Automotive Vehicle Mechanics and its Modelling

Project Title: Motorbike

Submitted By:

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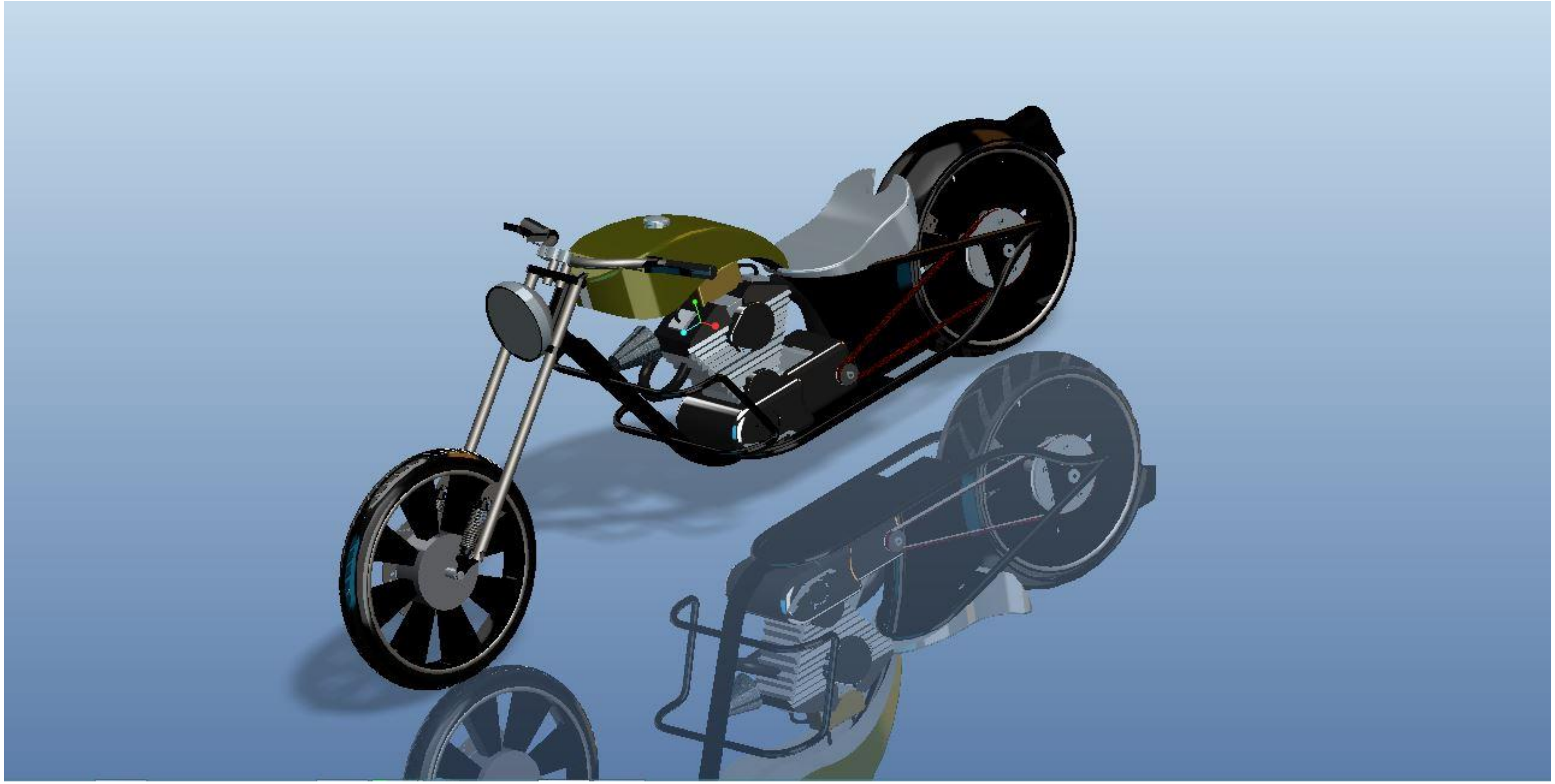
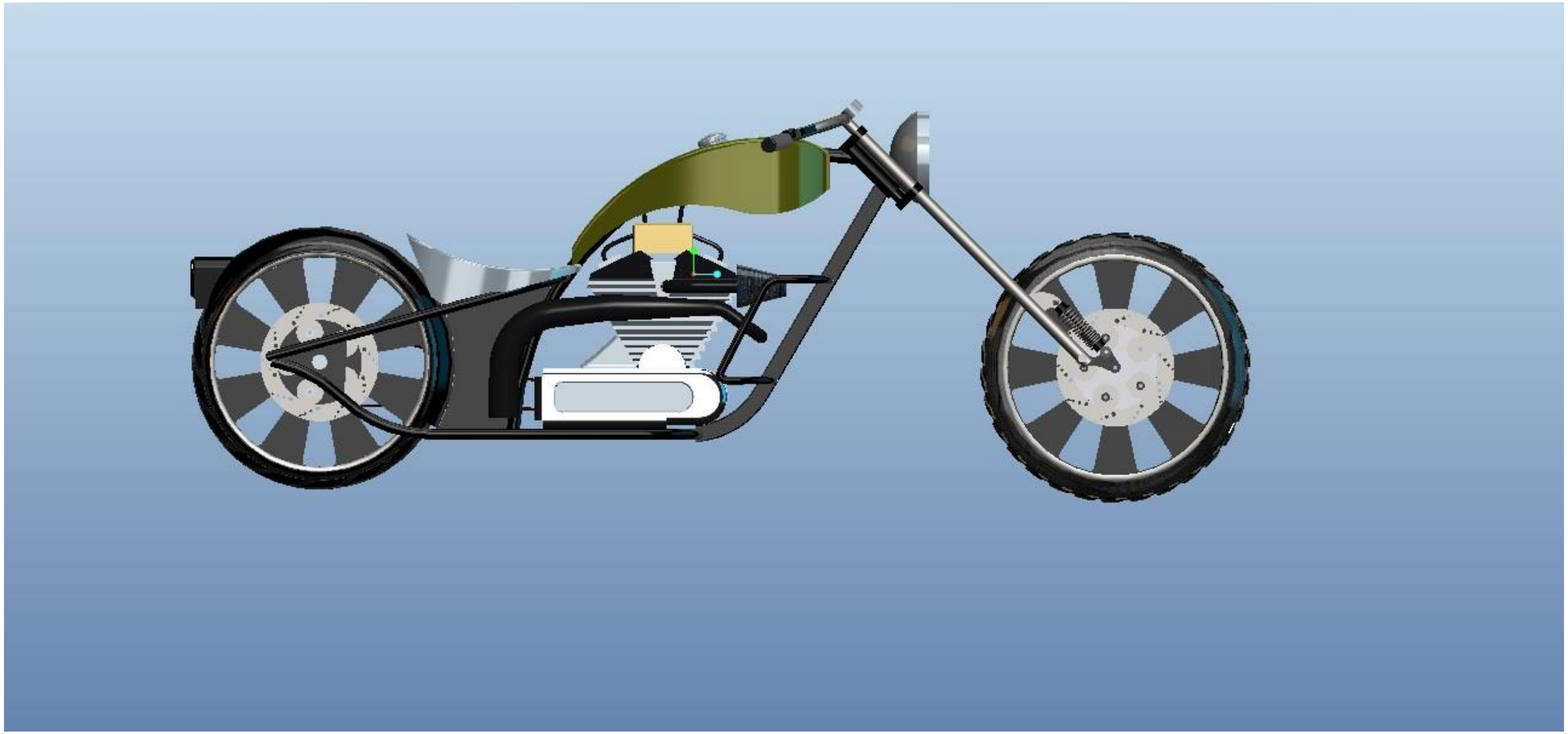
Introduction

- What makes bikes interesting is when studying the handling behaviour of a two-wheeled vehicle, rolling motions and, to a lesser extent, gyroscopic moments must not be neglected.
- The mass of the driver, who controls the vehicle not only by acting on the steering but also displacing his body, can be a substantial fraction of the total mass.
- Moreover, a two-wheeled vehicle is intrinsically unstable. The driver thus has to perform as a stabilizer for the capsize mode.
- Finally, the body of the driver, acting as an aerodynamic brake or control surface, contributes in a substantial way to aerodynamic forces.

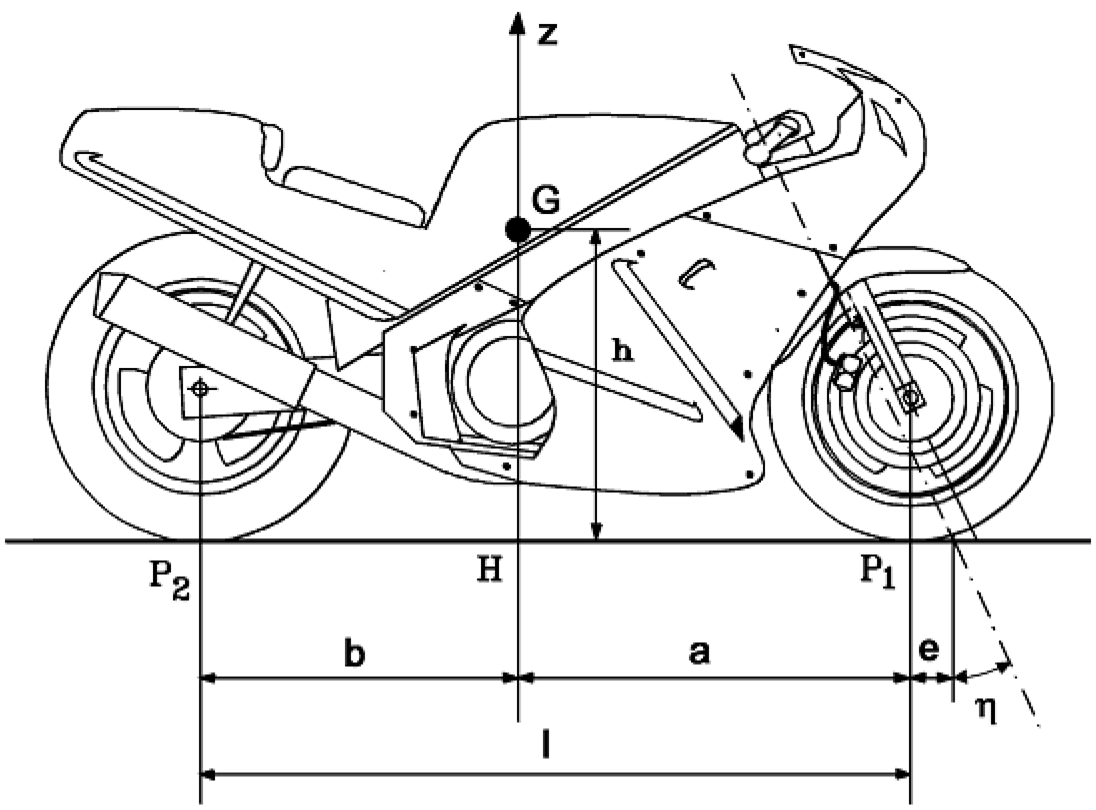
3 D model of Motorbike





Prepared using ProE

Governing Equations of the Motorbike



The translational and rotational kinetic energies are respectively:

$$\mathcal{T}_t = \frac{1}{2} m V_{\mathrm{G}}^2$$

$$\mathcal{T}_r = \frac{1}{2} \begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{Bmatrix}^T \begin{bmatrix} J_x & 0 & J_{xz} \\ 0 & J_y & 0 \\ J_{xz} & 0 & J_z \end{bmatrix} \begin{Bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{Bmatrix}$$

The kinetic energy of the steering system due to steering motion is

$$\mathcal{T}_{r_1} = \frac{1}{2} \mathbf{\Omega}_1{}^T \mathbf{J}_1 \mathbf{\Omega}_1$$

In locked controls motion the kinetic energy of the ith wheel is

$$\mathcal{T}_{r_i} = \frac{1}{2} \mathbf{\Omega}_{wi}^T \mathbf{J}_{wi} \mathbf{\Omega}_{wi}$$

Locked Control Model

- The locked controls Lagrangian function is

$$\mathcal{L} = \frac{1}{2}m\left(\dot{X}^2 + \dot{Y}^2\right) + \frac{1}{2}\dot{\phi}^2 J_x^* + \frac{1}{2}\dot{\psi}^2\left[J_z\cos^2(\phi) + J_y^*\sin^2(\phi)\right] +$$

$$+\dot{\psi}\dot{\phi}J_{xz}\cos(\phi) + mh\left[\dot{X}\dot{\psi}\sin(\phi) - \dot{Y}\dot{\phi}\cos(\phi)\right]\cos(\psi) +$$

$$+mh\left[\dot{X}\dot{\phi}\cos(\phi) + \dot{Y}\dot{\psi}\sin(\phi)\right]\sin(\psi) + \frac{1}{2}V^2\left(\frac{J_{p_1}^2}{R_{e_1}^2} + \frac{J_{p_2}^2}{R_{e_2}^2}\right) +$$

$$+V\dot{\psi}\sin(\phi)\left(\frac{J_{p_1}}{R_{e_1}} + \frac{J_{p_2}}{R_{e_2}}\right) - mgh\cos(\phi) \ .$$

- The Equations of motion are:

$$m\dot{V} = Q_x \ ,$$

$$m\dot{v} + mV\dot{\psi} - mh\ddot{\phi} = Q_y \ ,$$

$$J_z\ddot{\psi} + J_{xz}\ddot{\phi} + m\phi h\dot{V} + V\dot{\phi}\left(\frac{J_{p1}}{R_{e_1}} + \frac{J_{p2}}{R_{e_2}}\right) + \dot{V}\phi\left(\frac{J_{p1}}{R_{e_1}} + \frac{J_{p2}}{R_{e_2}}\right) = Q_\psi \ ,$$

$$J_x\ddot{\phi} + J_{xz}\ddot{\psi} - mh\dot{v} - mhV\dot{\psi} - V\dot{\psi}\left(\frac{J_{p1}}{R_{e_1}} + \frac{J_{p2}}{R_{e_2}}\right) - mgh\phi = Q_\phi \ .$$

- The Generalized forces are given by

$$Q_x = F_{x_1} + F_{x_2} + F_{xa} \ ,$$
$$Q_y = F_{y_1} + F_{y_2} + F_{ya} \ ,$$
$$Q_\psi = F_{y_1}a - F_{y_2}b + M_{z_1} + M_{z_2} - F_{xa}h\sin(\phi) + M_{za} \ ,$$
$$Q_\phi = -hF_{ya}\cos(\phi) - F_{za}h\sin(\phi) + M_{xa} \ .$$

- First Equation is thus

$$m\dot{V} = F_{x_1} + F_{x_2} + F_{xa}$$

- The other three equations are given by:

$$\begin{bmatrix} m & 0 & -mh \\ 0 & J_z & J_{xz} \\ -mh & J_{xz} & J_x \end{bmatrix} \begin{Bmatrix} \dot{v} \\ \ddot{\psi} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} -Y_v & mV - Y_r & 0 \\ -N_v & -N_r & N_g \\ -L_v & -mhV - VN_g & 0 \end{bmatrix} \begin{Bmatrix} v \\ \dot{\psi} \\ \dot{\phi} \end{Bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & -Y_\phi \\ 0 & 0 & mh\dot{V} + \dot{V}\left(\frac{J_{p1}}{R_{e_1}} + \frac{J_{p2}}{R_{e_2}}\right) - N_\phi \\ 0 & 0 & -mgh - L_\phi \end{bmatrix} \begin{Bmatrix} y \\ \psi \\ \phi \end{Bmatrix} = \begin{Bmatrix} Y_\delta \delta + F_{y_e} \\ N_\delta \delta + M_{z_e} \\ 0 \end{Bmatrix},$$

Free Control Model

- The Lagrangian function for a system with free controls is

$$\begin{aligned}\Delta\mathcal{L} &= \frac{1}{2}J_{z1}\dot{\delta}^2 + \dot{\delta}\dot{\psi}\left[J_{z1}\cos(\eta) + J_{xz1}\sin(\eta)\cos(\phi)\right] + \\ &+ \dot{\delta}\dot{\phi}\left[-J_{z1}\sin(\eta) + J_{xz1}\cos(\eta)\right] + A_1\delta\dot{\psi}^2 + A_2\delta\dot{\phi}^2 + \\ &+ A_3\delta\dot{\psi}\dot{\phi} - V\frac{J_{p1}}{R_{e1}}\delta\left[\dot{\psi}\cos(\phi)\sin(\eta) + \dot{\phi}\cos(\eta)\right] .\end{aligned}$$

- The four equations describing the lateral free controls motions are

$$\begin{bmatrix} m & 0 & -mh & 0 \\ & J_z & J_{xz} & J_{z1}\cos(\eta) + J_{xz1}\sin(\eta) \\ & & J_x & -J_{z1}\sin(\eta) + J_{xz1}\cos(\eta) \\ & \text{symm.} & & J_{z1} \end{bmatrix} \begin{Bmatrix} \dot{v} \\ \ddot{\psi} \\ \ddot{\phi} \\ \ddot{\delta} \end{Bmatrix} +$$

$$+ \begin{bmatrix} -Y_v & mV - Y_r & 0 & -Y_{\dot{\delta}} \\ -N_v & -N_r & N_g & -N_{\dot{\delta}} - S^* \\ -L_v & -mhV - VN_g & 0 & -VC^* \\ -M_v & -M_r + VS^* & +VC^* & -M_{\dot{\delta}} + c_\delta \end{bmatrix} \begin{Bmatrix} v \\ \dot{\psi} \\ \dot{\phi} \\ \dot{\delta} \end{Bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 & -Y_\phi & -Y_\delta \\ 0 & 0 & mh\dot{V} + \dot{V}\left(\frac{J_{p1}}{R_{e1}} + \frac{J_{p2}}{R_{e2}}\right) - N_\phi & -N_\delta - \dot{V}S^* \\ 0 & 0 & -mgh - L_\phi & -\dot{V}C^* \\ 0 & 0 & -M_\phi & -M_\delta \end{bmatrix} \begin{Bmatrix} y \\ \psi \\ \phi \\ \delta \end{Bmatrix}$$

$$= \begin{Bmatrix} Y_\delta \delta + F_{y_e} \\ N_\delta \delta + M_{z_e} \\ 0 \\ M_g \end{Bmatrix},$$

Analysis of the following systems considered:

- Engine to slip modelling for motorbike control design
- A Motorbike Tire Model for Dynamic Simulations
- Stability under Locked and Free control

Engine to slip modelling for motorcycle control design

- TC – Traction control aims at regulating the slip of the rear tyre using the throttle opening and/or the engine spark-advance

- ETSTF – Engine Torque-to-slip transfer function

- An appropriate model of the engine-to-slip dynamics is required for TC since the slip of the rear tyre is influenced by the engine torque

- Engine-to-slip dynamics of a plant

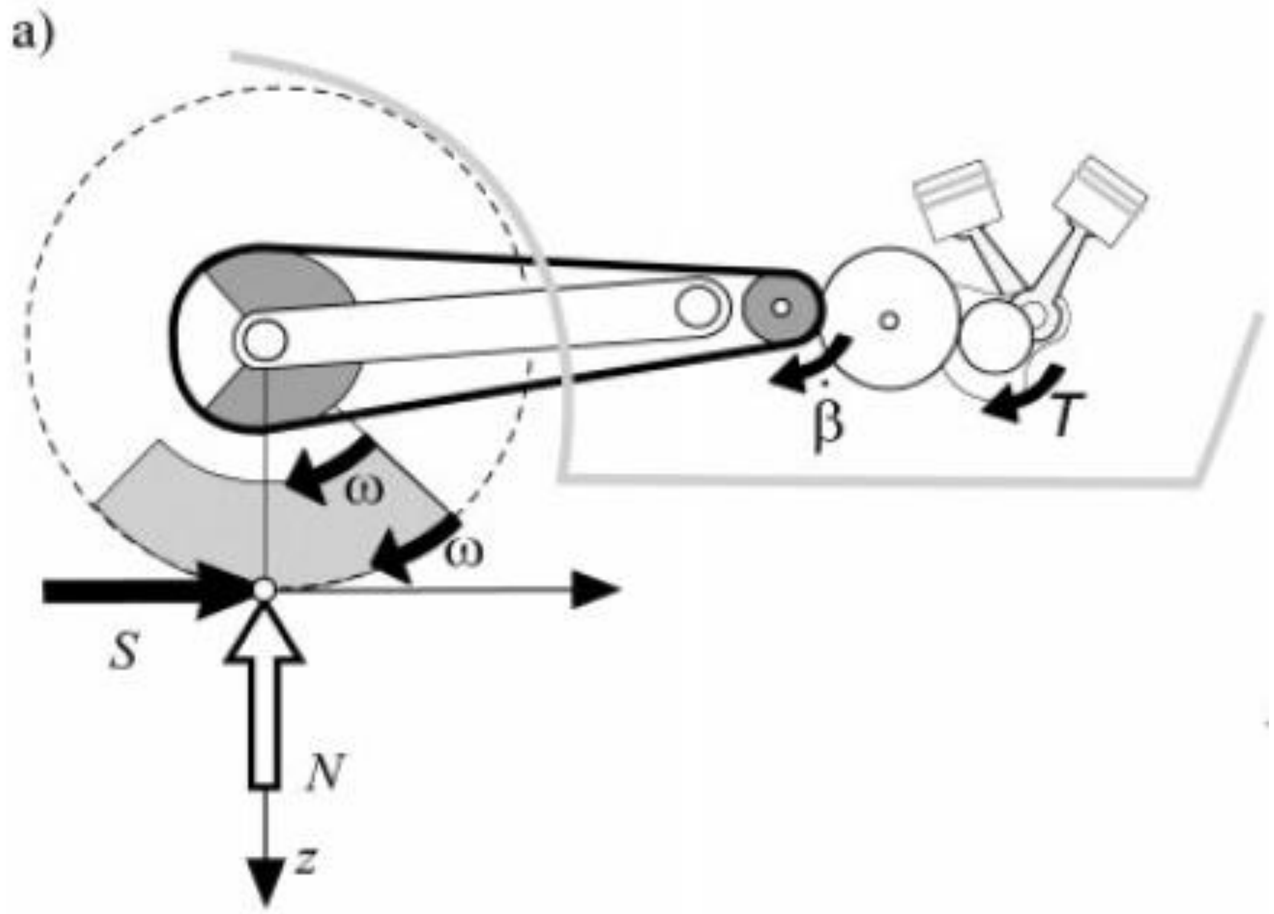
MODELS FOR THE ETSTF :-

- a single-wheel model with a rigid tyre and a rigid transmission

- a single-wheel model with a compliant tyre

- a single-wheel model which features a compliant tyre and a compliant sprocket absorber

A single-wheel model with a rigid tyre and a rigid Transmission



Torque equilibrium at the centre of the wheel:-

$(I_w + I_t)\,\dot{\omega} = \tau * T - R * S$

I_w : spin inertia of rear wheel
I_t : transmission inertia of the rear wheel
$\dot{\omega}$: spin rate of real wheel
R : Rolling radius
T : engine torque at crank shaft
S : longitudinal force

$\tau = \tau_p * \tau_g * \tau_f$
τ : whole transmission ratio
τ_p : between crankshaft and primary shaft of gear box
τ_f : between output shaft and rear wheel
τ_g : between primary and output shaft of the gear box

Longitudinal slip is given as

$$k = \frac{\boldsymbol{\omega}R - V}{V}$$

V = longitudinal speed

$S_k = S_{ss} + k_k\ (k - k_{ss})*N$

S_k : Actual force

S_{ss} : steady state longitudinal force

K_{ss} : steady state slip

k_k : slope of the longitudinal slip curve the liearised point

Therefore,

$$(I_w + I_t)\ \dot{\omega} = \tau*T - R\ [\ S_{ss} + k_k\ (\frac{\boldsymbol{\omega}R - V}{V} - k_{ss})\ N]$$

Now to find the ETSTF ,

$\dot{x} = A*x + B*T$
$K = C*x$

A, B & C are state space matrices
x is a state vector

Hence, ETSTF is given as

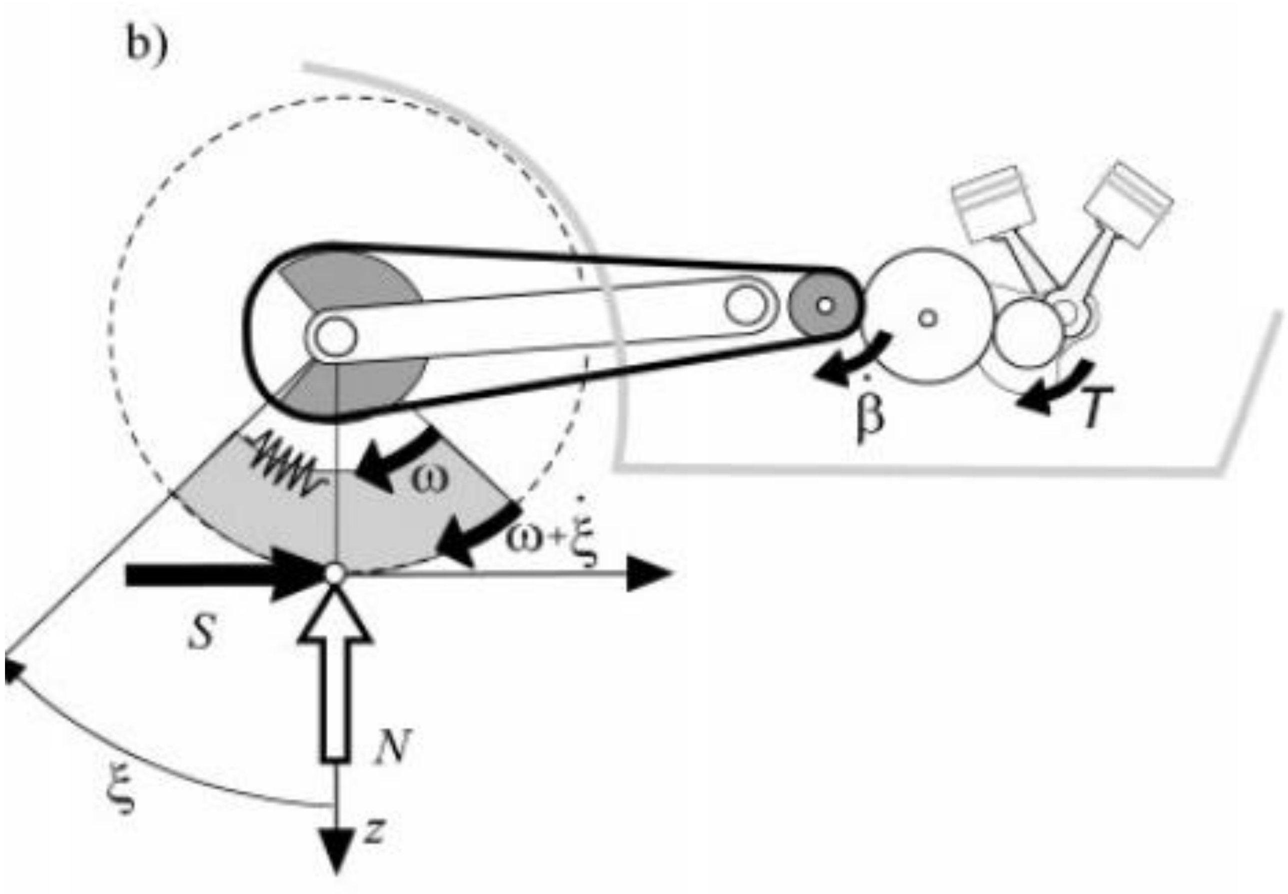
$$H(s) = \frac{k}{T}(s)$$

After substituting the state space matrices

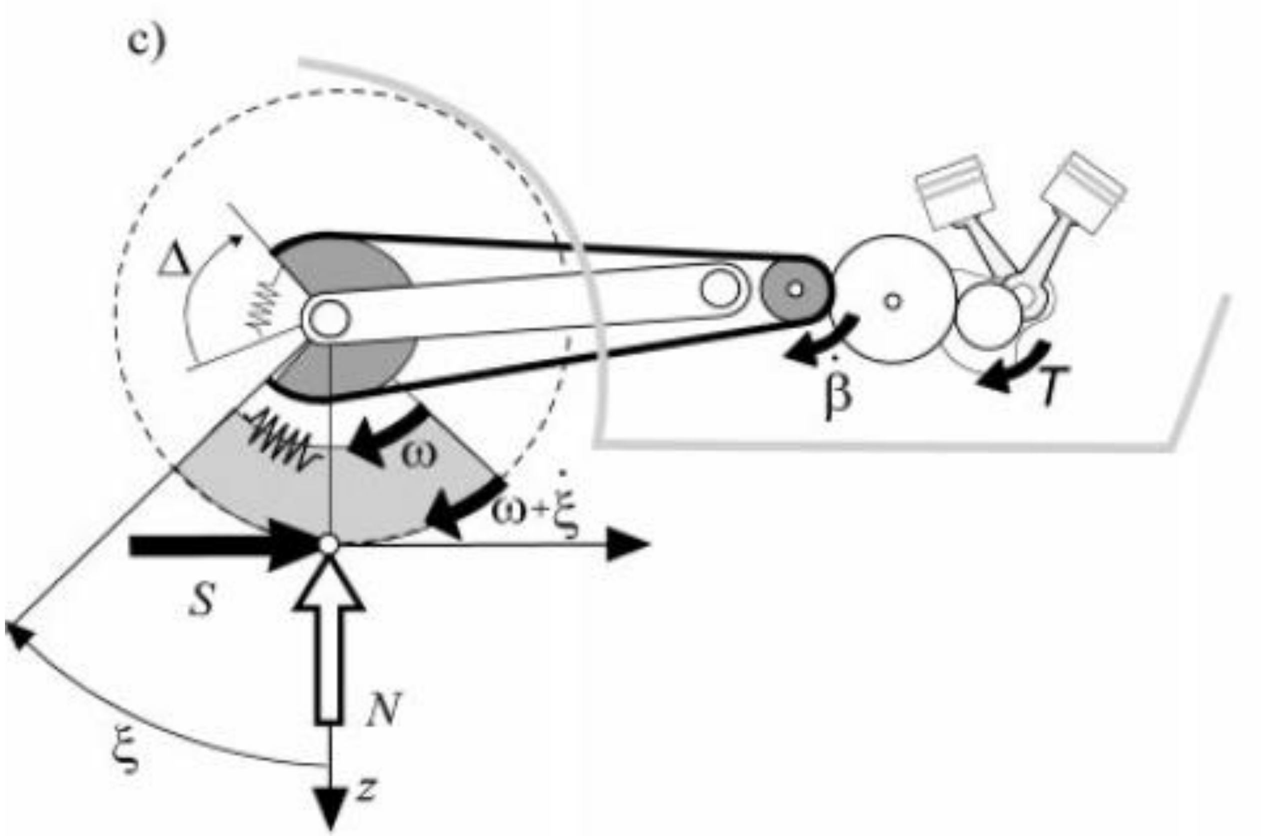
$$H(s) = \frac{b_0}{a_1 S + a_0}$$

$b_0 = \tau * R$
$a_1 = (I_w + I_t) * V$
$a_0 = R^2 * k_k * N$

A single-wheel model with a compliant tyre



Single-wheel model which features a compliant tyre and a compliant sprocket absorber



Conclusions :

- Feasibility, design and performance of the TC are determined by engine-to-slip dynamics
- Tyre’s circumferential compliance is essential to model the main resonance of the plant
- Absorber flexibility improves the model consistency

A Motor Cycle Tire Model for Dynamic Simulations

Tire Geometry and Position of contact Patch

The position of contact points depends both on geometry of carcass and camber angle

$$^{w}\mathbf{C}_0 = \begin{Bmatrix} 0 \\ y_0(\varphi) \\ z_0(\varphi) \end{Bmatrix}$$

where

$$\varphi = -\arctan\left(\frac{dz_0}{dy_0}\right)$$

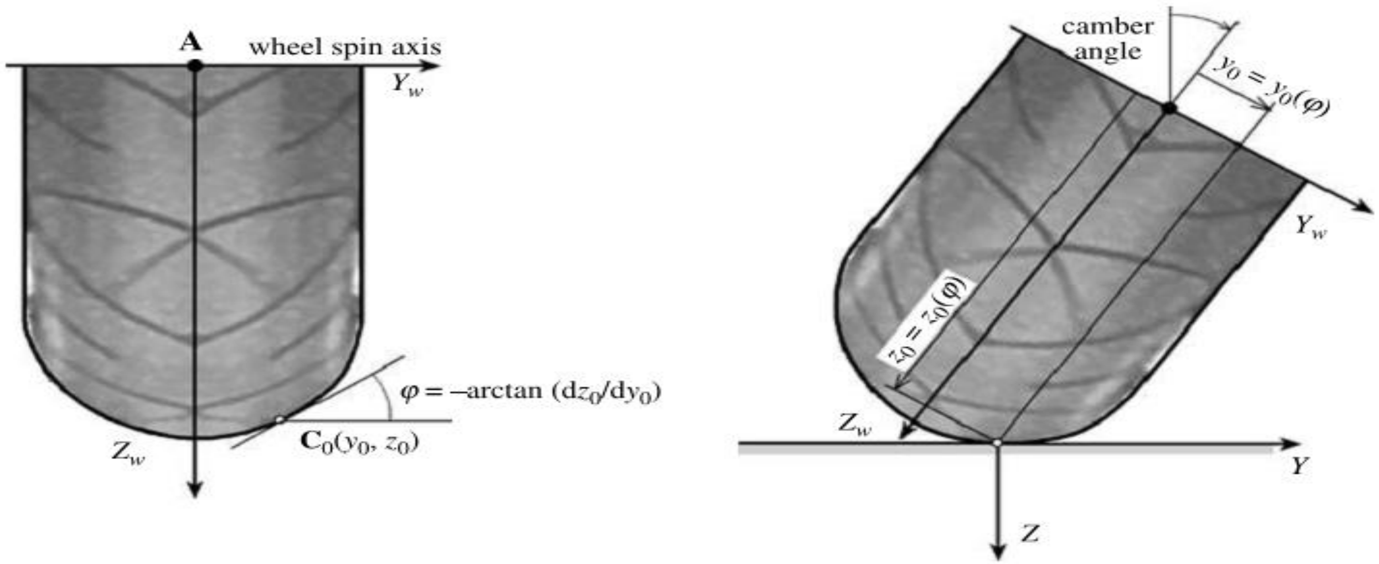


Figure 1. Description of profile and identification of the contact point as a function of roll angle.

The Elasticity of carcass and static behaviour of tire

$$ {}^{w}\mathbf{C} = \begin{Bmatrix} 0 \\ y_0 + \zeta_L \\ y_0 + \zeta_R \end{Bmatrix} $$

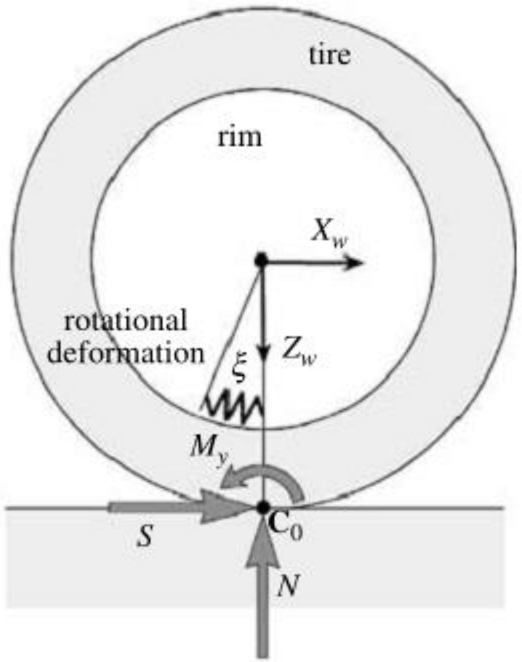
The vertical force N and Horizontal force F is given as

$$ \begin{aligned} N &= N_{\text{elastic}}(\zeta_R, \zeta_L, \varphi) = -F Z_W \cos(\varphi) - F Y_W \sin(\varphi) \\ &= -K_R(1 + \alpha\,\zeta_R)\,\zeta_R \cos(\varphi) - K_L \zeta_L \sin(\varphi), \end{aligned} $$

$$ \begin{aligned} F &= F_{\text{elastic}}(\zeta_R, \zeta_L, \varphi) = -F Z_W \sin(\varphi) + F Y_W \cos(\varphi) \\ &= -K_R(1 + \alpha\,\zeta_R)\zeta_R \sin(\varphi) + K_L \zeta_L \cos(\varphi), \end{aligned} $$

Relation between longitudinal force S, deformation is

$$ S = S_{\text{elastic}}(\xi, \varphi) = K_\xi \xi $$



Tread Rubber Sliding and Steady-state Tire Forces

Longitudinal slip is defined as the ratio between slip velocity and forward velocity

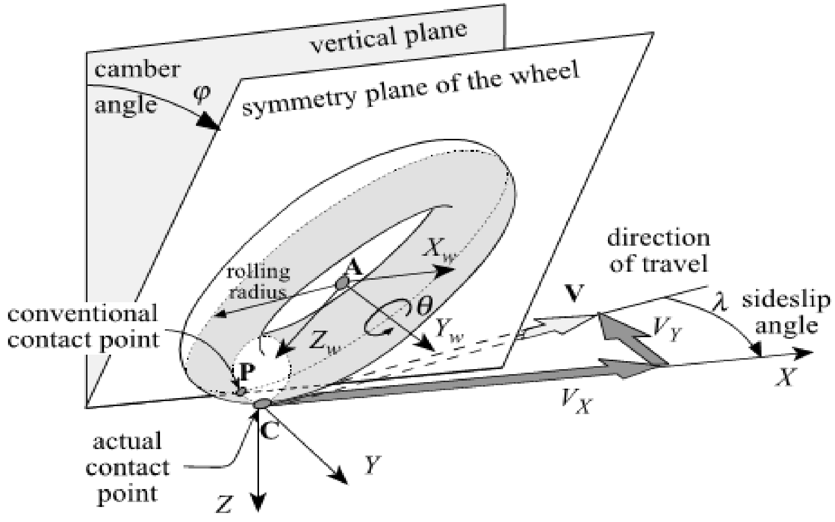
$$\kappa = -\frac{V_{SX}}{X}$$

Using wheel kinematics

$$\kappa = -\frac{V_X \; - \; V_R}{V_X} = -1 - \frac{r\omega}{V_{A,X} + \dot{\Psi} \; \sin(\varphi)}$$

The sideslip angle is defined as the angle between direction of travel and intersection of wheel plane with the road

$$\tan(\lambda) = -\frac{V_{SY}}{V_X}$$



The sideslip angle can be calculated using wheel kinematics

$$\lambda = -\arctan\left[\frac{V_{A,Y} - \dot{\varphi}\, r\, \cos(\varphi)}{V_{A,X} + \dot{\psi}\, r\, \sin(\varphi)}\right]$$

The longitudinal thrust as a function of the vertical load and longitudinal slip:

$$S = \text{magicS}(\kappa, N) = N D_x \sin(C_x \arctan\{B_x\kappa - E_x [B_x\kappa - \arctan(B_x \kappa)]\})$$

The equivalent camber angle is

$$\varphi_{\mathrm{eq}} = \varphi + \frac{k_\lambda}{k_\varphi}\lambda_i$$

Lateral force can be calculated as

$$F = \mathrm{magic}_F(\varphi_{\mathrm{eq}}, N) = N D_y \sin(C_y \arctan\{B_y\varphi_{\mathrm{eq}} - E_y[B_y\varphi_{\mathrm{eq}} - \arctan(B_y\varphi_{\mathrm{eq}})]\})$$

The yaw Moment can be calculated as

$$M_z = M_{Tz}(\varphi) - t(\lambda)F = m_r\varphi N + t_0\left(1 - \left|\frac{\lambda}{\lambda_{\max}}\right|\right)F$$

Twisting moment is proportional to camber angle and pneumatic trail

$t = t_0(1 - |\lambda/\lambda_{\max}|)$ varies linearly with the sideslip angle.

Rolling Resistance moment is My = Nu

Coupling between Elastic and sliding phenamena

Relaxation equations are given as

$$\frac{\sigma_X}{V_X}\dot{S} + \dot{S} = S_{st}(\kappa, \lambda, \varphi, N) \qquad \frac{\sigma_Y}{V_X}\dot{F} + F = F_{st}(\kappa, \lambda, \varphi, N)$$

The instantaneous slip is

$$\kappa_i = -1 - \frac{V_{C,R}}{V_{C,X}} = -1 - \frac{[z_0(\varphi) + \zeta_R](\dot{\theta} + \dot{\xi})}{V_{A,X} + \dot{\psi}\,\{[z_0(\varphi) + \zeta_R]\,\sin(\varphi) - [y_0(\varphi) + \zeta_L]\,\cos(\varphi)\}}$$

it depends not only on the wheel kinematics , but also on the carcass geometry (y0,z0) and tire deformations

By linearize equation of vertical and lateral forces

$$N = -K_R\zeta_R \qquad F = -K_R\zeta_R\varphi + K_L\zeta_L = N\varphi - K_L\zeta_L$$

By Linearize expression of side slip

$$\lambda_i = \lambda - \frac{\dot{\zeta}_{\mathrm{L}}}{V_X}$$

From sliding force equation

$$F = k_\lambda \lambda + k_\varphi \varphi - k_\lambda \frac{\dot{F} - N\dot{\varphi}}{K_{\mathrm{R}} V_X}$$

By rearranging the terms

$$\frac{k_\lambda / K_{\mathrm{L}}}{V_x} \dot{F} + F = k_\lambda \lambda + k_\varphi \varphi + \frac{k_\lambda / K_{\mathrm{L}}}{V_X} N\dot{\varphi}$$

Linearize relaxation equation

$$\frac{\sigma_Y}{V_X} \dot{F} + F = k_\lambda \lambda + k_\varphi \varphi$$

Relaxation length can be found out

$$\sigma_Y = \frac{k_\lambda}{K_{\mathrm{L}}}$$

Where K_L is carcass stiffness , k_λ is sideslip stiffness

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Acknowledgement

- V. Srikanth, Staff (Heat Transfer Lab), IIT Hyderabad

Thank You