Term project in Vehicle Dynamics Course

DIRECTIONAL STABILITY OF ARTICULATED VEHICLE

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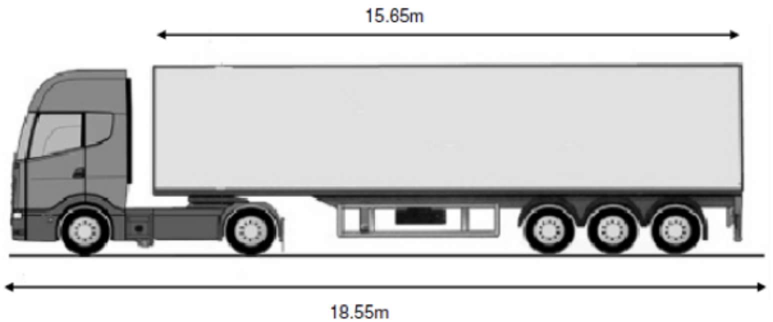
OBJECTIVES

- Introduction
- Stability of single vehicle
- Stability of one articulated vehicle
- Results

INTRODUCTION

- Articulated Vehicle ?

A vehicle which has a permanent or semi-permanent pivot joint in its construction, allowing the vehicle to turn more sharply.



- Directional stability ?

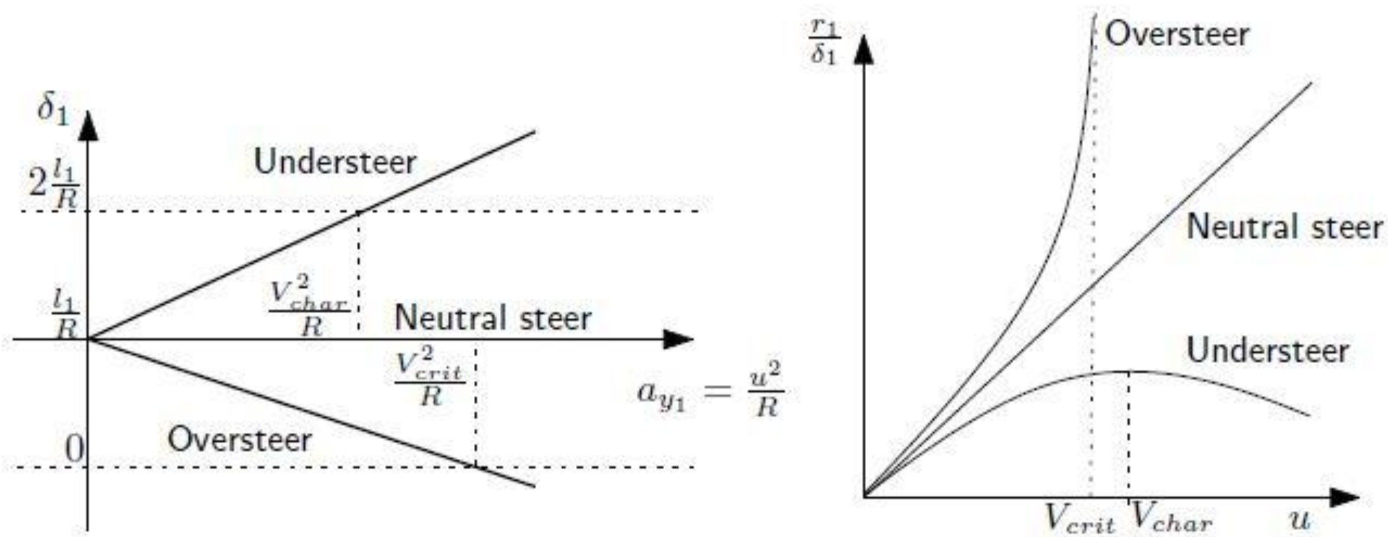
Stability of a moving body or vehicle about an axis which is perpendicular to its direction of motion.

Eg : Vehicle during yaw motion.

Factors influencing the stability?

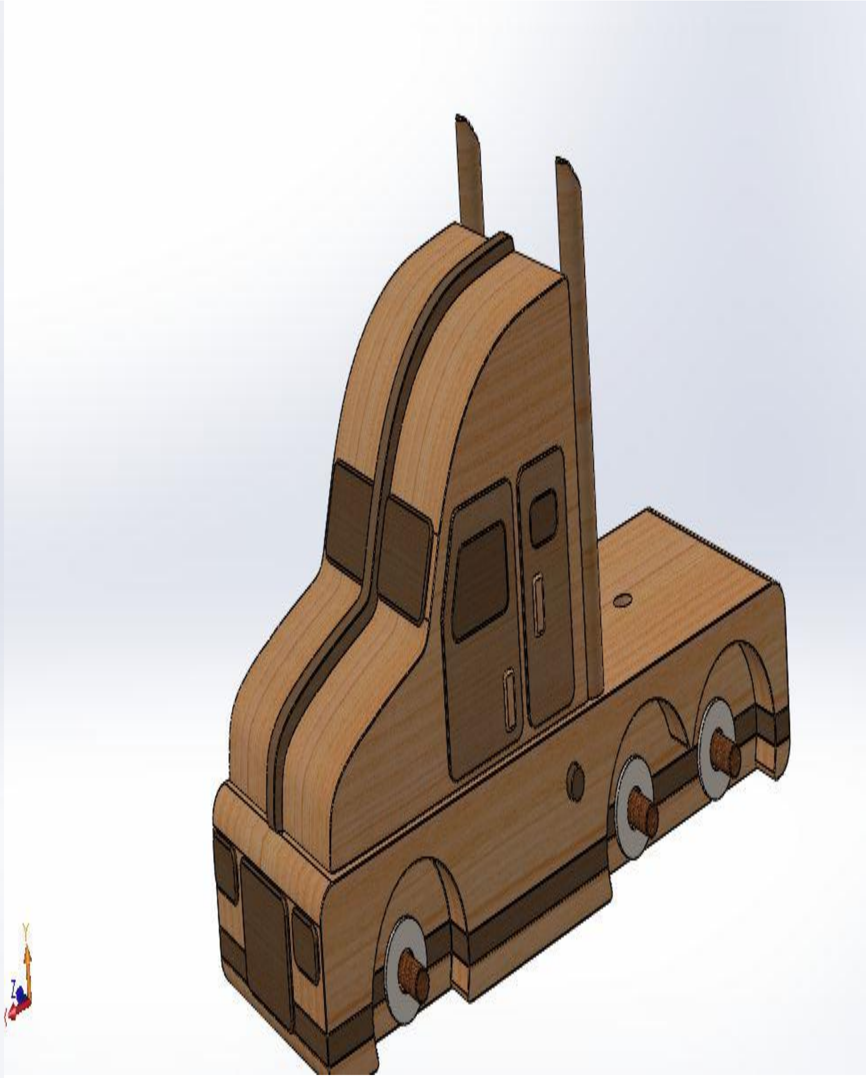
- Steering characteristics
  - Under steering.($\alpha_1 > \alpha_2$)
  - Neutral steering.($\alpha_1 = \alpha_2$)
  - Over steering. ($\alpha_1 < \alpha_2$)

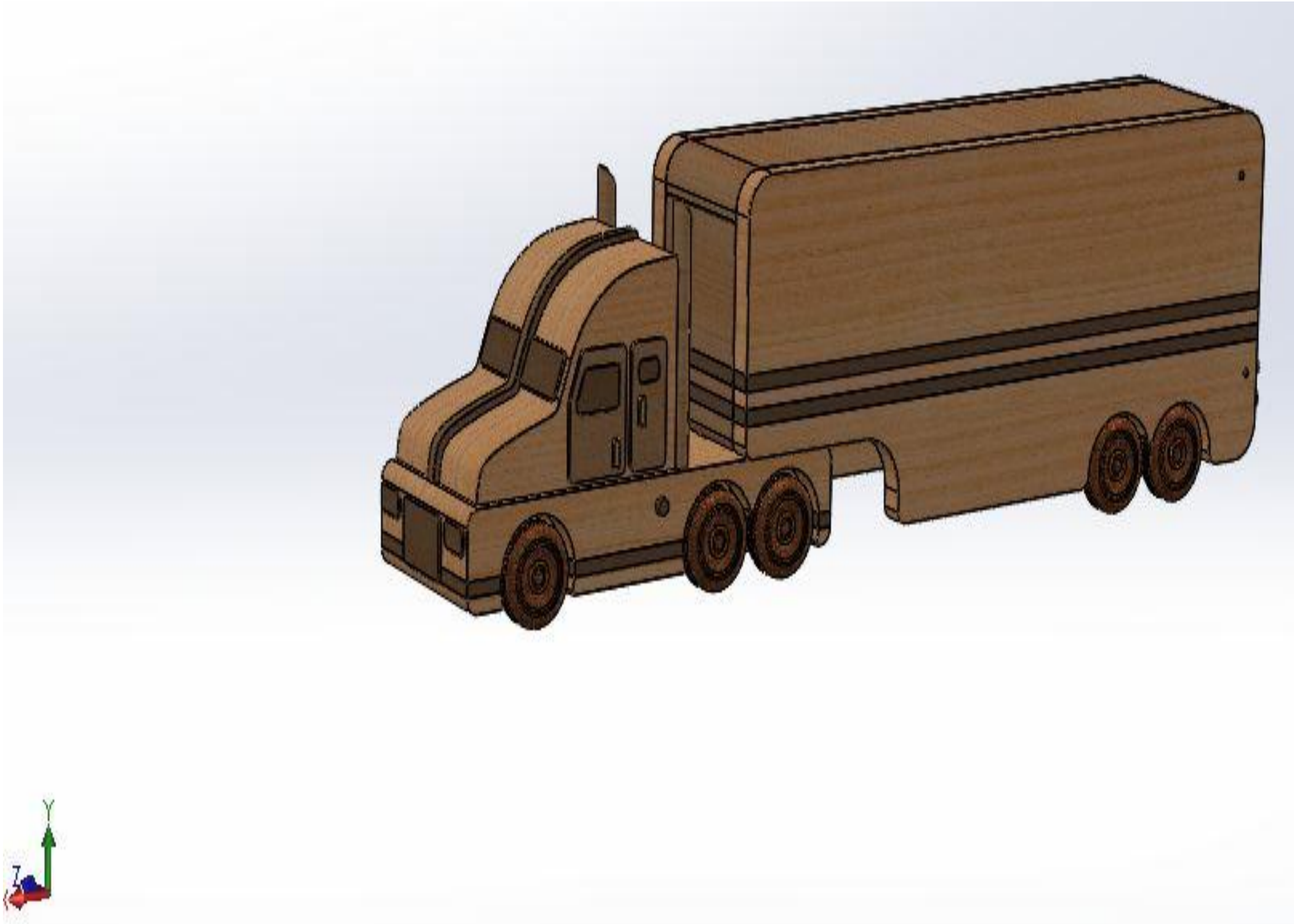
  Those are depends on front(α_1) and rear(α_2) tyre slip angles.



(a) Handling diagram (b) Yaw gain diagram

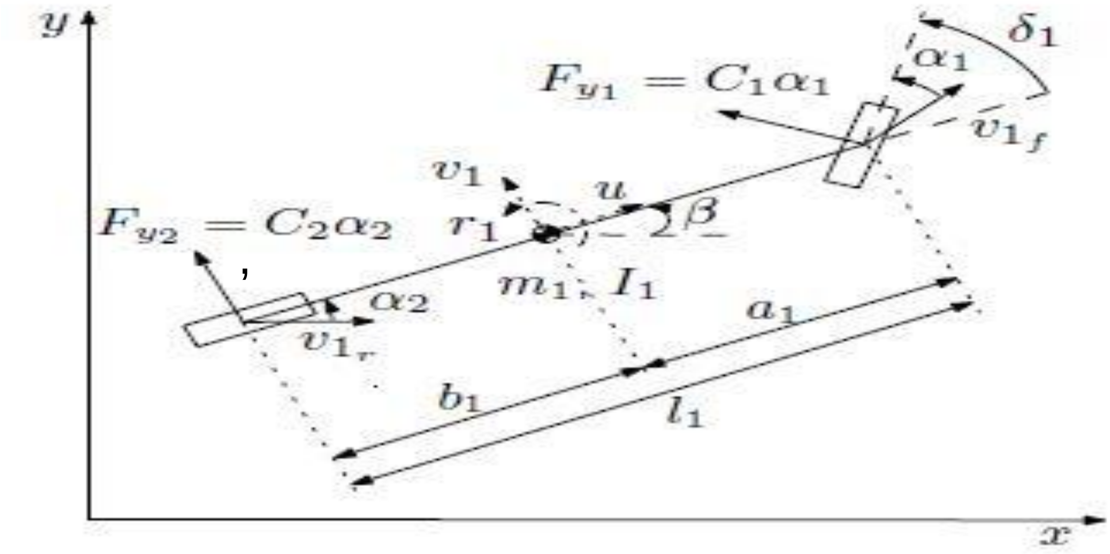
CAD MODEL





STABILITY OF SINGLE VEHICLE

Let us consider vehicle is moving with forward velocity u, then Equation of motion can be written as,



$$\begin{bmatrix} m_1 & 0 \\ 0 & I_1 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{r}_1 \end{bmatrix} = -\frac{1}{u}\begin{bmatrix} C & C_{s_1} + m_1 u^2 \\ C_{s_1} & C_{q_1^2} \end{bmatrix} \begin{bmatrix} v_1 \\ r_1 \end{bmatrix} + \begin{bmatrix} C_1 \\ a_1 C_1 \end{bmatrix} \delta_1$$

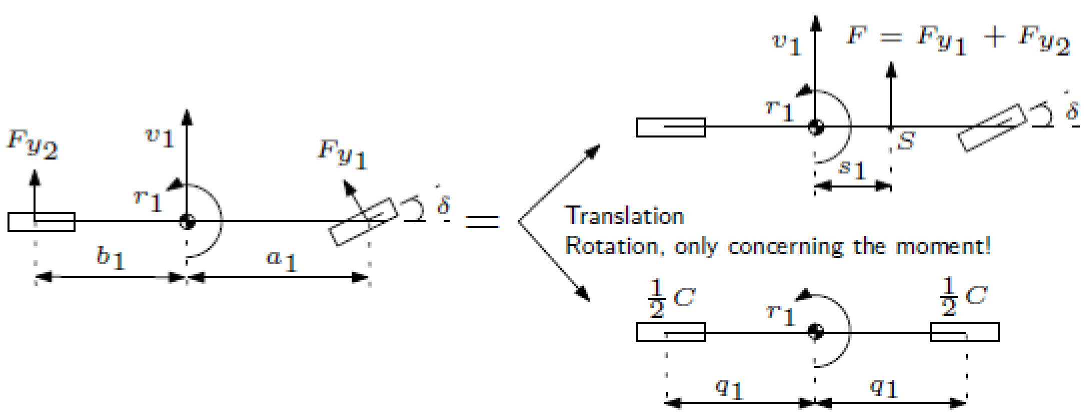
$v_1 = lateral\ velocity\ at\ centre\ of\ gravity$

$r_1 = Yaw\ velocity$

$$C = C_1 + C_2$$
$$C_{s1} = a_1 C_1 - b_1 C_2$$
$$C_{q1}^2 = a_1^2\, C_1 + b_1^2 C_2$$
$$I_1 = m_1 k_1^2$$

with,

C : the total cornering stiffness,
s_1 : the distance along the longitudinal axis of the vehicle from the centre of gravity to the neutral steer point *S*. The neutral steer point is the point on the vehicle where an external side force does not impose a yaw angle to the vehicle. This means that $s1 = 0$ corresponds to a neutrally steered vehicle, $s1 < 0$ to an under steered vehicle and $s1 > 0$ to an over steered vehicle,
q_1 : the length which corresponds to an average moment arm,
k_1 : the radius of gyration.



By solving the above differential equation, we get the characteristic equation of a single vehicle is given by:

$$c_0\lambda^2 + c_1\lambda + c_2 = 0$$

with,

$c_0 = 1$

$c_1 = C\dfrac{k_1^2 + q_1^2}{m_1 k_1^2 u}$

$c_2 = \dfrac{C}{m_1 k_1^2 u^2}\left(\dfrac{C(q_1^2 - s_1^2}{m_1} - s_1 u^2\right), since q_1^2 - s_1^2 = l_1^2 \dfrac{C_1}{C}\dfrac{C_2}{C}$

By Hurwitz criterion , stability of vehicle can be studied,

Therefore for stability,

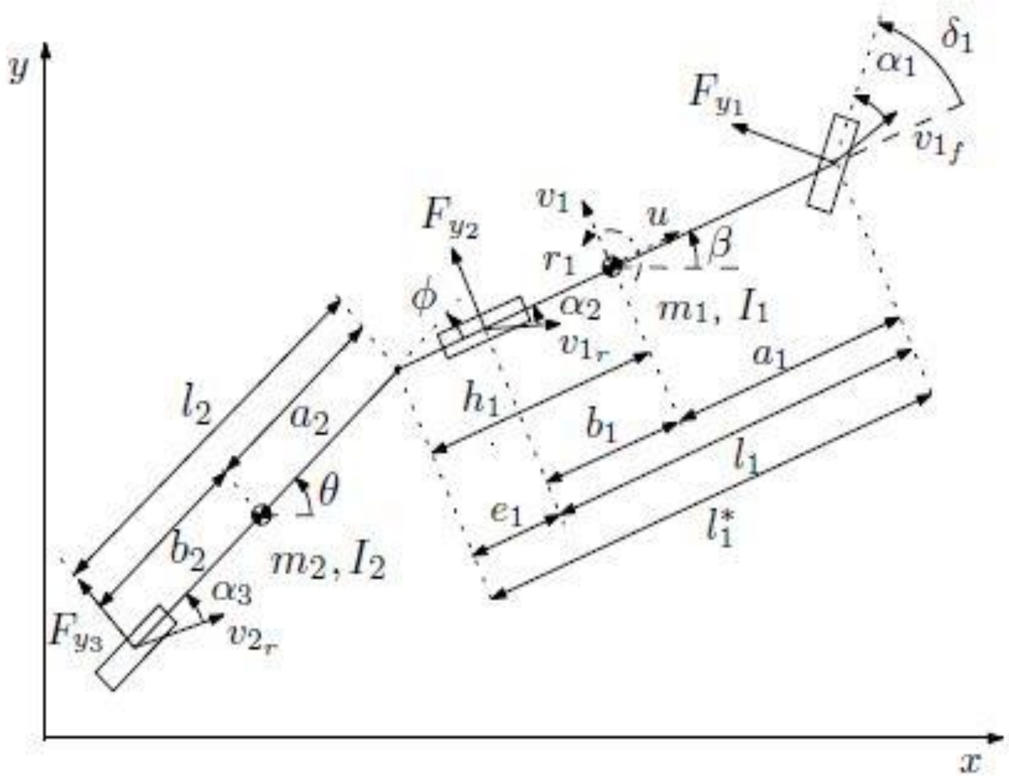
$c_0 > 0$: always fulfilled

$c_1 > 0$: always fulfilled

$$c_2 > 0: \begin{cases} s_1 < 0: always\ fulfilled \\ s_1 = 0: u < \infty \Rightarrow always\ fulfilled \\ s_1 > 0:\ u < V_{crit} = \sqrt{\dfrac{C(q_1^2 - s_1^2)}{m_1 s_1}} \end{cases}$$

$H_1 > 0: always\ fulfilled$

Stability of one articulated vehicle



There are two case,

1) $e1 < 0$, Hitch point is located in front of the rear axle of the towing vehicle.
Ex : Tractor-semitrailer

2) $e1>0$, Hitch point is located behind the rear axle of the towing vehicle.
Ex : Truck-centre axle trailer

Equation of motion can be written as,

$$\begin{bmatrix} m_1+m_2 & -m_2(h_1+a_2) & -m_2a_2 & 0 \\ -m_2h_1 & I_1+m_2h_1(h_1+a_2) & m_2h_1a_2 & 0 \\ -m_2a_2 & I_2+m_2a_2(h_1+a_2) & I_2+m_2a_2^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{r}_1 \\ \ddot{\phi} \\ \dot{\phi} \end{bmatrix} = -\frac{1}{u}$$

$$\begin{bmatrix} C+C_3 & Cs_1-C_3(h_1+l_2)+(m_1+m_2)u^2 & -C_3l_2 & -C_3u \\ Cs_1-C_3h_1 & Cq_1^2+C_3h_1(h_1+l_2)-m_2h_1u^2 & C_3h_1l_2 & C_3h_1u \\ -C_3l_2 & C_3l_2(h_1+l_2)-m_2a_2u^2 & C_3l_2^2 & C_3l_2u \\ 0 & 0 & -u & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ r_1 \\ \dot{\phi} \\ \phi \end{bmatrix} + \begin{bmatrix} C_1 \\ a_1C_1 \\ 0 \\ 0 \end{bmatrix} \delta_1,$$

where,

$v_1 = lateral\ velocity\ at\ centre\ of\ gravity$

$r_1 = Yaw\ velocity$

$\phi = Relative\ articulation\ angle\ between\ the\ trailer\ and\ towing\ vehicle$

$\theta = \beta + \phi$

By solving above differential equation we arrive at the characteristic equation as:

$$c_0\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4 = 0$$

Where,

$$\left.\begin{aligned}
c_0 = {} & m_1 m_2 u\left[m_1 k_1^2(k_2^2+a_2^2)+m_2 k_2^2(k_1^2+h_1^2)\right] \\
c_1 = {} & m_1 m_2\left[C_3(b_2^2+k_2^2)(k_1^2+h_1^2)+C(a_2^2+k_2^2)(k_1^2+q_1^2)\right] \\
& +m_1^2 C_3 k_1^2 l_2^2+m_2^2 C k_2^2(q_1^2+h_1^2+2h_1 s_1) \\
c_2 = {} & \frac{1}{u}\left[u^2\left\{m_1 C_3 l_2\left(m_1 k_1^2+m_2\frac{b_2}{l_2}(k_1^2+h_1^2)\right)-m_2 C(s_1+h_1)\left(m_2 k_2^2+m_1\frac{s_1}{s_1+h_1}(k_2^2+a_2^2)\right)\right\}\right. \\
& \left.+m_1 C C_3 l_2^2(k_1^2+q_1^2)+m_2 C\left\{C_3(q_1^2+h_1^2+2h_1 s_1)(k_2^2+b_2^2)+C(q_1^2-s_1^2)(k_2^2+a_2^2)\right\}\right] \\
c_3 = {} & \frac{CC_3 l_2}{u^2}\left[u^2\left\{m_1(q_1^2+k_1^2-s_1 l_2)+m_2\frac{b_2}{l_2}(q_1^2+h_1^2+2h_1 s_1)-m_2\frac{1}{l_2}(s_1+h_1)(b_2^2+k_2^2)\right\}\right. \\
& \left.+Cl_2(q_1^2-s_1^2)\right] \\
c_4 = {} & \frac{CC_3 l_2}{u}\left[-u^2\left\{m_2\frac{b_2}{l_2}(s_1+h_1)+m_1 s_1\right\}+C(q_1^2-s_1^2)\right]
\end{aligned}\right\}.$$

$$\left.\begin{aligned}
Cs_2 &= l_1^* C_1 + e_1 C_2 \\
s_2 &= h_1 + s_1 \\
Cq_2^2 &= l_1^{*2} C_1 + e_1^2 C_2 \\
q_2^2 &= q_1^2 - s_1^2 + s_2^2 \\
&= q_1^2 + h_1^2 + 2h_1 s_1
\end{aligned}\right\},$$

l_1^* = Distance between the front axle and hitch point

By Hurwitz criterion, stability of vehicle can satisfied under the conditions of,

$$\left.\begin{array}{ll} c_0 > 0 & : \text{always fulfilled} \\ c_1 > 0 & : \text{always fulfilled} \\ c_2 > 0 & : \begin{cases} D_{c_2} < 0 & : \text{always fulfilled} \\ D_{c_2} = 0 & : u < \infty \Rightarrow \text{ always fulfilled} \\ D_{c_2} > 0 & : u < V_{crit,c2} \end{cases} \\ c_3 > 0 & : \begin{cases} D_{c_3} < 0 & : \text{always fulfilled} \\ D_{c_3} = 0 & : u < \infty \Rightarrow \text{ always fulfilled} \\ D_{c_3} > 0 & : u < V_{crit,c3} \end{cases} \\ c_4 > 0 & : \begin{cases} D_{c_4} < 0 & : \text{always fulfilled} \\ D_{c_4} = 0 & : u < \infty \Rightarrow \text{ always fulfilled} \\ D_{c_4} > 0 & : u < V_{crit,c4} \end{cases} \\ H_1 > 0 & : \text{always fulfilled} \\ H_3 > 0 & : c_1 c_2 c_3 - c_0 c_3^2 - c_4 c_1^2 > 0 \end{array}\right\},$$

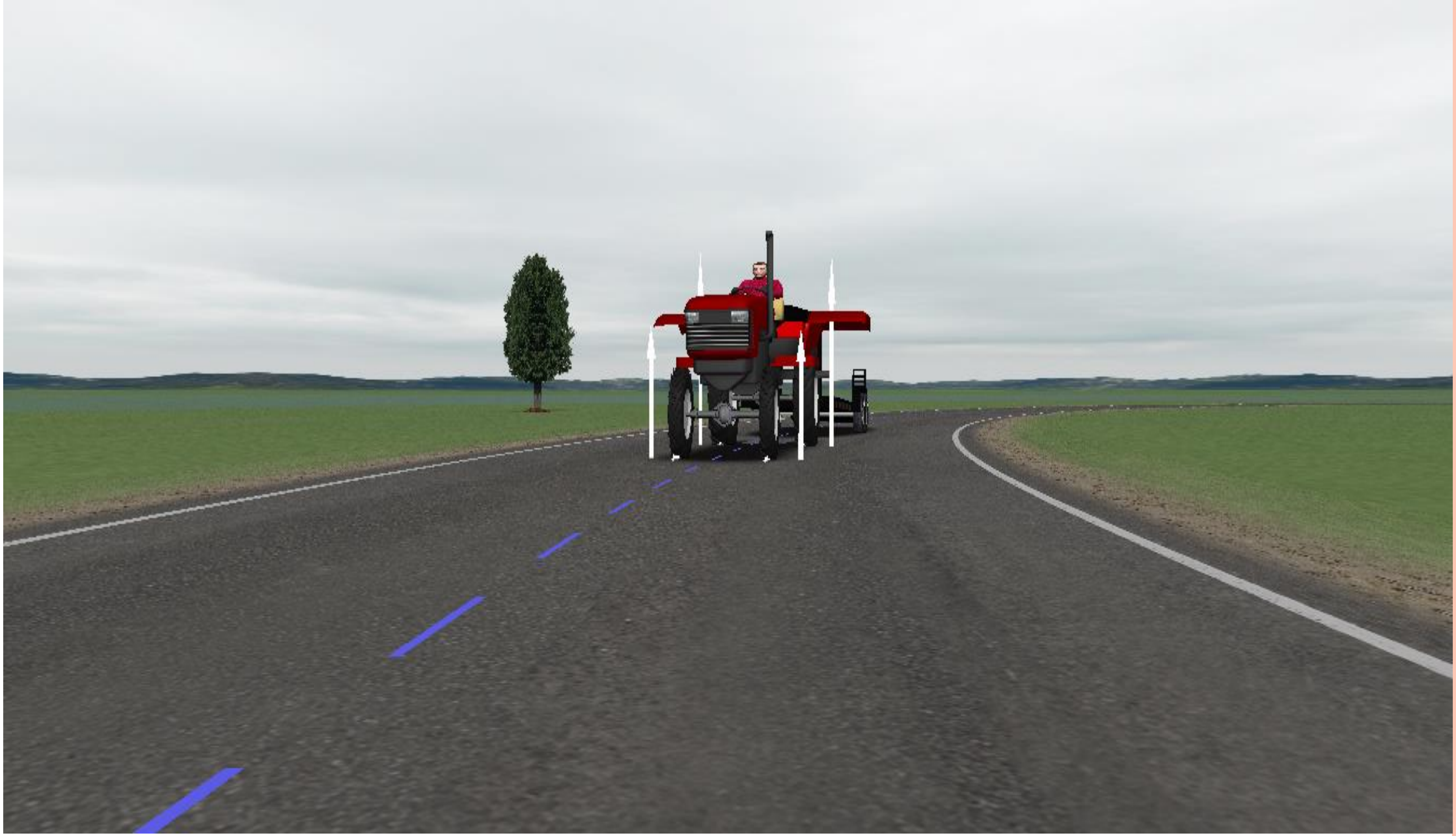
With,

$$\left.\begin{array}{lll} V_{crit,c_2} & = & \sqrt{\frac{m_1 C C_3 l_2^2 (k_1^2+q_1^2)+m_2 C\left\{C_3 q_2^2(k_2^2+b_2^2)+C(q_1^2-s_1^2)(k_2^2+a_2^2)\right\}}{-m_1 C_3 l_2\left\{m_1 k_1^2+m_2\frac{b_2}{l_2}(k_1^2+h_1^2)\right\}+m_2 C s_2\left\{m_2 k_2^2+m_1\frac{s_1}{s_2}(k_2^2+a_2^2)\right\}}} \\ V_{crit,c_3} & = & \sqrt{\frac{C l_2(q_1^2-s_1^2)}{-m_1(q_1^2+k_1^2-s_1 l_2)-m_2\frac{b_2}{l_2}q_2^2+m_2\frac{s_2}{l_2}(b_2^2+k_2^2)}} \\ V_{crit,c_4} & = & \sqrt{\frac{C(q_1^2-s_1^2)}{m_2\frac{b_2}{l_2}s_2+m_1 s_1}}, \end{array}\right\},$$

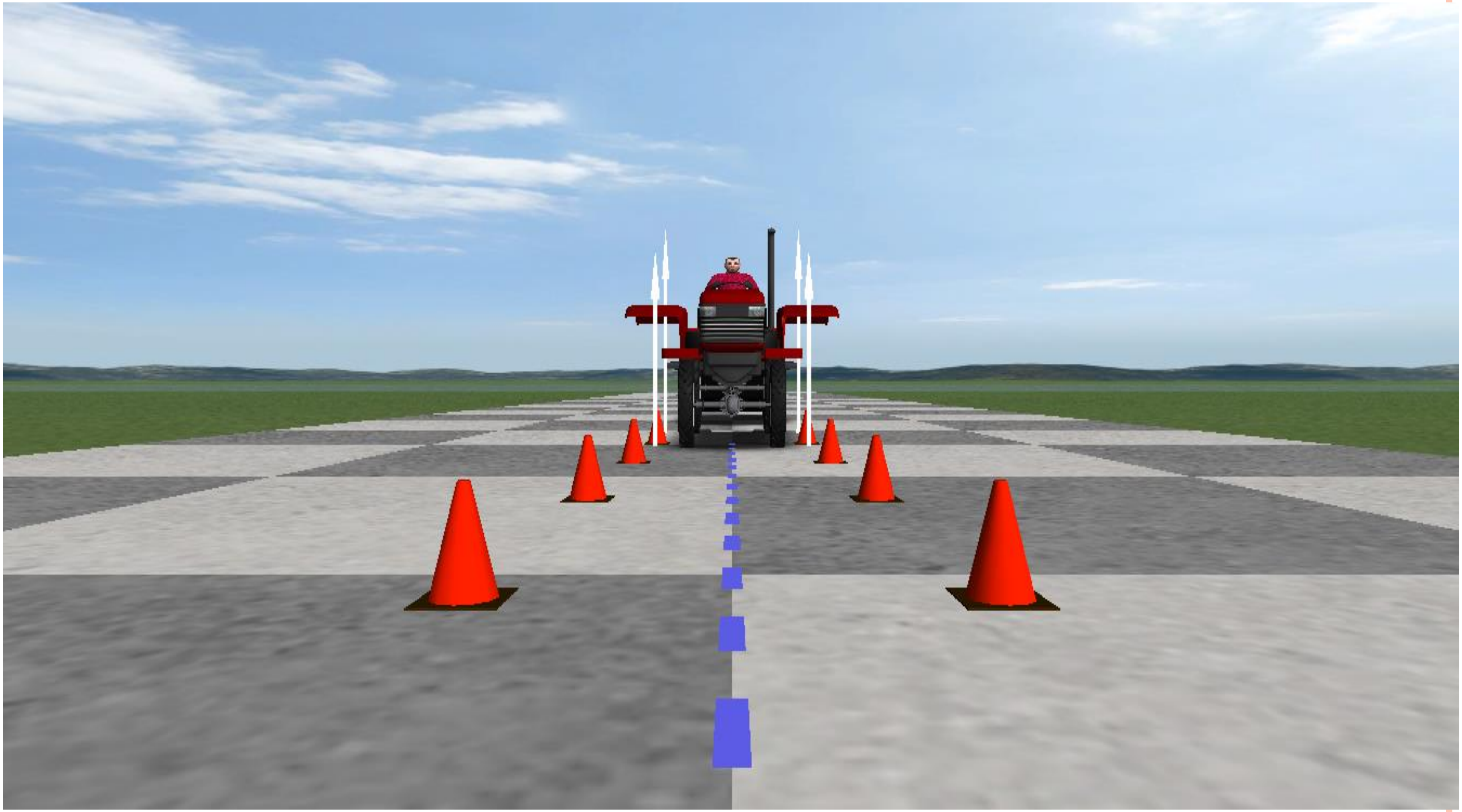
where, D_{c2} , D_{c2} , D_{c2} the denominators of the critical speeds of $V_{crit,c2}$, $V_{crit,c3}$ *and* speeds $V_{crit,c4}$ respectively.

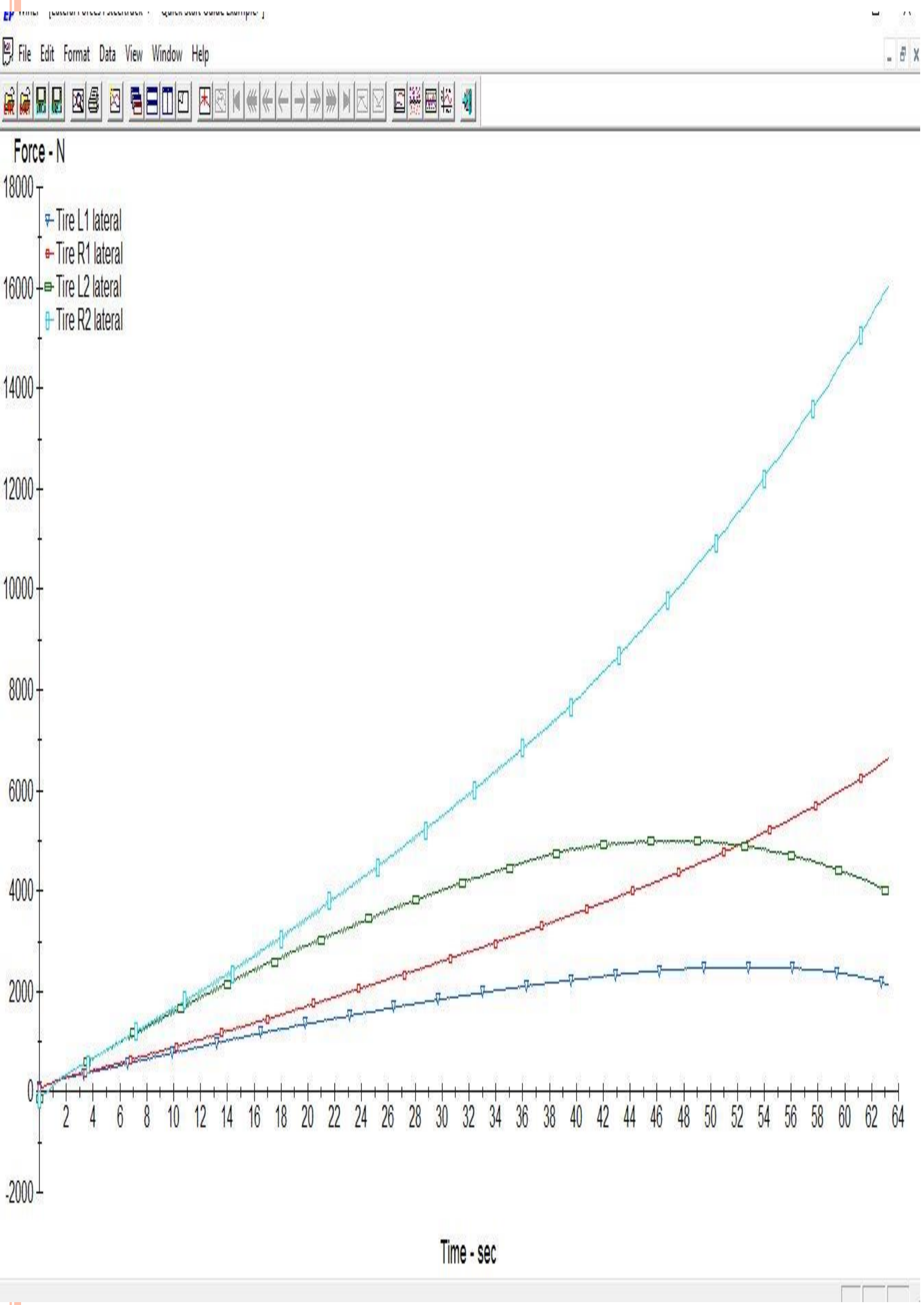
RESULTS

- Observed stability of vehicle during understeer case through car-sim software within the radius of 100m.



- Observed double lane change test at different speeds and concluded that at high velocity vehicle becomes unstable.

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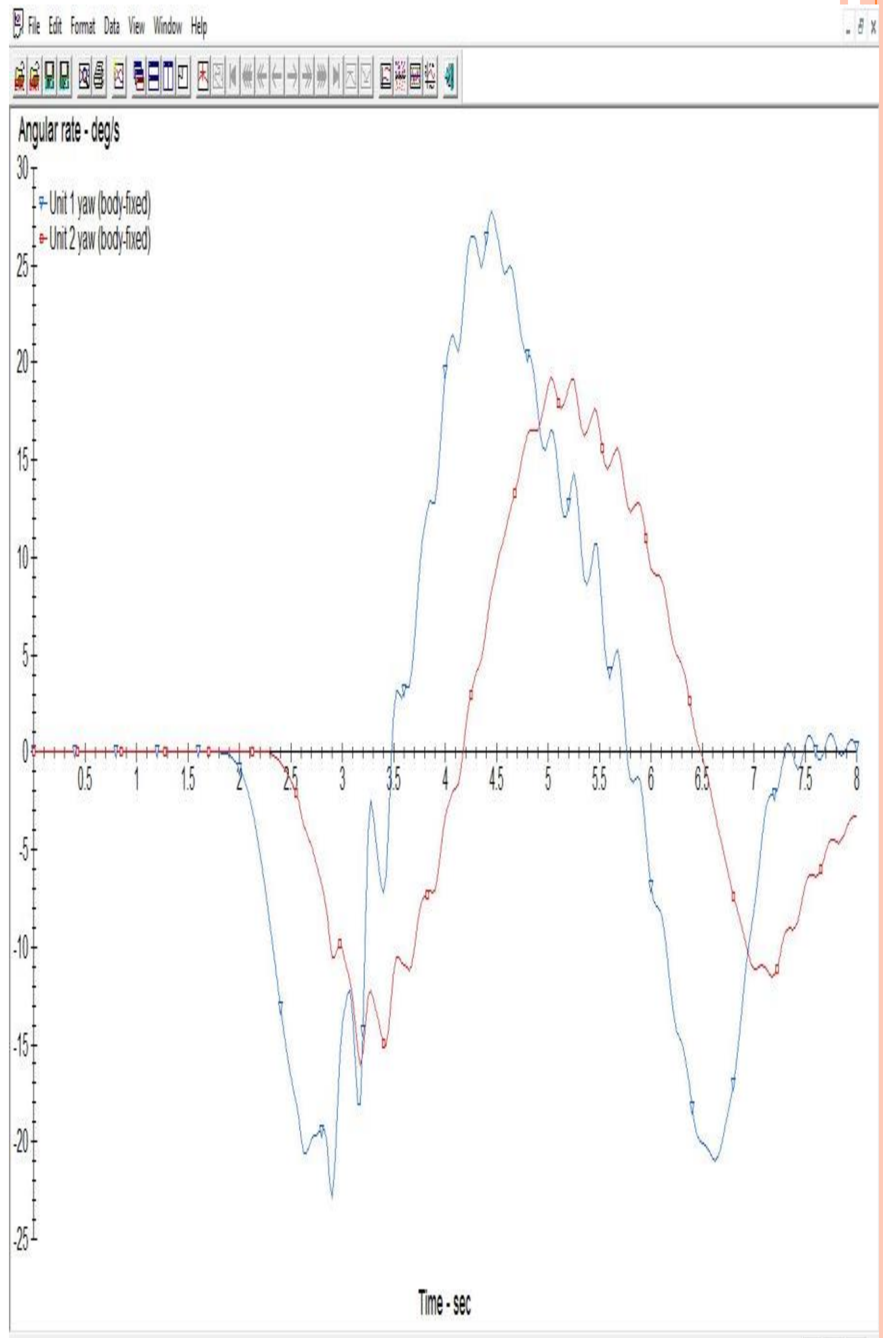
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