Hunting stability analysis of railway systems

Term Project in Vehicle Dynamics Course@IIT Hyderabad

Borrowed from – **'Hunting stability analysis of a railway vehicle system using a novel non-linear creep model' by Yung Chang Cheng**

By

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What is hunting oscillation?

- Wheels of 'traction railway system' have a cone angle.
- At high speed , adhesion force is in-sufficient in maintaining stability and the wheels start to oscillate.
- This oscillation about the mean position where it is 'trying' to find the equilibrium position is called hunting.
- We pretty much know the obvious implications of these oscillation.

Approach to hunting oscillation solution

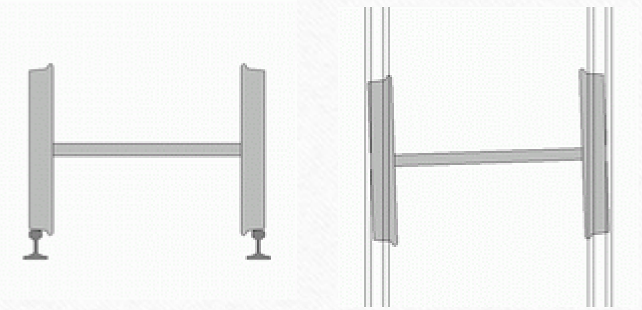
- Various models
  - a. Linear
  - b. Non-linear

Figure 1

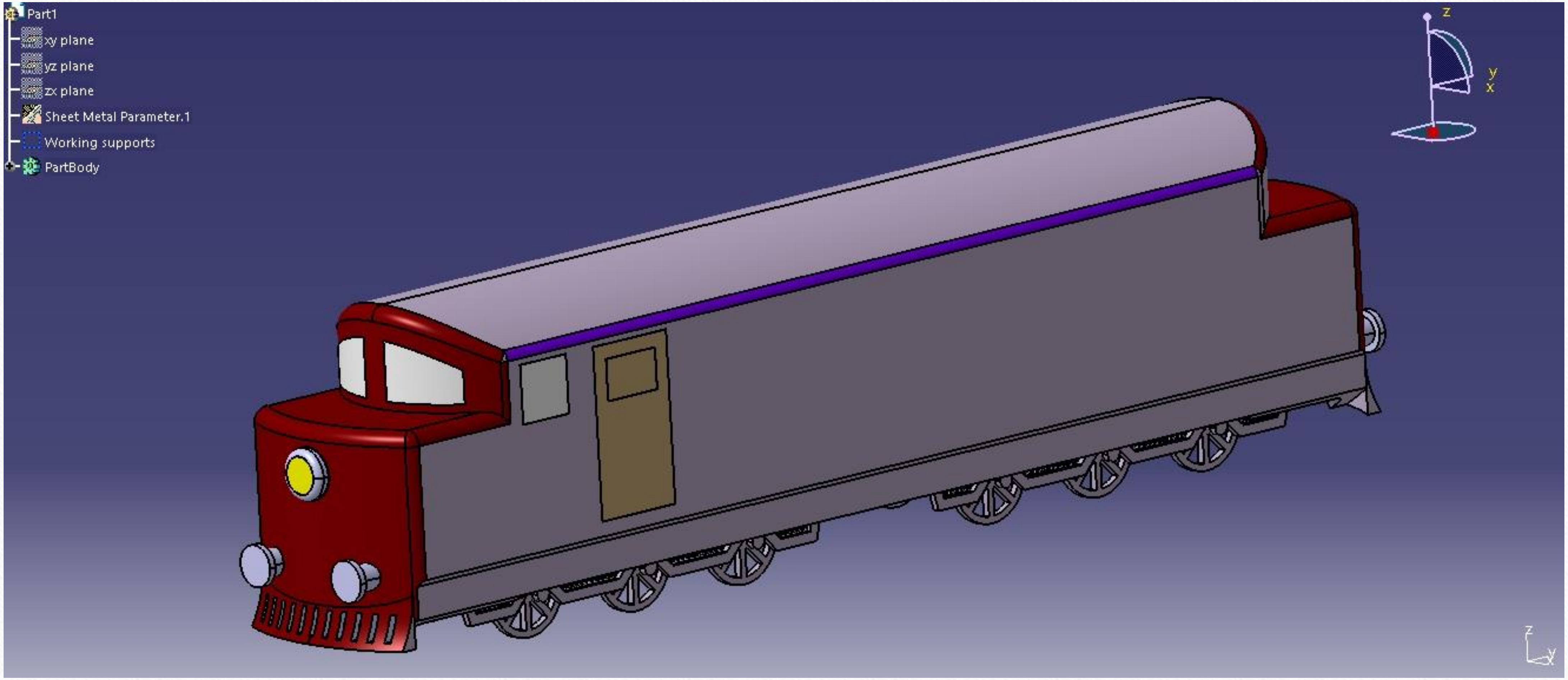
- A modified non-linear concept is propose by the author.

- Why linear to non-linear?

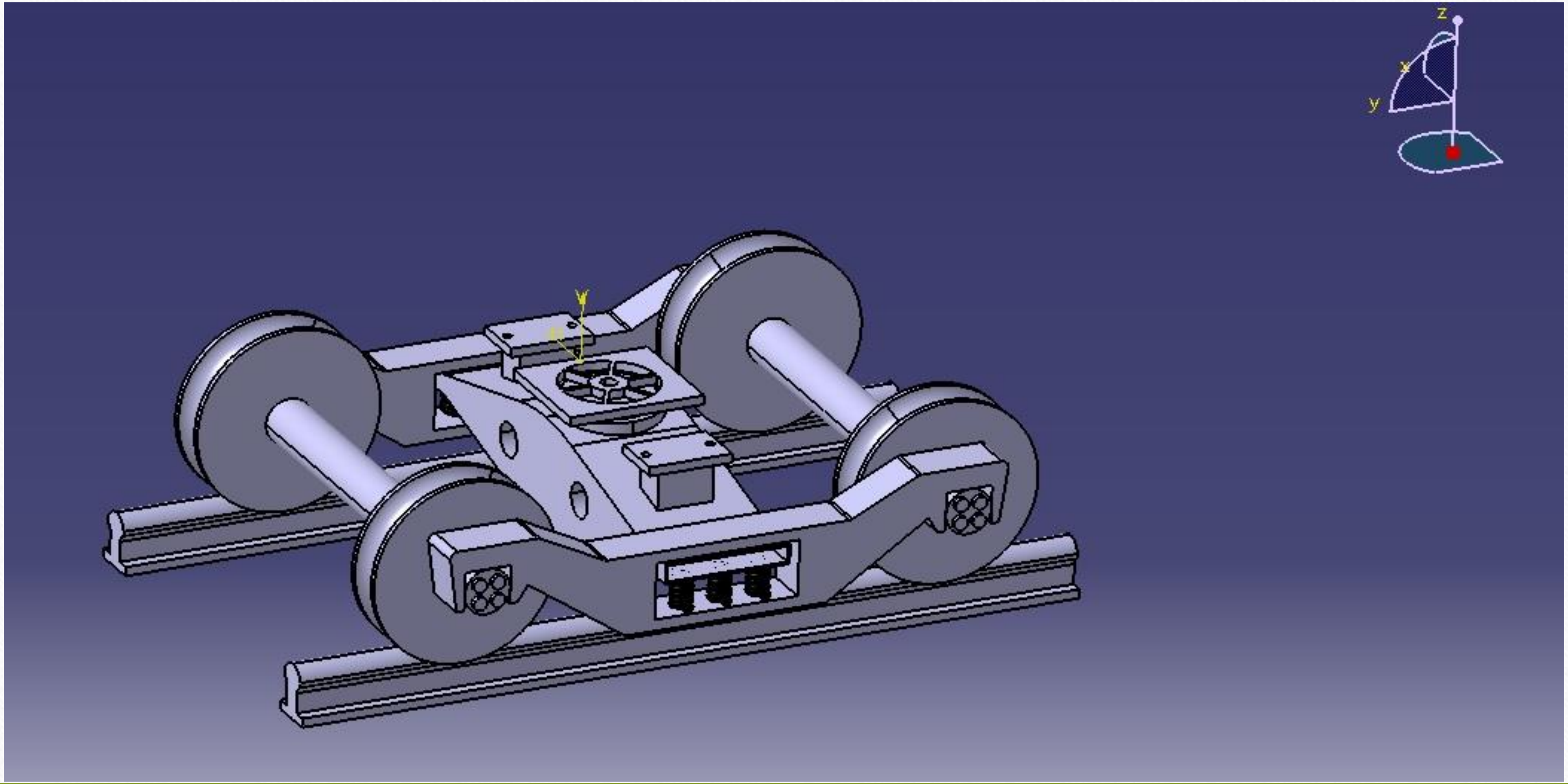
Figure 2



Cad Model of Locomotive



Cad Model of suspension system



Car Body Model

It has three parts :

1. Wheel
2. Bogie/Truck
3. Car

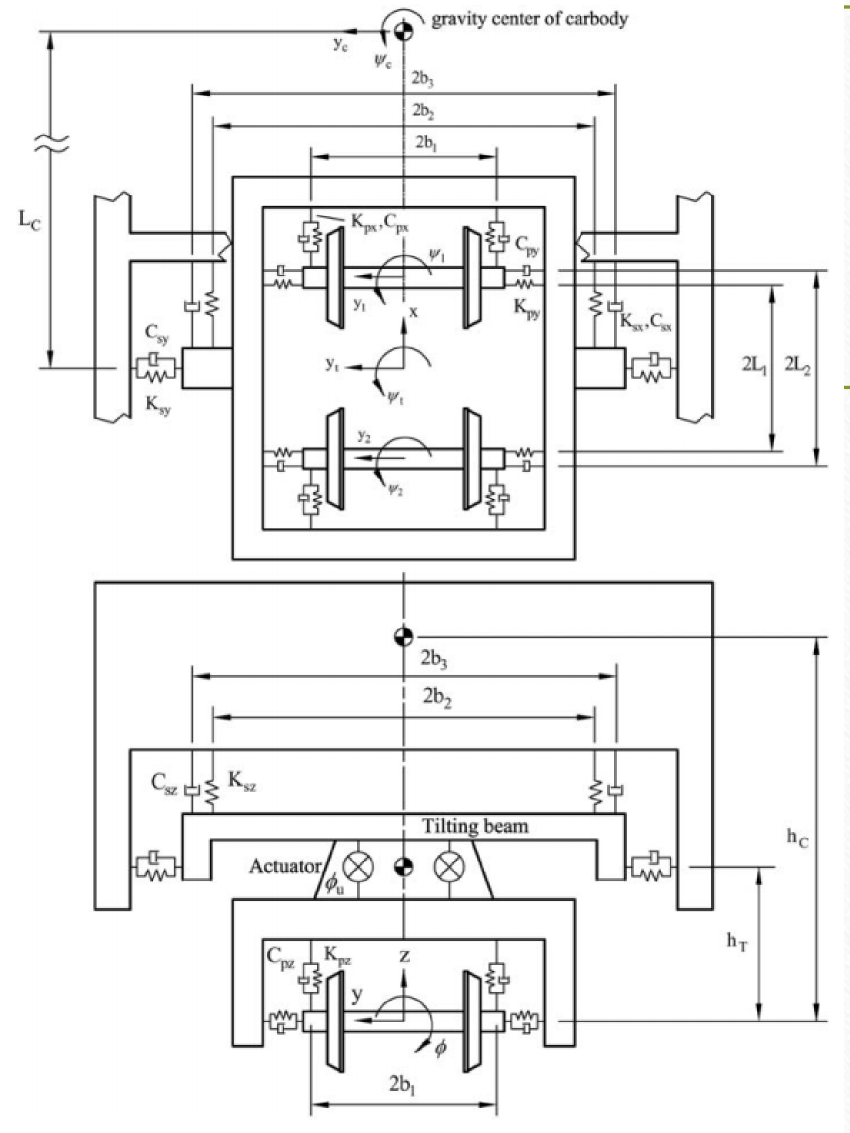


Figure 3

Governing equations of wheel-sets

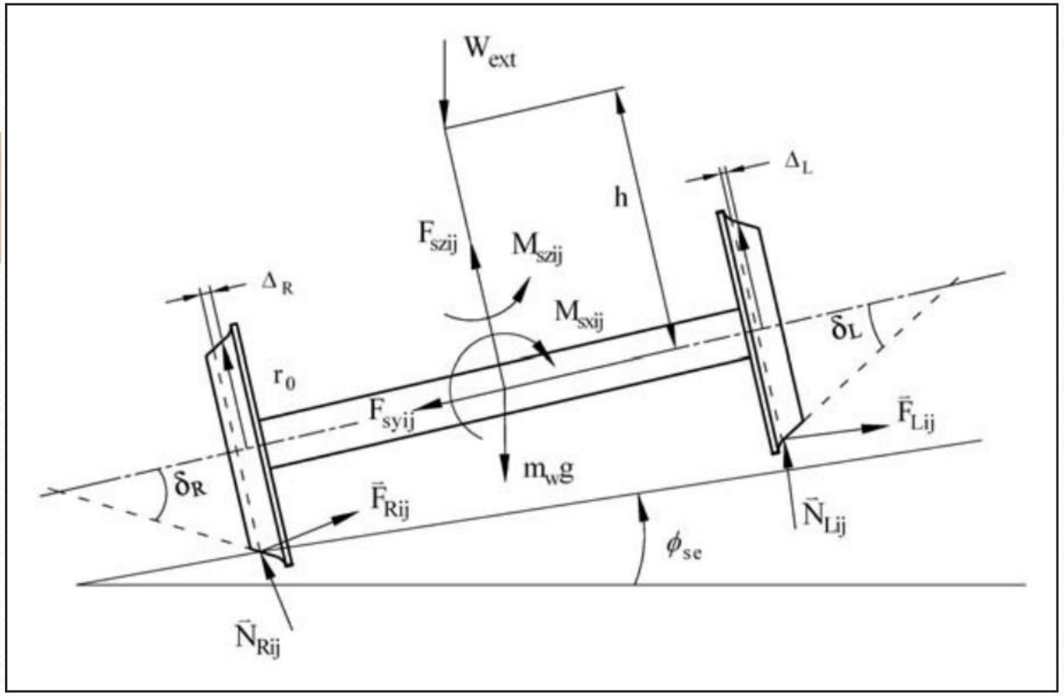


Figure 4

$$m_t\ddot{y}_{ti} = F_{syti} + \left(\frac{V^2}{gR} - \phi_{se}\right)m_t g$$

$$m_t\ddot{z}_{ti} = F_{szti} - \left(1 + \frac{V^2}{gR}\phi_{se}\right)m_t g$$

$$I_{tx}\ddot{\phi}_{ti} = M_{sxti}$$

$$I_{tz}\ddot{\psi}_{ti} = M_{szti}$$

$$m_c\ddot{y}_c = F_{syc} + \left(\frac{V^2}{gR} - \phi_{se}\right)m_c g$$

$$m_c\ddot{z}_c = F_{szc} - \left(1 + \frac{V^2}{gR}\phi_{se}\right)m_c g$$

$$I_{cx}\ddot{\phi}_c = M_{sxc}$$

Governing equations of wheel-sets

Equations for non-linear creep forces

$$m_w\left(\ddot{y}_{wij}-\frac{V^2}{R}\right)=F^n_{Lyij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+F^n_{Ryij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+N_{Lyij}+N_{Ryij}+F_{syij}+\frac{V^2}{gR}W_{ext}$$
$$-(W_{ext}+m_w g)\phi_{se} \qquad (27)$$

$$m_w\left(\ddot{z}_{wij}+\frac{V^2}{R}\phi_{se}\right)=F^n_{Lzij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+F^n_{Rzij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+N_{Rzij}+N_{Lzij}+F_{szij}-\frac{V^2}{gR}W_{ext}\phi_{se}$$
$$-(W_{ext}+m_w g) \qquad (28)$$

$$I_{wz}\ddot{\psi}_{wij}=-I_{wy}\frac{V}{r_0}\dot{\phi}_{wij}+R_{Rxij}F^n_{Ryij}$$
$$(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$-R_{Ryij}F^n_{Rxij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+R_{Lxij}F^n_{Lyij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$-R_{Lyij}F^n_{Lxij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+(R_{Rxij}N_{Ryij}+R_{Lxij}N_{Lyij})+M_{szij}$$
$$+M^n_{Lzij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$
$$+M^n_{Rzij}(y_{wij},\dot{y}_{wij},\psi_{wij},\dot{\psi}_{wij})$$

Governing equations of wheel-sets

Equations for non-linear creep forces(Continued)

$$
\begin{aligned}
I_{wx}\ddot{\phi}_{wij} = I_{wy}\frac{V}{r_0}\left(\dot{\psi}_{wij} - \frac{V}{R}\right) \\
&+ \left[R_{Ryij}F^n_{Rzij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij})\right. \\
&\left. - R_{Rzij}F^n_{Ryij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij})\right] \\
&+ \left[R_{Lyij}F^n_{Lzij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij})\right. \\
&\left. - R_{Lzij}F^n_{Lyij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij})\right] \\
&+ (R_{Lyij}N_{Lzij} + R_{Ryij}N_{Rzij}) \\
&- (R_{Rzij}N_{Ryij} + R_{Lzij}N_{Lyij}) \\
&+ M^n_{Lxij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij}) \\
&+ M^n_{Rxij}(y_{wij}, \dot{y}_{wij}, \psi_{wij}, \dot{\psi}_{wij}) \\
&+ M_{sxij} + M_{exij}
\end{aligned}
$$

Assumptions

- $\alpha_{ij} = 1$ (to convert non-linear to linear)
- $\phi_{wij} = (\lambda y_{ij})/a$ (roll angle of front and rear wheels)
- $\phi_{se} = 0$ (super-elevation angle of curved track)
- $W = W_{ext}$ (external load)

Proposed linear model

$$
\begin{aligned}
m_w \ddot{y}_{wi} = &-\frac{2f_{11}}{V}\dot{y}_{wi} + 2f_{11}\psi_{wi} \\
&-\frac{2f_{12}}{V}\dot{\psi}_{wi} - W\frac{\lambda}{a}y_{wi} - \frac{2r_o f_{11}}{V}\left(\frac{\lambda}{a}\right)\dot{y}_{wi} \\
&-2K_{py}y_{wi} - 2C_{py}\dot{y}_{wi} - (-1)^i 2K_{py}L_1\psi_t \\
&-(-1)^i 2C_{py}L_2\dot{\psi}_t + 2K_{py}y_t + 2C_{py}\dot{y}_t
\end{aligned}
$$

$$
\begin{aligned}
I_{wz}\ddot{\psi}_{wi} = &-\frac{2af_{33}\lambda}{r_o}y_{wi} + \frac{2f_{12}}{V}\dot{y}_{wi} \\
&+(-2f_{12} + a\lambda W)\psi_{wi} \\
&-\left(\frac{2a^2 f_{33}}{V} + \frac{2f_{22}}{V}\right)\psi_{wi} \\
&+\left(-\frac{I_{wy}V}{r_o} + \frac{2r_o f_{12}}{V}\right)\left(\frac{\lambda}{a}\right)\dot{y}_{wi} + M_{szi}
\end{aligned}
$$

Proposed linear model(continued)

$$
\begin{aligned}
M_{szti} = {} & (-4K_{py}L_1^2 - 4K_{px}b_1^2 - 2K_{sx}b_2^2)\psi_{ti} \\
& + (-4C_{py}L_2^2 - 4C_{px}b_1^2 - 2C_{sx}b_3^2)\dot{\psi}_{ti} \\
& + 2K_{py}L_1 y_{wi1} + 2C_{py}L_2\dot{y}_{wi1} \\
& + 2K_{px}b_1^2\psi_{wi1} + 2C_{px}b_1^2\dot{\psi}_{wi1} \\
& - 2K_{py}L_1 y_{wi2} - 2C_{py}L_2\dot{y}_{wi2} \\
& + 2K_{px}b_1^2\psi_{wi2} + 2C_{px}b_1^2\dot{\psi}_{wi2} \\
& + 2K_{sx}b_2^2\psi_c + 2C_{sx}b_3^2\dot{\psi}_c
\end{aligned}
$$

Stability Analysis using Lyapunov

- How to use it in this model?

$$\ddot{y} = a\dot{y} + by + c\dot{\psi} + d\psi$$
$$\ddot{\psi} = A\dot{y} + By + E\dot{\psi} + D\psi$$

$$\dot{y} = p$$
$$\dot{p} = ap + by + cq + d\psi$$
$$\dot{q} = \psi$$
$$\dot{q} = Ap + By + Eq + D\psi$$

Note:
Values of constants are avoided to simplify the solutions

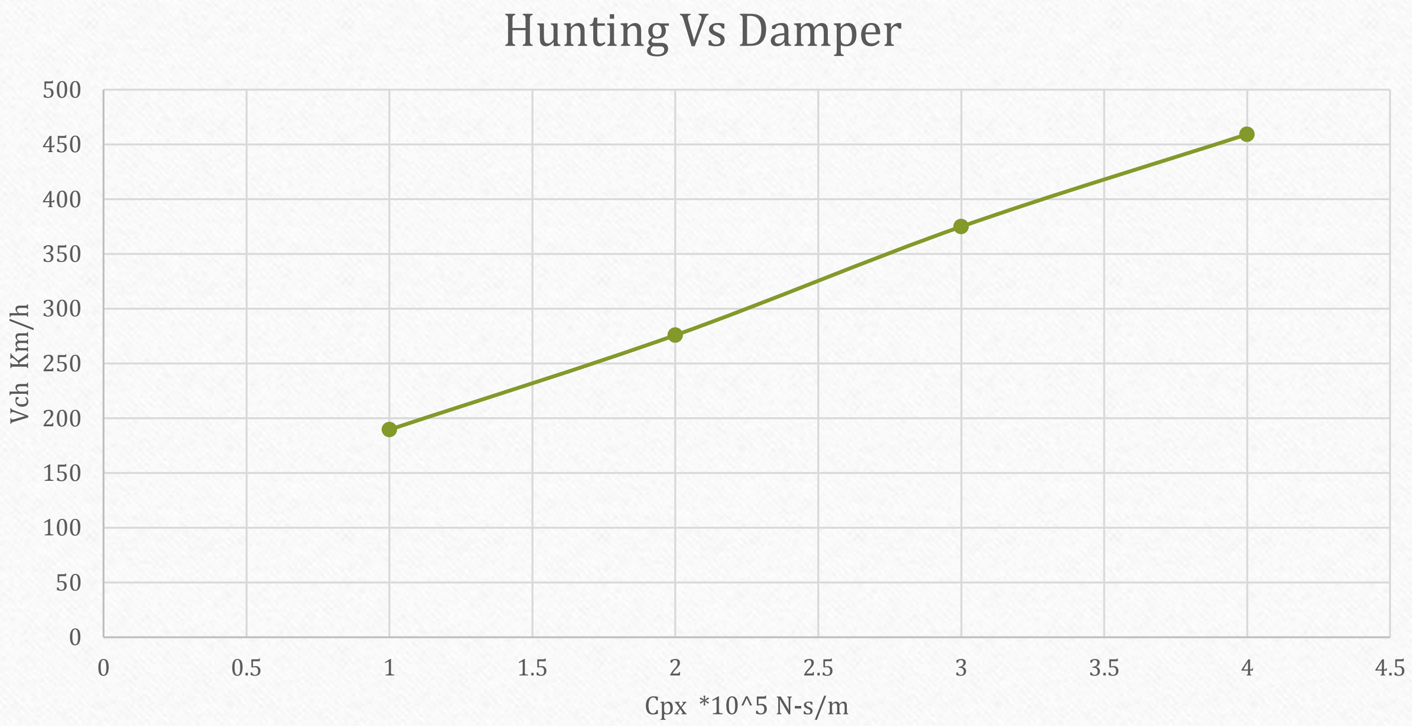
Methodology

- Since the equations are linearized we find Eigen values
- Applying the stability conditions Re(λ)<0
- We arrive at an inequality to find the range of V_{ch} hunting Velocity
- We check the dependency of V_{ch} on K_{px} , K_{py} , K_{pz} C_{px} , C_{py} , C_{pz}

Plots

Hunting Vs Stiffness
250
200
150
100
50
0
Vch Km/h
0
2
4
6
8
10
12
14
Kpx * 10^6 N/m

Plots



Hunting Vs Damper
500
450
400
350
300
250
200
150
100
50
0
Vch Km/h
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Cpx *10^5 N-s/m

References

- Figure1- https://www.google.co.in/search?q=indian+railway+suspension&source=lnms&tbm=isch&sa=X&ved=0ahUKEwjskoiTqJbMAhUKjJQKHY6bCxIQ_AUIBygB&biw=1097&bih=541#imgrc=OUE5yJesFlyAeM%3A
- Figure 2- https://en.wikipedia.org/wiki/Hunting_oscillation
- Figure 3 & 4- **Hunting stability analysis of a railway vehicle system using a novel non-linear creep model**- Yung Chang Cheng

THANK YOU