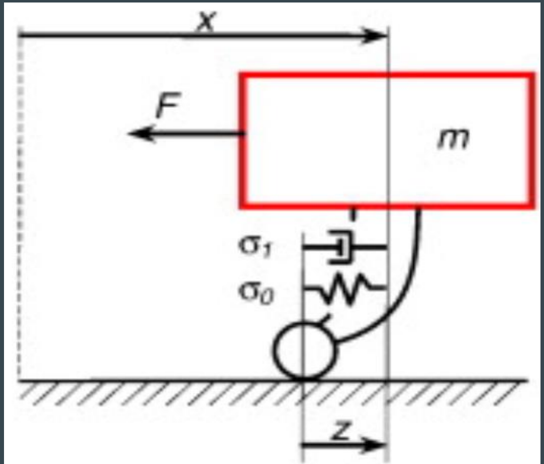
μ_c = *The normalized Coulomb friction*

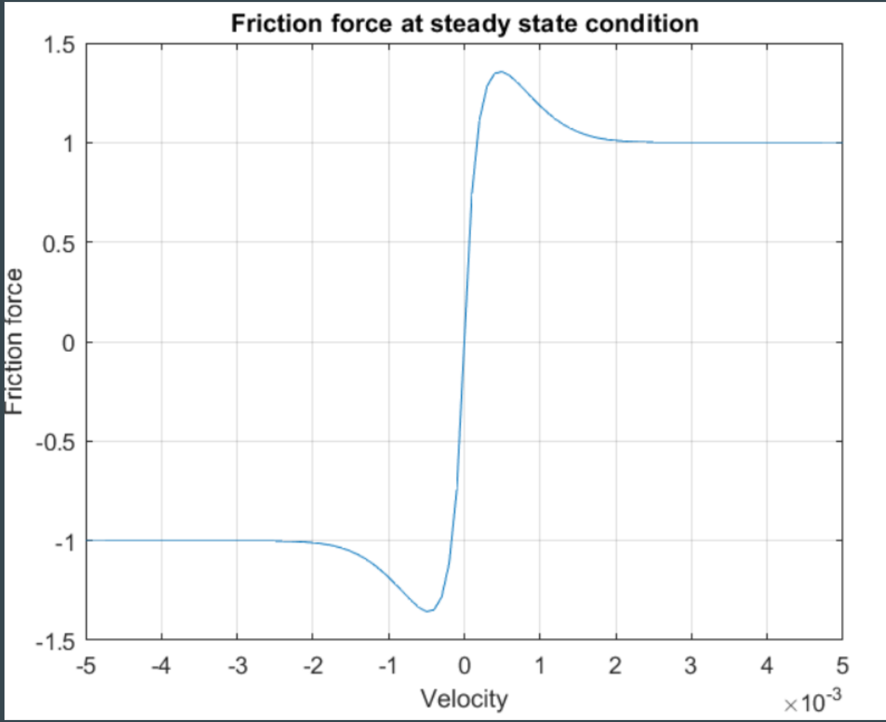
μ_s = *The normalized static friction*

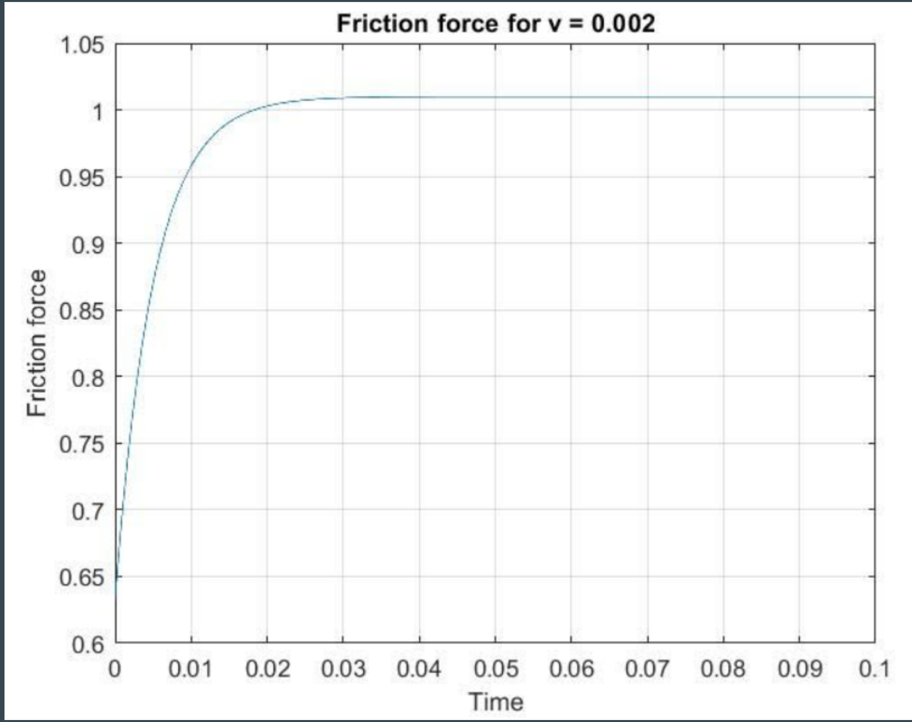
F_n = *Normal force*

$v_r = r\omega - v$ (relative velocity)

z = *internal friction state*




Friction force at steady state condition
Friction force
Velocity
×10^-3
1.5
1
0.5
0
-0.5
-1
-1.5
-5
-4
-3
-2
-1
0
1
2
3
4
5


Friction force for v = 0.002
Friction force
Time
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

Distributed LuGre Model

- Distributed models assume the existence of an area of contact (or patch) between the tire and the road. This patch represents the projection of the part of the tire that is in contact with the road.
- The LuGre model is applied here, because of its apparent accuracy, and simple and compact mathematical form.
- Equation used

$$dz(\zeta,t)/dt = v_r - (\sigma_o z|v_r|)/(g\, v_r)$$

$$F = \int_o^l dF(\zeta,t)$$

$$dF(\zeta,t) = (\sigma_o z(\zeta,t) + \sigma_1 dz(\zeta,t)/dt + \sigma_2 v_r)\, dF_n(\zeta,t)$$

$$dF_n(\zeta) = f_n(\zeta) d\zeta$$

$$\boldsymbol{dF(\zeta,t) = \int_0^L (\sigma_o z(\zeta,t) + \sigma_1 dz(\zeta,t)/dt + \sigma_2 v_r)\, f(\zeta,t) d\zeta}$$

$$dz(\zeta,t)/dt = (dz/d\zeta)(d\zeta/dt) + dz/dt$$

Final equation

$$(dz(\zeta,t)/d\zeta) * (|r\omega|) + (dz(\zeta,t)/dt) = v_r - [(\sigma_o z|v_r|) * (z(\zeta,t))]/(g\, v_r)$$

Steady state equation

$$dz(\zeta,t)/d\zeta = (1/|r\omega|) * [v_r - (\sigma_o z|v_r|)/(g\, v_r)]$$

Here $\zeta = |r\omega|$

$f_n(\zeta)$= Normal force density function

$dF_n(\zeta,t)$= differential friction force in element $d\zeta$

BC's

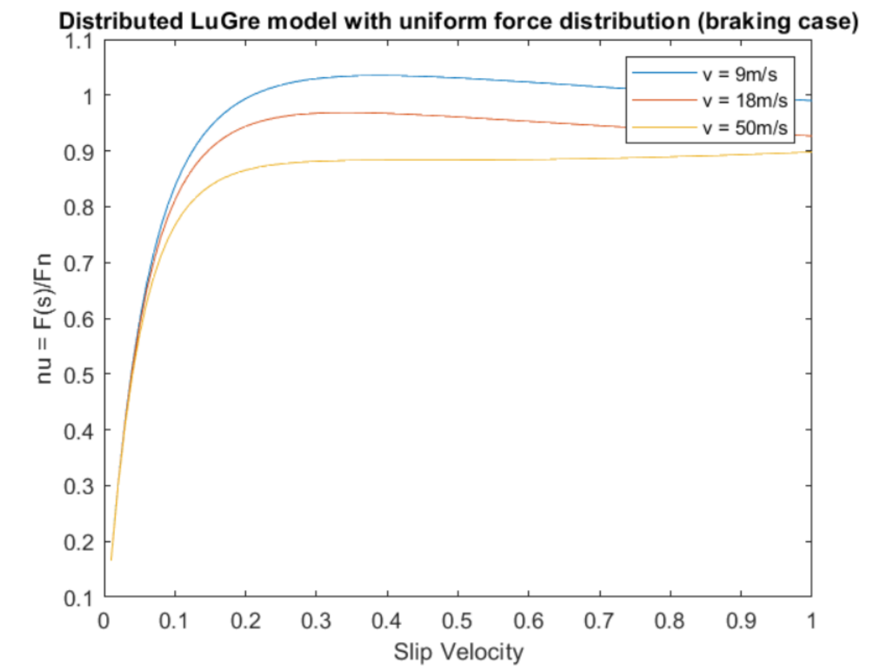
$$z(\zeta,0) = z(0,t) = 0$$

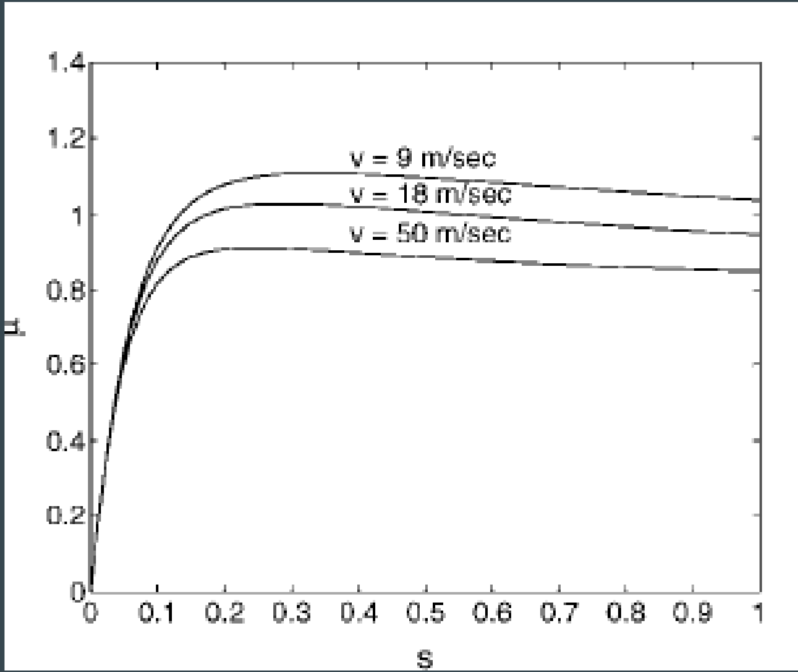
Distributed LuGre Model

Uniform distribution $f_n = F_n/L, 0 \leq \zeta \leq L$

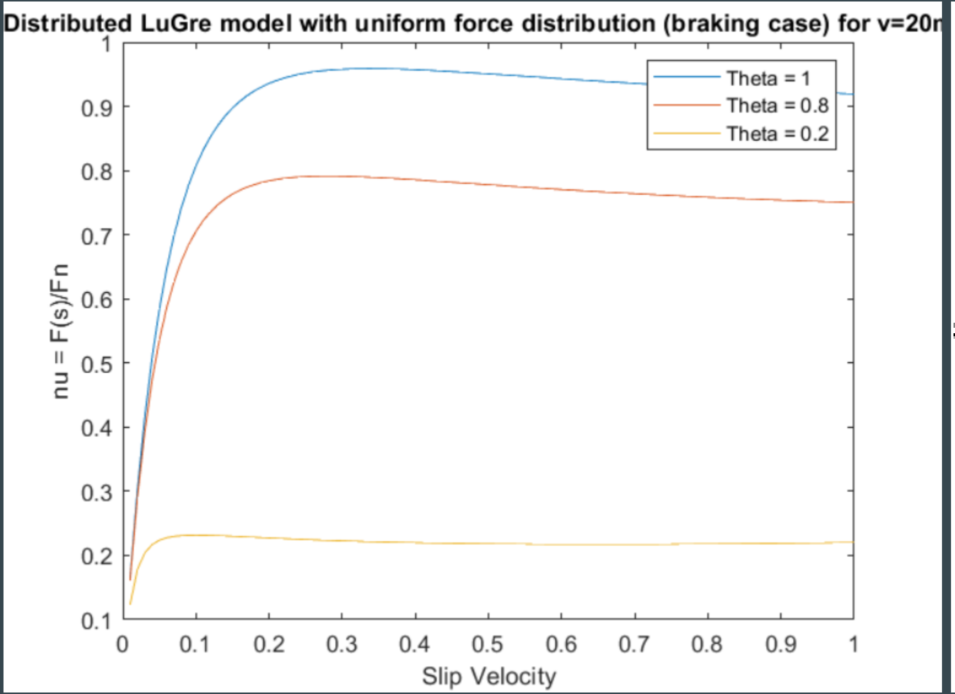
Braking case

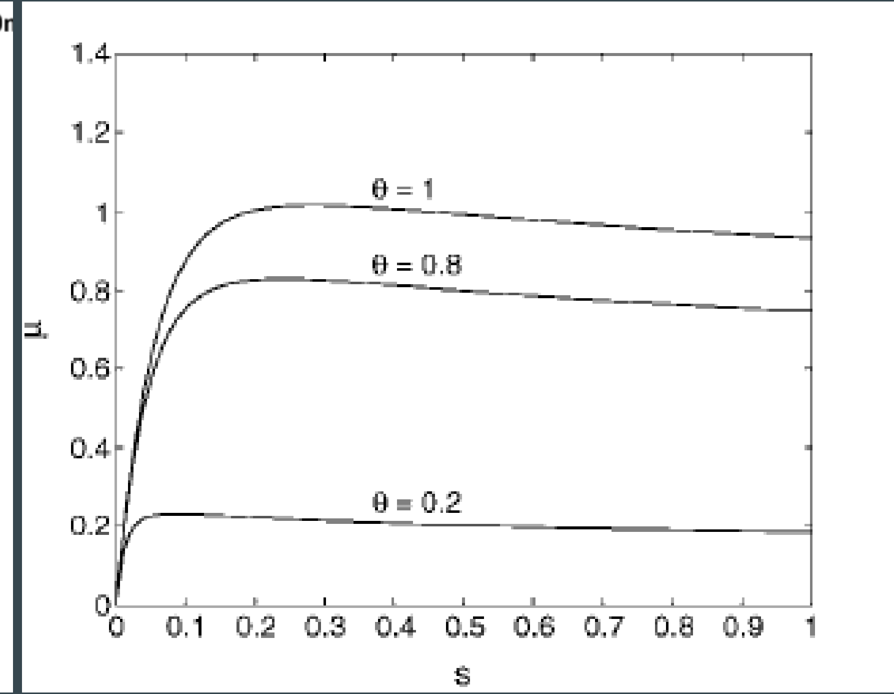
$$\mu = F_b(s)/F_n = (sgn(v_r)g(s))[1 + (g(s)\,|1+s|\,(e^{-\sigma_0 L|s|/g(s)|1+s|} - 1)/\sigma_0 L\,|s|)] + \sigma_2 vs$$





Distributed LuGre Model

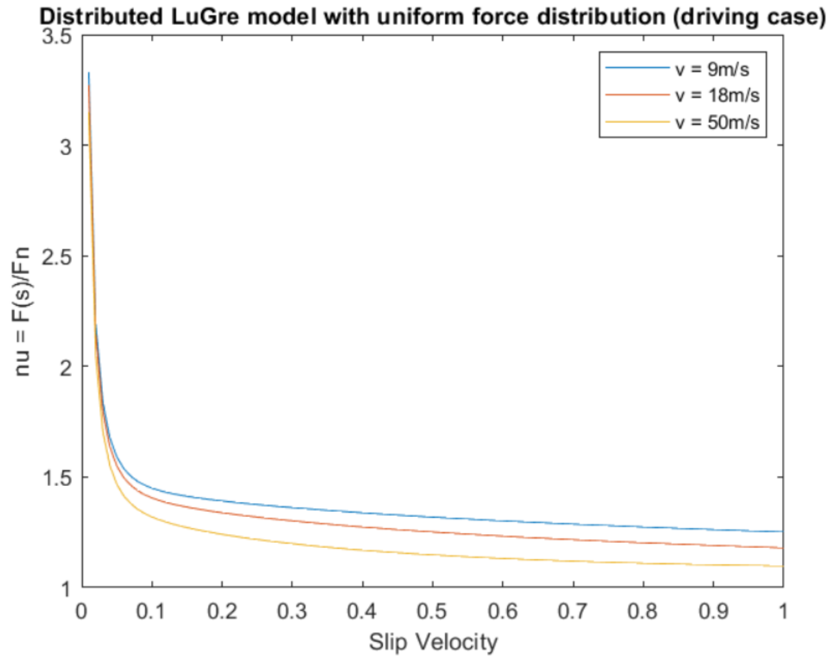




Distributed LuGre Model

Driving case

$$\mu = F_b(s)/F_n = (sgn(v_r)g(s))[1 + (g(s)\,|1+s|\,(e^{-\sigma_0 L|s|/g(s)} - 1)/\sigma_0 L\,|s|)] + \sigma_2 r \omega s$$



GUI DEMO

Thank You

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