# VEHICLE DYNAMICS PROJECT

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# PROJECT TOPIC

# SUSPENSION SYSTEM IN AUTOMOBILE SYSTEM

# ACKNOWLEDGMENT


- We would like to thank **Dr Ashok Kumar Pandey** sir and Veerbhadra reddy sir for providing us this opportunity and helping us in this project.

# INTRODUCTION

- A vehicle suspension system is a complex vibration system having multiple degrees of freedom . The purpose of the suspension system is to isolate the vehicle body from the road inputs.

- Suspension systems serve a dual purpose —

- Contributing to the vehicle's road holding/handling and braking for good active safety and driving pleasure, and
- Keeping vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations,etc.

# MOTIVATION

Study of Suspension system aims at benefiting-

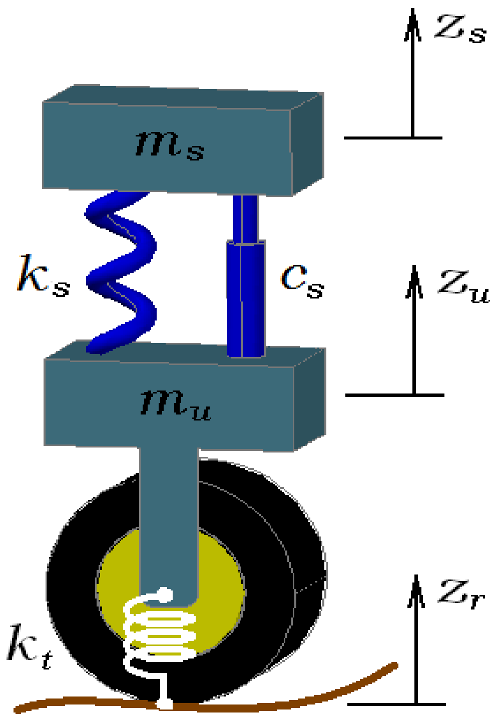
1:- Ride comfort

2:- Long life of vehicle

3:-Stability of vehicle

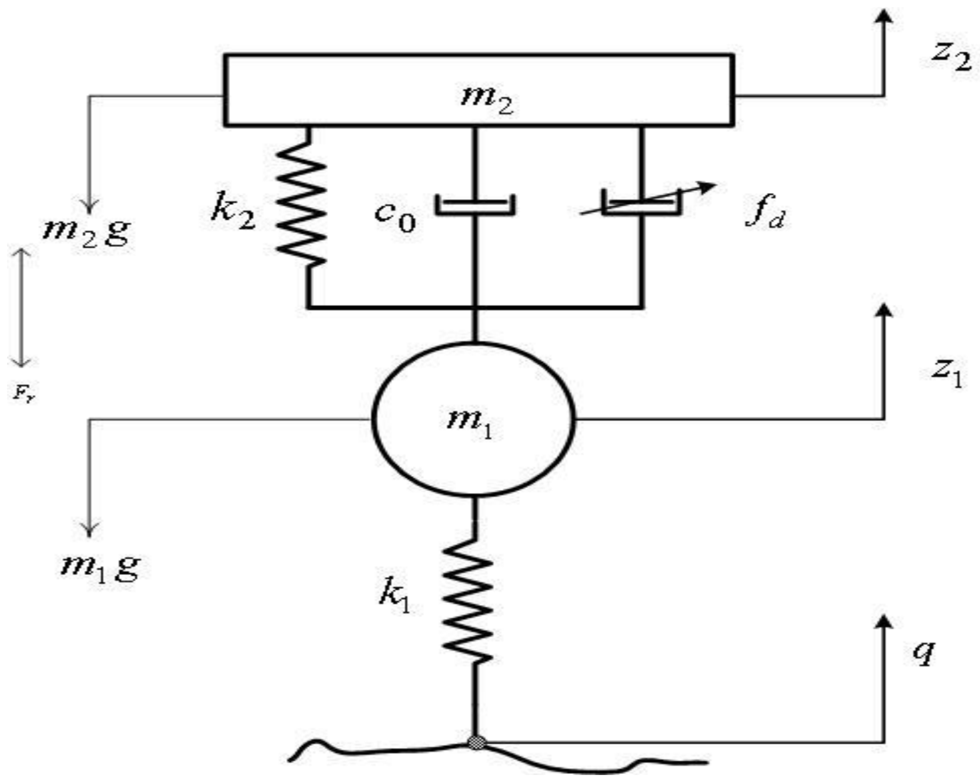
# TYPES OF SUSPENSIONS

- **Passive Suspensions**- Traditional springs and dampers are referred to as passive suspensions — most vehicles are suspended in this manner.



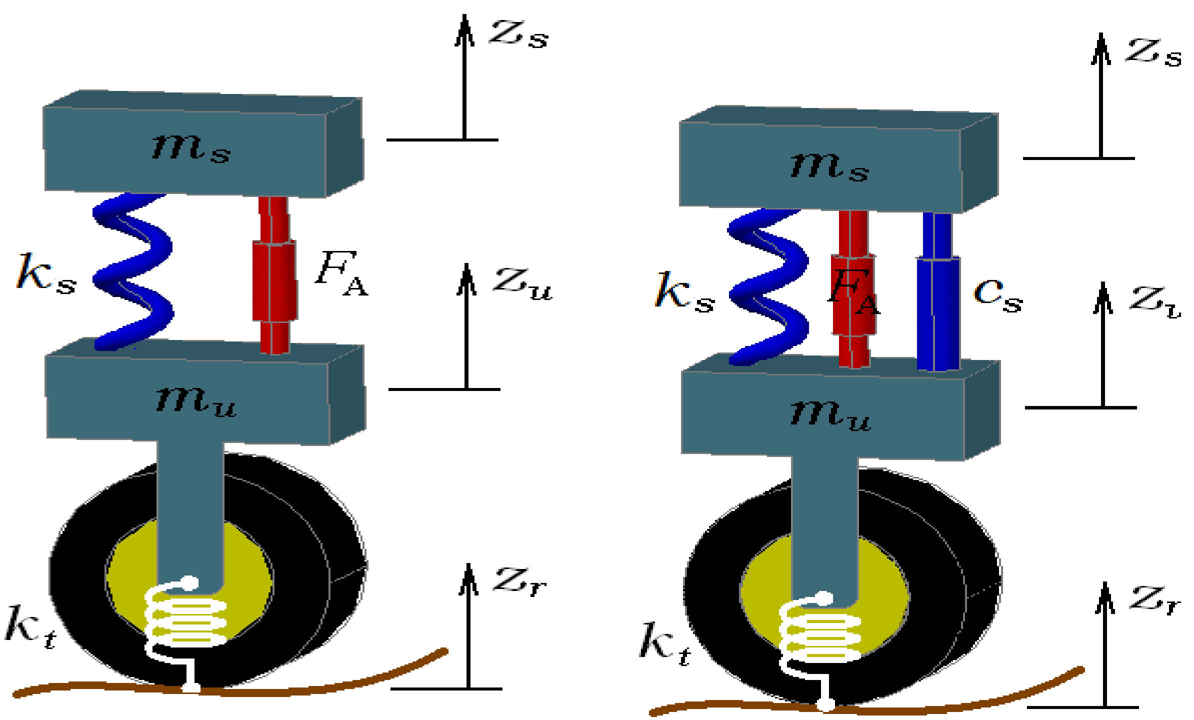
# SEMI-ACTIVE SUSPENSION

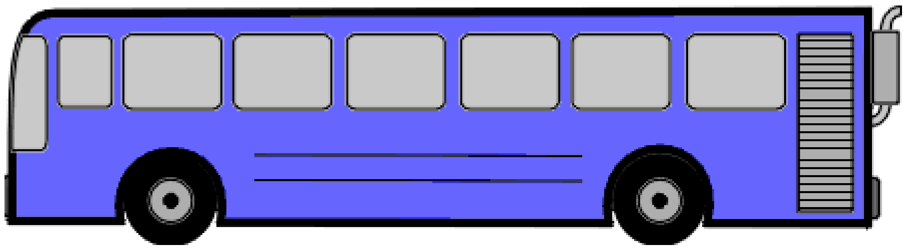
- Semi-active suspensions include devices such as air springs and switchable shock absorbers, various self-leveling  solutions, as well as systems like hydro pneumatic ,  hydro Elastic, and hydragas suspensions.



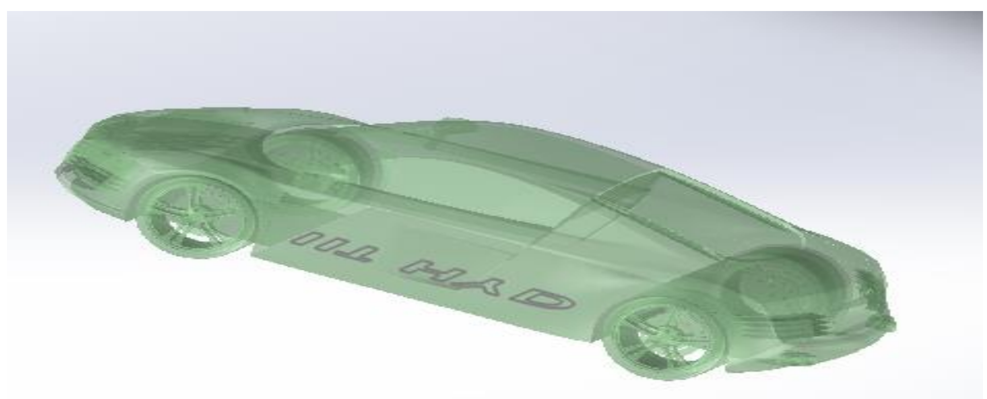
# ACTIVE SUSPENSION

- Fully active suspension systems use electronic monitoring of vehicle conditions, coupled with the means to impact vehicle suspension and behavior in real time to directly control the motion of the car.



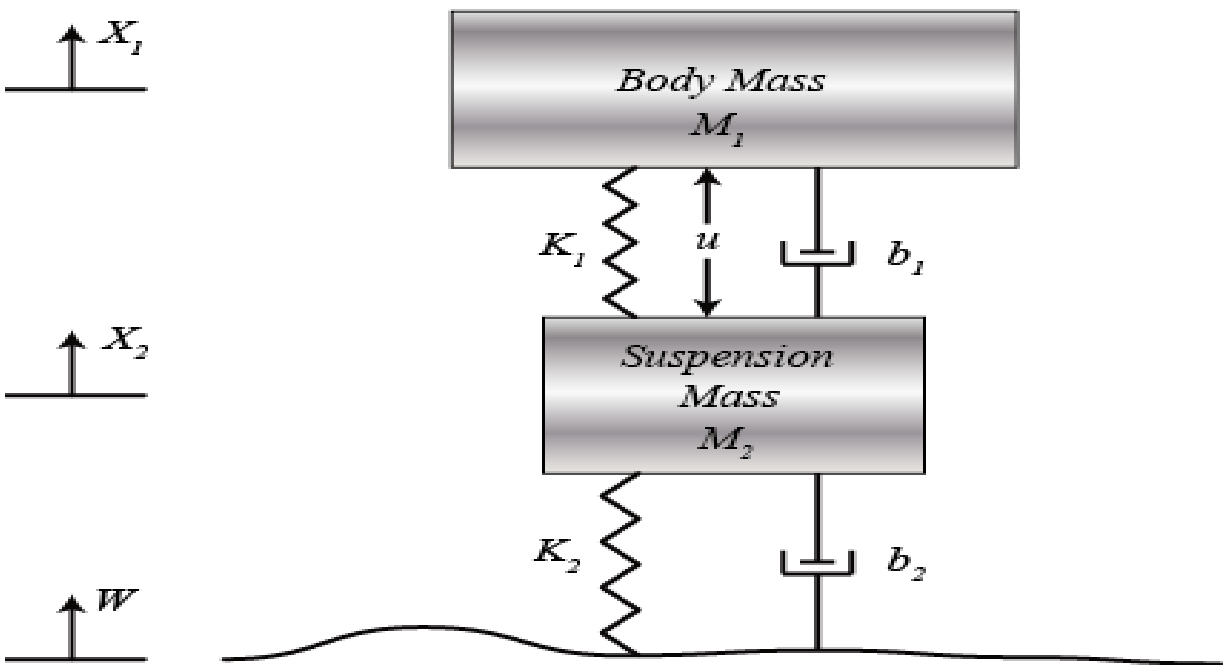


# Active suspension quarter car/bus model



# Mathematical Modeling

Model of Bus Suspension System (1/4 Bus)



# Parameters

- (M1) 1/4 bus body mass - 2500 kg
- (M2) suspension mass - 320 kg

- (K1) spring constant of suspension system-80,000 N/m
- (K2) spring constant of wheel and tire - 500,000 N/m

- (B1) damping constant of suspension system -350 N.s/m
- (B2) damping constant of wheel and tire 15,020 N.s/m

- (U) Control force

# Mathematical model

$$M_1 X_1 + K_1(X_1 - X_2) + B_1(X_1 - X_2) = U$$

$$M_2 \ddot{X}_2 - K_1(X_1 - X_2) + K_2(X_2 - W) + B_2(\dot{X}_2 - \dot{W}) = -U$$

Assuming zero initial condition, Laplace transform gives

$$(M_1 s^2 + B_1 s + K_1)X_1(s) - (B_1 s + K_1)X_2(s) = U(s)$$

$$(B_1 s + K_1)X_1(s) + (M_2 s^2 + (B_1 + B_2)s + (K_1 + K_2))X_2(s) = (B_2 s + K_2)W(s) - U(s)$$

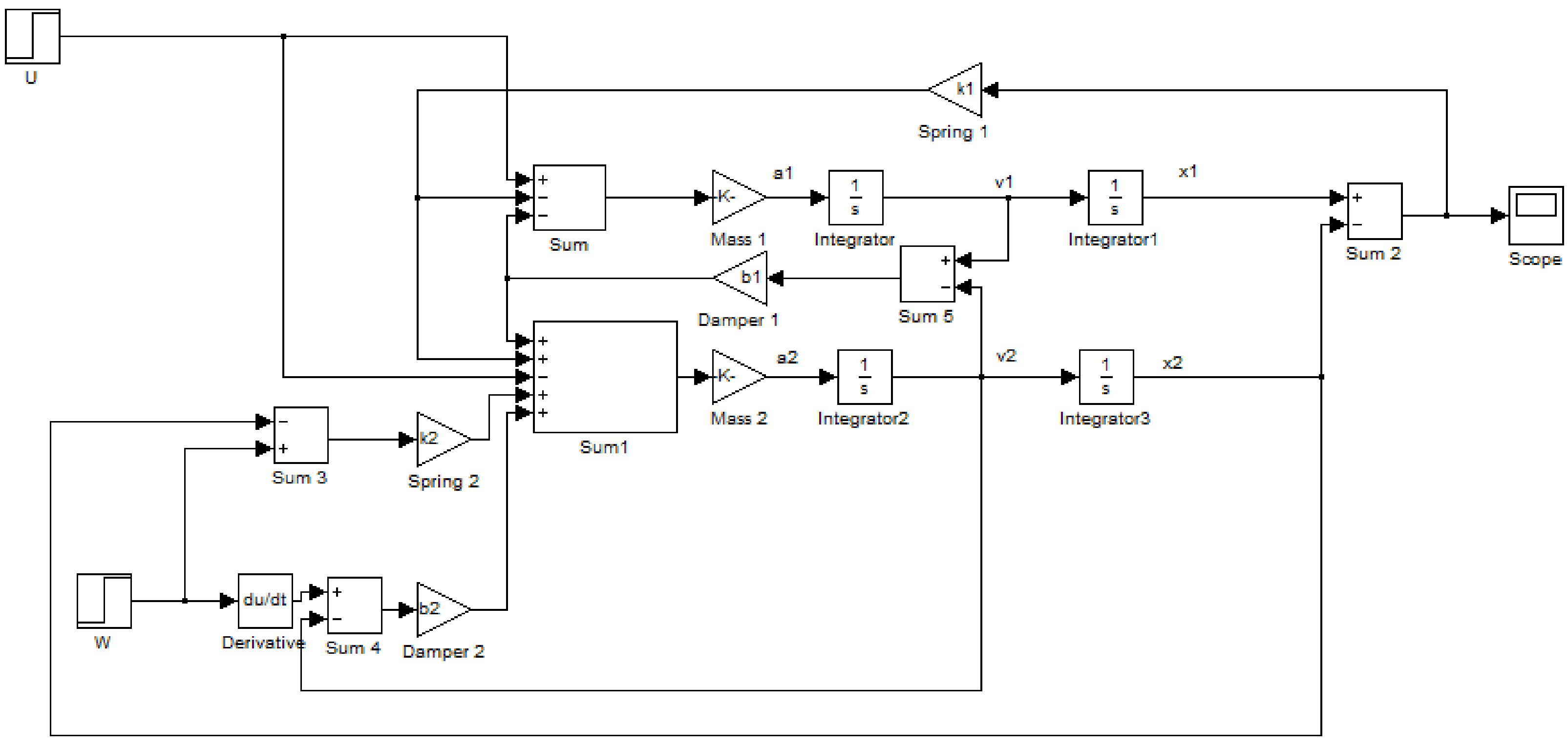
$$\begin{bmatrix} (M_1 s^2 + B_1 s + K_1) & -(B_1 s + K_1) \\ -(B_1 s + K_1) & M_2 s^2 + (B_1 + B_2)s + (K_1 + K_2) \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} U(s) \\ (B_2 s + K_2)W(s) - U(s) \end{bmatrix}$$

Transfer function

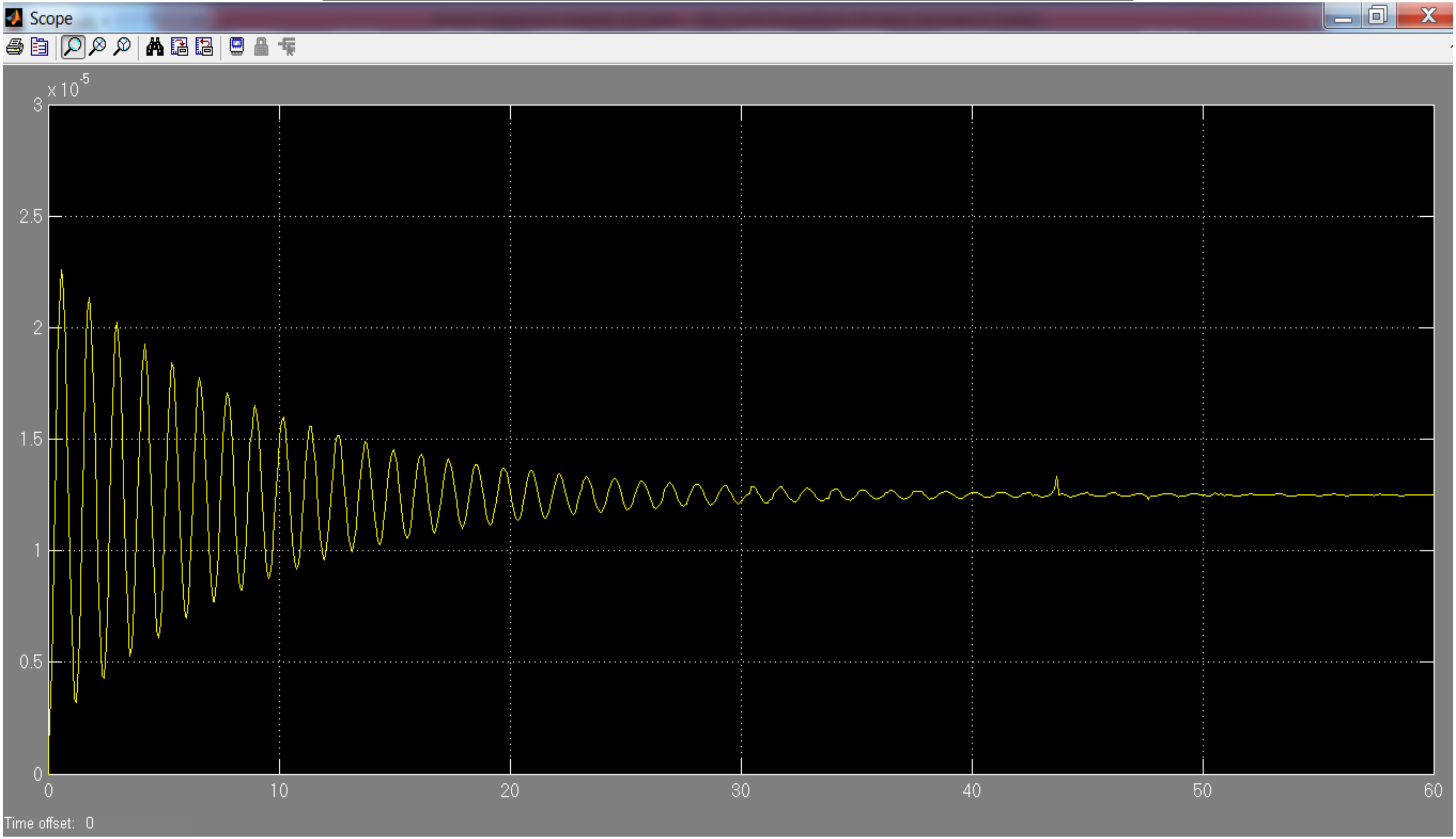
$$G_1(s) = \frac{X_1(s) - X_2(s)}{U(s)} = \frac{(M_1 + M_2)s^2 + B_2 s + K_2}{\Delta}$$

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)} = \frac{-M_1 B_2 s^3 - M_1 K_2 s^2}{\Delta}$$

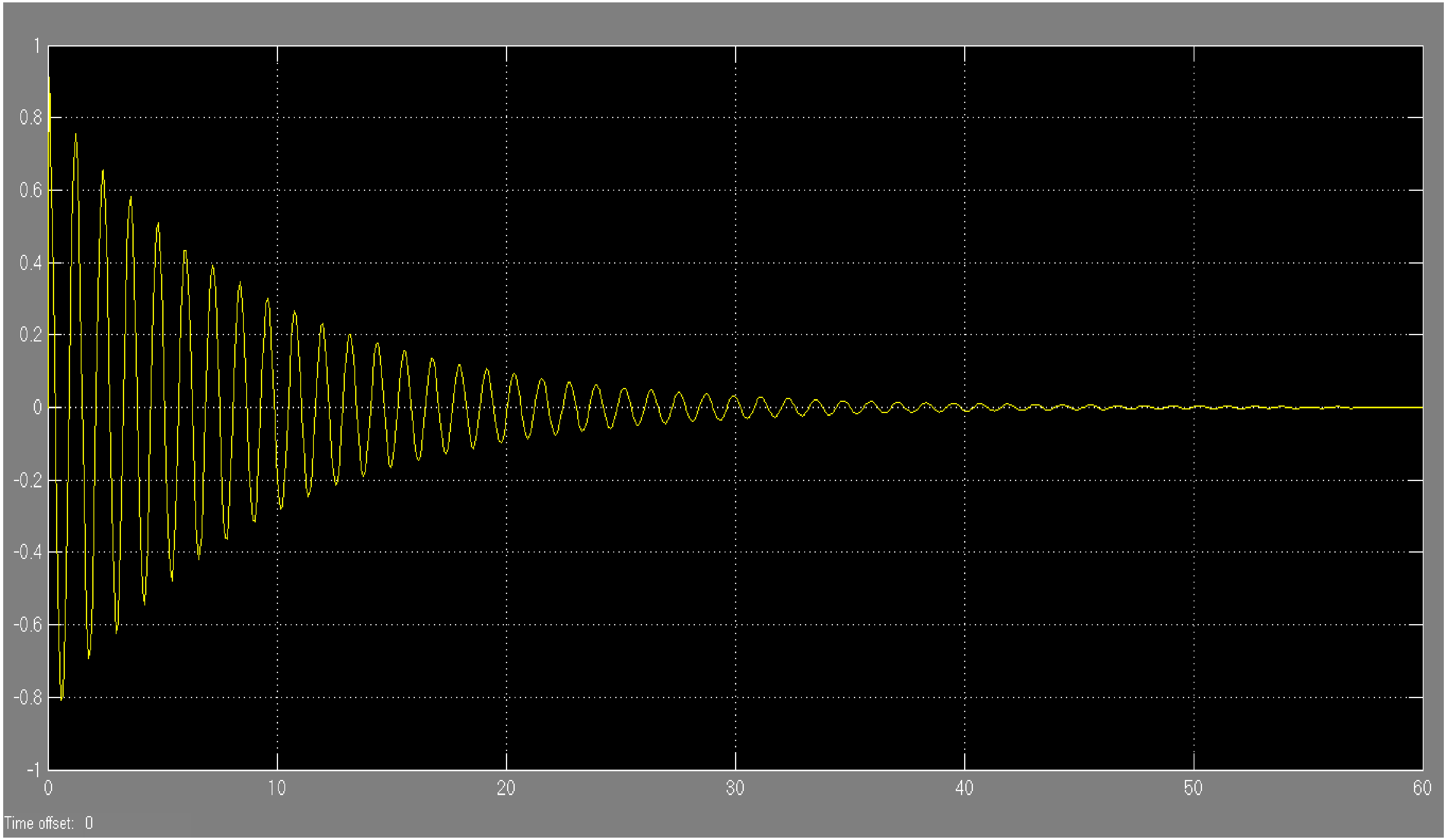
# Matlab Simulink model



# Amplitude vs time(s)



# With Bump



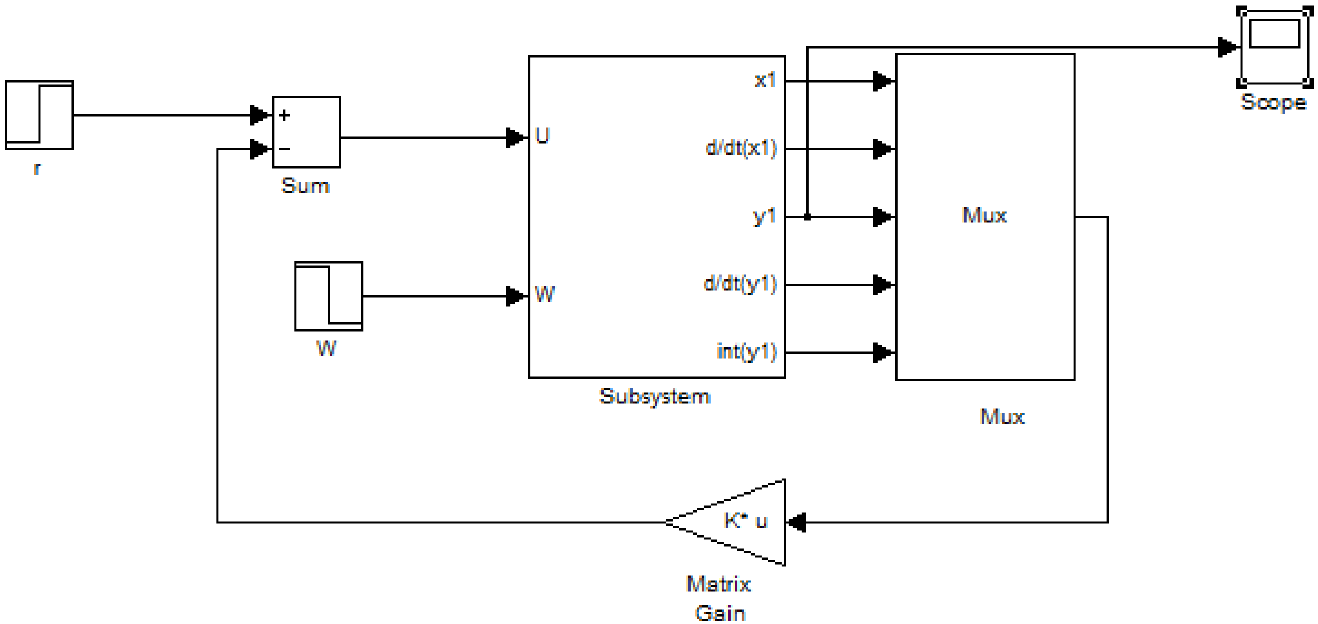
# Control System

- Requirement
- Ex – When Bus run 10 cm high bus body should not oscillate more than +/- 5 mm and stop oscillating with in 5 sec

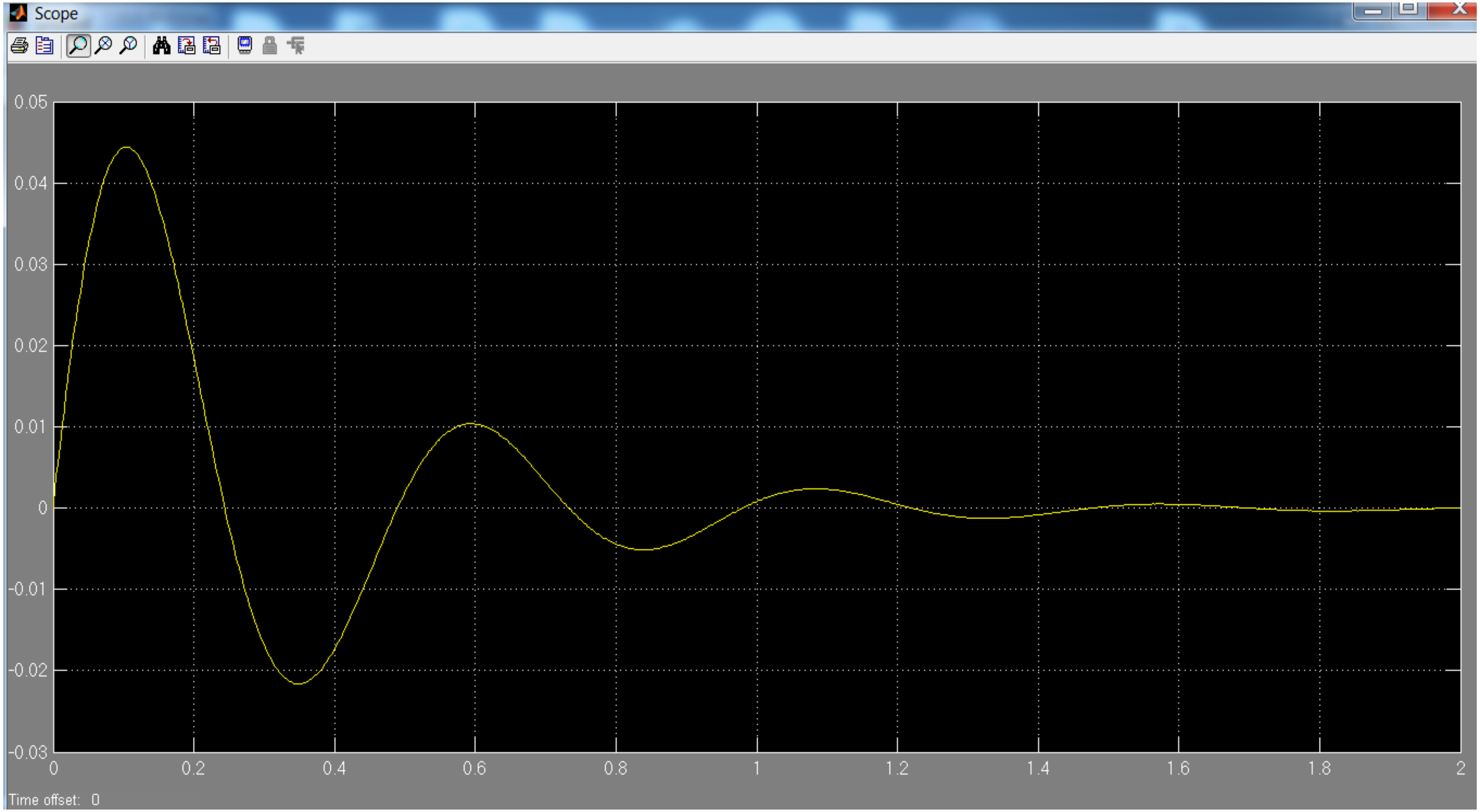
# Characteristics of P, I, and D Controllers

| CL RESPONSE | RISE TIME | OVERSHOOT | SETTLING TIME | S-S ERROR |
|---|---|---|---|---|
| Kp | Decrease | Increase | Small Change | Decrease |
| Ki | Decrease | Increase | Increase | Eliminate |
| Kd | Small Change | Decrease | Decrease | No Change |

# Matlab Simulink model

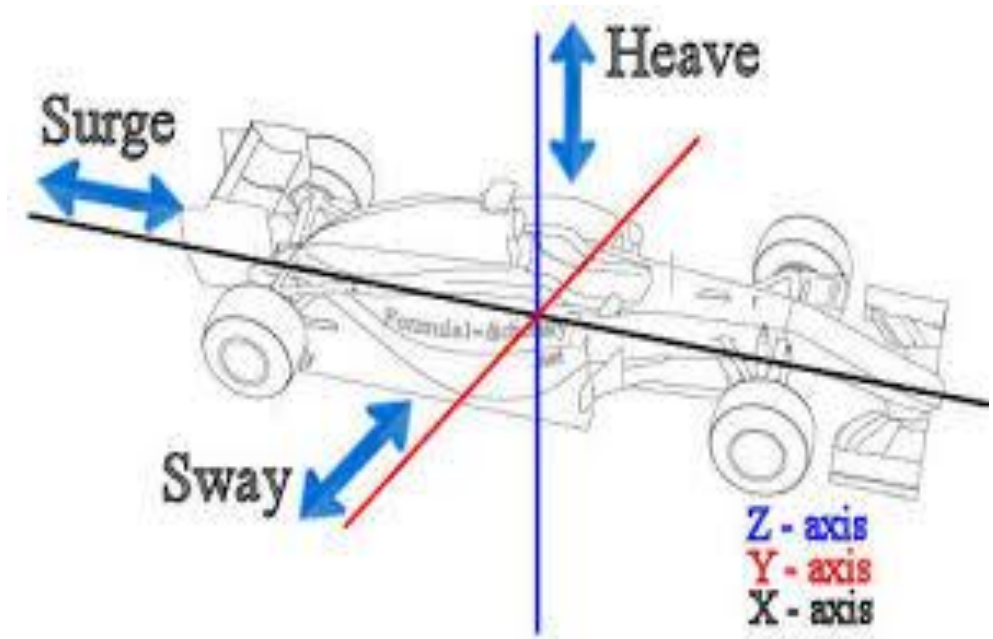


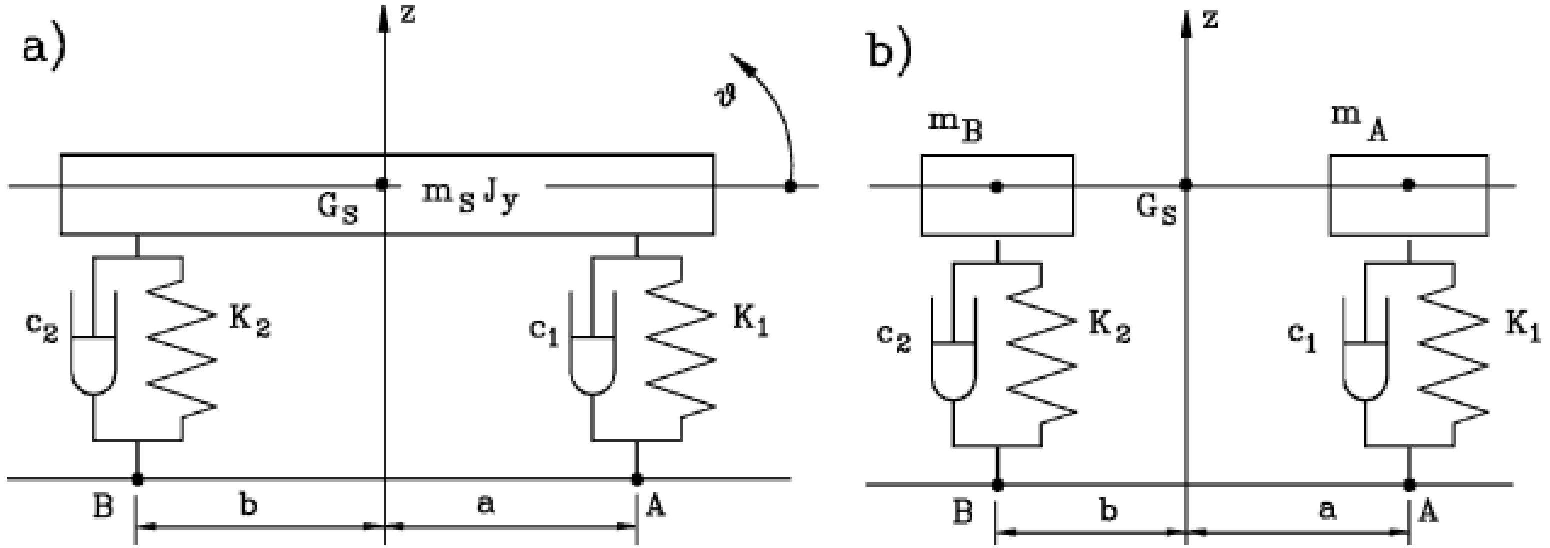
# Result


Scope
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time offset: 0

# HEAVE AND PITCH MOTION

- The translational motion of the vehicle along the Z-axis is called  “HEAVE “ or “BOUNCE”
- The rotation of the vehicle about X-axis is called “PITCH”.






HALF AND QUARTER MODELS

Equations of motion in half model,

$$\begin{bmatrix} m_S & 0 \\ 0 & J_y \end{bmatrix} \begin{Bmatrix} \ddot{Z}_s \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -ac_1 + bc_2 \\ -ac_1 + bc_2 & a^2 c_1 + b^2 c_2 \end{bmatrix} \begin{Bmatrix} \dot{Z}_s \\ \dot{\theta} \end{Bmatrix} +$$

$$+ \begin{bmatrix} K_1 + K_2 & -aK_1 + bK_2 \\ -aK_1 + bK_2 & a^2 K_1 + b^2 K_2 \end{bmatrix} \begin{Bmatrix} Z_s \\ \theta \end{Bmatrix} =$$

$$= \begin{Bmatrix} c_1 \dot{h}_A + c_2 \dot{h}_B + K_1 h_A + K_2 h_B \\ -ac_1 \dot{h}_A + bc_2 \dot{h}_B - aK_1 h_A + bK_2 h_B \end{Bmatrix} .$$

The coordinate transformation matrix for beam model to a pair of quarter cars is,

$$\begin{bmatrix} Zs \\ \theta \end{bmatrix} = \frac{1}{l}\begin{bmatrix} b & a \\ -1 & 1 \end{bmatrix}\begin{bmatrix} Z_A \\ Z_B \end{bmatrix}$$

The mass matrix in new coordinates is,

$$M' = T^T M T$$

Now, the equations of motion for quarter model is,

$$\frac{m_S}{l^2}\begin{bmatrix} b^2+r_y^2 & ab-r_y^2 \\ ab-r_y^2 & a^2+r_y^2 \end{bmatrix}\begin{Bmatrix} \ddot{Z}_A \\ \ddot{Z}_B \end{Bmatrix}+\begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix}\begin{Bmatrix} \dot{Z}_A \\ \dot{Z}_B \end{Bmatrix}+$$

$$+\begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}\begin{Bmatrix} Z_A \\ Z_B \end{Bmatrix}=\begin{Bmatrix} c_1\dot{h}_A+K_1h_A \\ c_2\dot{h}_B+K_2h_B \end{Bmatrix},$$

To calculate the natural frequencies of the system,

put ,

$$\begin{bmatrix} Z_A \\ Z_B \end{bmatrix} = \begin{bmatrix} Z_{AO} \\ Z_{BO} \end{bmatrix} e^{iwt}$$

The characteristic equation becomes ,

$$\det \left| \frac{-w^2 m}{l} \begin{bmatrix} b^2 + r_y^2 & ab - r_y^2 \\ ab - r_y^2 & a^2 + r_y^2 \end{bmatrix} + \begin{bmatrix} K1 & 0 \\ 0 & K2 \end{bmatrix} \right| = 0$$

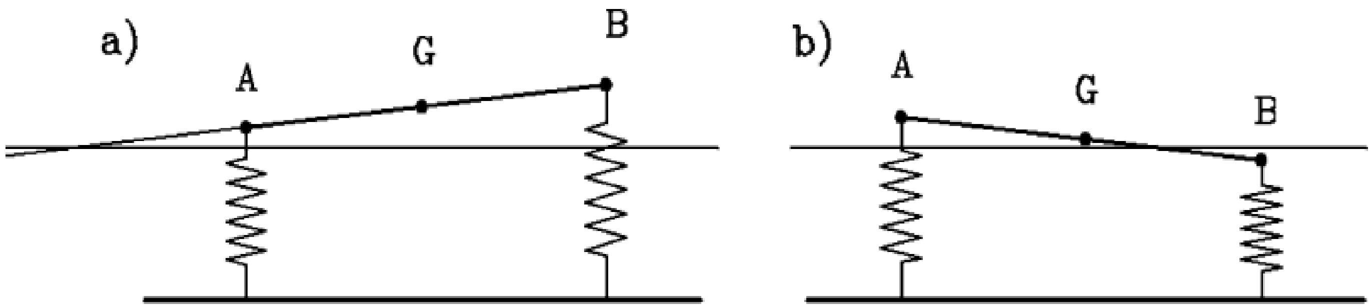
The roots (natural frequencies) of the equation are ,

$$\omega_i = \frac{\sqrt{\left(b^2 + r_y^2\right) K_2 + \left(a^2 + r_y^2\right) K_1 \pm \Delta}}{r_y \sqrt{2 m_S}}$$

The corresponding eigen vectors (mode shapes) are,

$$\mathbf{q}_i = \left\{ \begin{array}{c} \dfrac{\left(b^2 + r_y^2\right) K_2 - \left(a^2 + r_y^2\right) K_1 \mp \Delta}{2K_1 \left(ab - r_y^2\right)} \\ 1 \end{array} \right\}$$

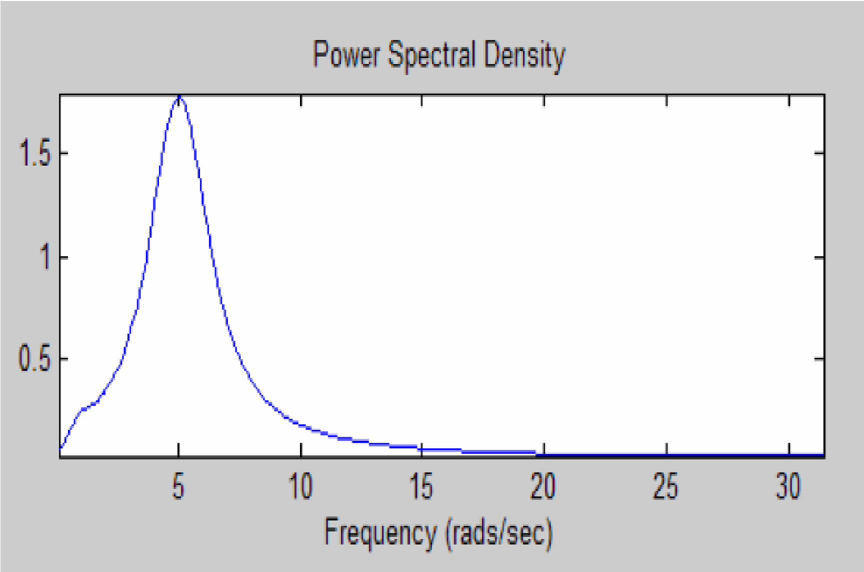
Different mode shapes are ,



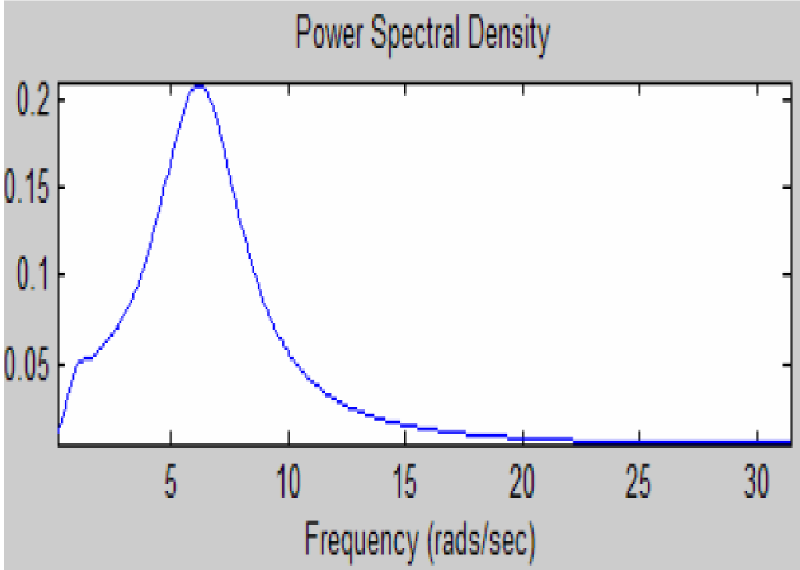
m=2000 kg , J y=2500 kgm2, k1=k2=30000 N/m, c1=c2=3000Ns/ m,a=1.5m,b=1m

| | ANALYTICAL | MATLAB | SIMULINK |
|---|---|---|---|
| Wn1 | 5.1399 | 5.1403 | 4.9676 |
| Wn2 | 6.5251 | 6.5252 | 6.1681 |
| First mode | -2.0963 | -2.0957 | |
| Second mode | 0.5963 | 0.5965 | |

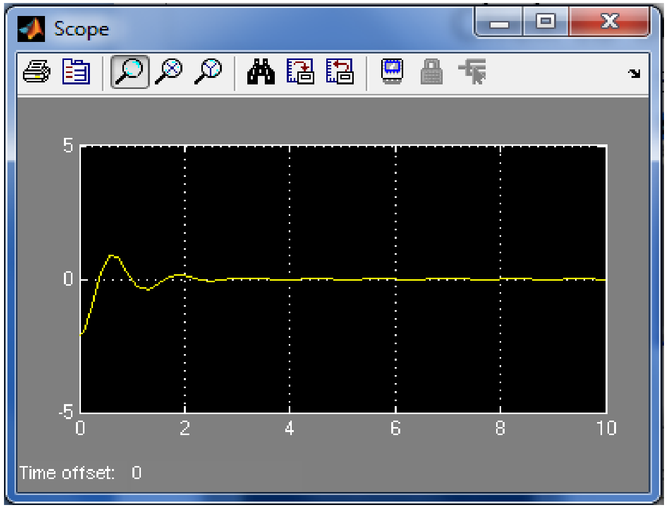
FIRST MODE :



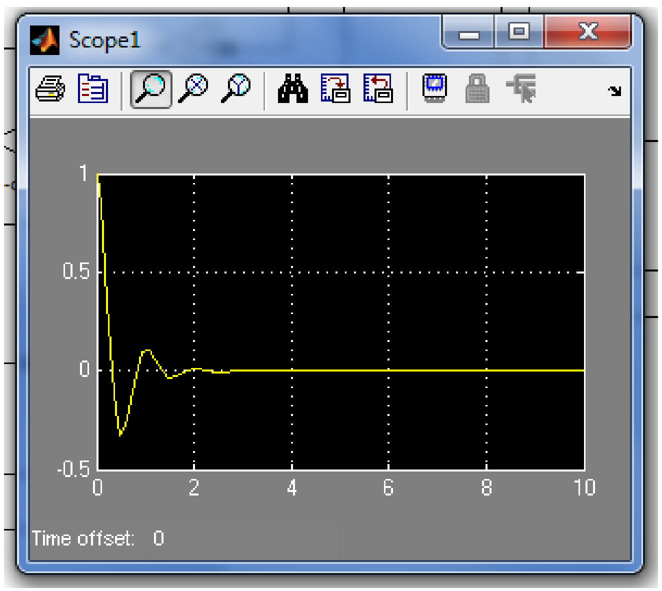

SECOND MODE :



## FIRST MODE :

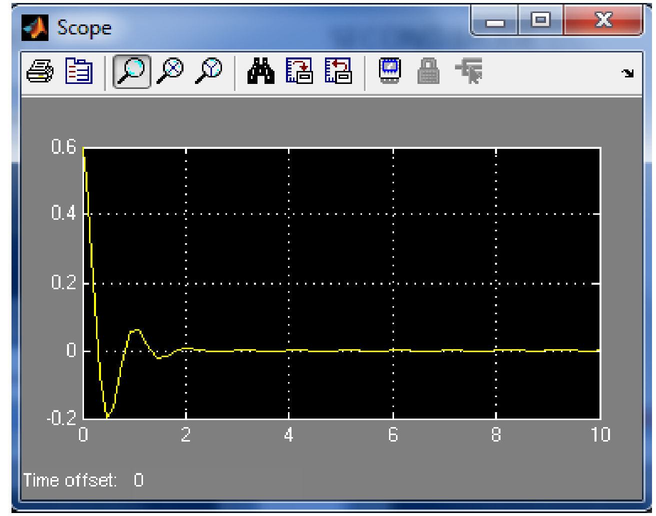



Time history for bounce

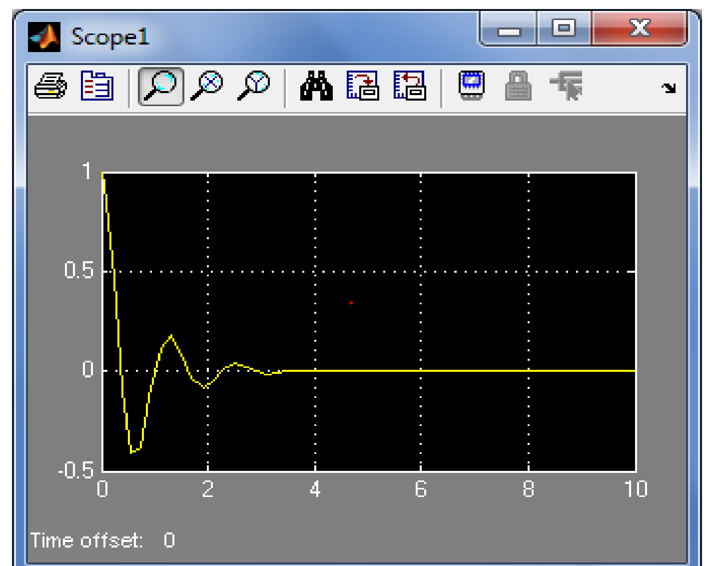



Time history for pitch

## SECOND MODE :



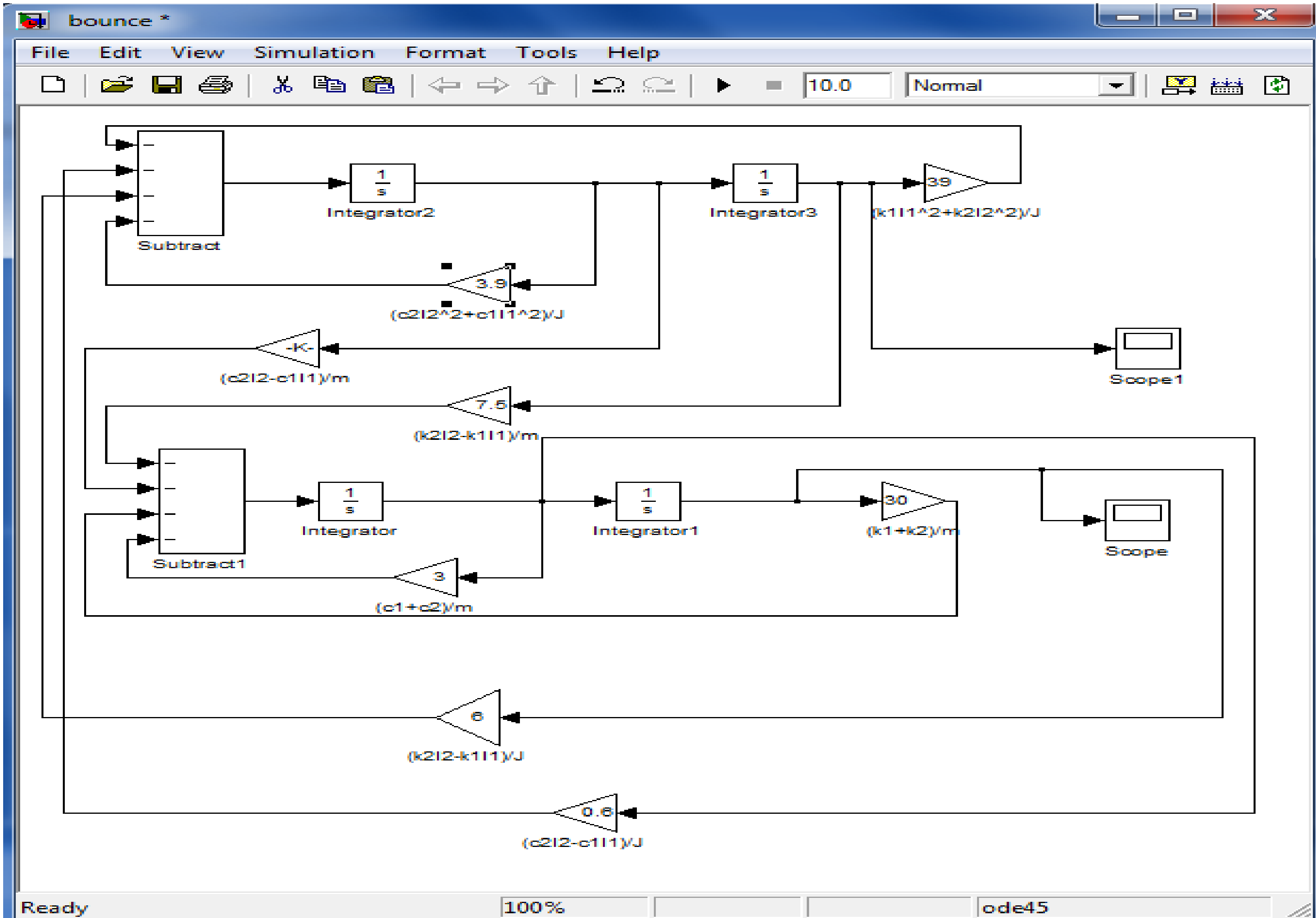

Time history for bounce




Time history for pitch

$$m\ddot{x} + (c_1 + c_2)\dot{x} + (c_2 l_2 - c_1 l_1)\dot{\theta} + (k_1 + k_2)x + (k_2 l_2 - k_1 l_1)\theta = k_1 y_1 + k_2 y_2 + c_1 \dot{y}_1 + c_2 \dot{y}_2$$

$$J\ddot{\theta} + (c_2 l_2 - c_1 l_1)\dot{x} + (c_2 l_2^2 + c_1 l_1^2)\dot{\theta} + (k_2 l_2 - k_1 l_1)x + (k_1 l_1^2 + k_2 l_2^2)\theta = k_2 l_2 y_2 - k_1 l_1 y_1 + c_2 l_2 y_2 - c_1 l_1 \dot{y}_1$$



Upon taking laplace transform to above equations we will arrive at the domain equations of steady response of the system as follows,

$$[ms^2 + (c_1 + c_2)s + k_1 + k_2]X(s) + [(c_2 l_2 - c_1 l_1)s + (k_2 l_2 - k_1 l_1)]\theta \mathrm{C}(s) = (k_1 + c_1 s)Y_1(s) + (k_2 + c_2 s)Y_2(s)$$

$$[(c_2 l_2 - c_1 l_1)s + k_2 l_2 - k_1 l_1]X(s) + [Js^2 + (c_2 l_2^2 + c_1 l_1^2)s + k_1 l_1^2 + k_2 l_2^2]\theta(s) = -(c_1 l_1 s + k_1 l_1)Y_1(s) + (c_2 l_2 s + k_2 l_2)Y_2(s)$$

Using Cramer’s rule and applying the principle of superposition, we can find the transfer functions for both bounce and pitch motions for each input Y1 and Y2

# TRANSFER FUNCTIONS USING MATLAB :

```
      7.5e006 s^3 + 1.088e008 s^2 + 6.75e008 s + 3.375e009
------------------------------------------------------------
5e006 s^4 + 3.45e007 s^3 + 4.013e008 s^2 + 1.125e009 s + 5.625e009
```

Transfer function = $\dfrac{X(s)}{Y_1(s)}$

```
      -6e006 s^3 - 8.25e007 s^2 - 4.5e008 s - 2.25e009
------------------------------------------------------------
5e006 s^4 + 3.45e007 s^3 + 4.013e008 s^2 + 1.125e009 s + 5.625e009
```

Transfer function = $\dfrac{\Theta(s)}{Y_1(s)}$

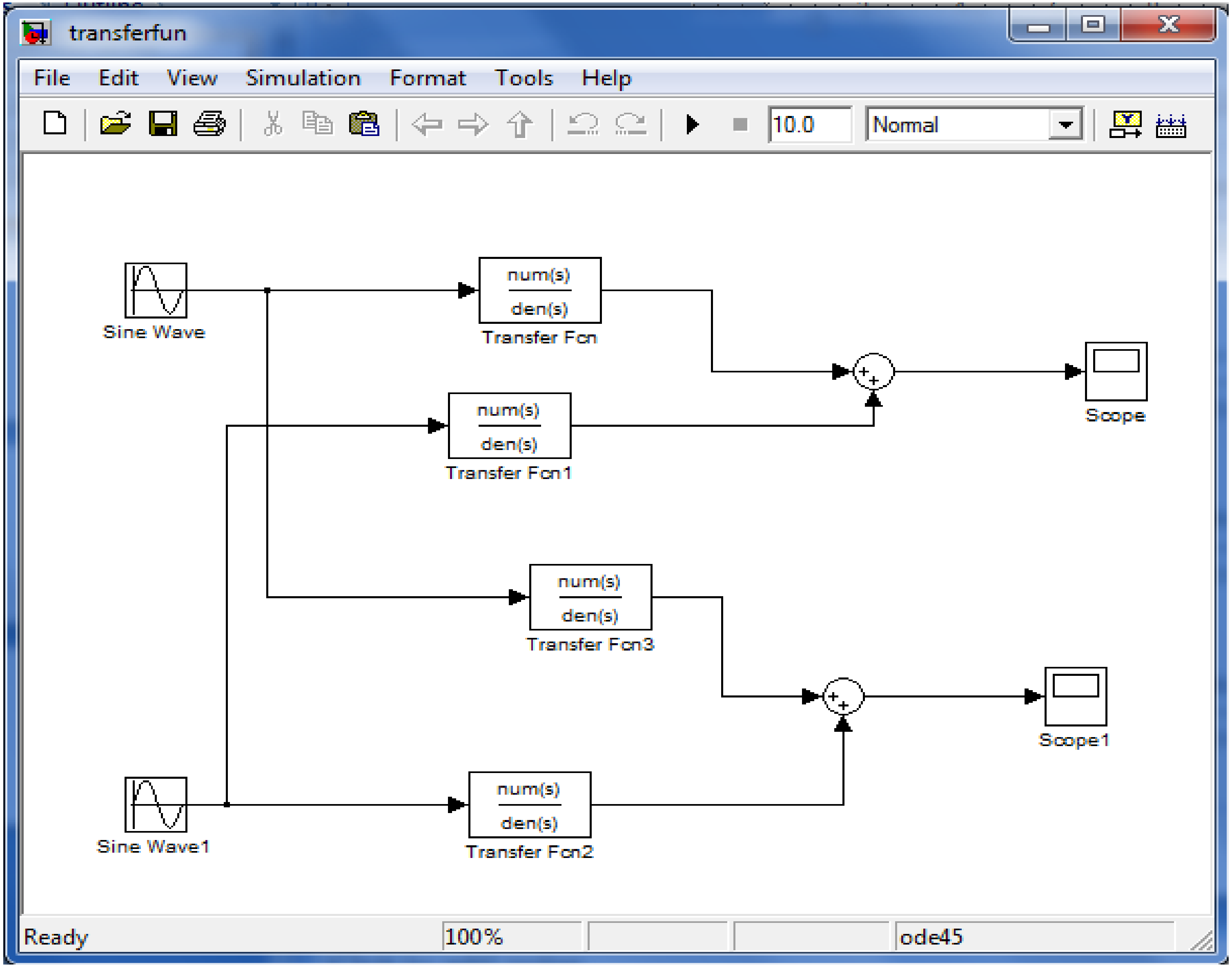
```
      7.5e006 s^3 + 9.75e007 s^2 + 4.5e008 s + 2.25e009
------------------------------------------------------------
5e006 s^4 + 3.45e007 s^3 + 4.013e008 s^2 + 1.125e009 s + 5.625e009
```

Transfer function = $\dfrac{X(s)}{Y_2(s)}$

```
      9e006 s^3 + 1.125e008 s^2 + 4.5e008 s + 2.25e009
------------------------------------------------------------
5e006 s^4 + 3.45e007 s^3 + 4.013e008 s^2 + 1.125e009 s + 5.625e009
```

Transfer function = $\dfrac{\Theta(s)}{Y_2(s)}$

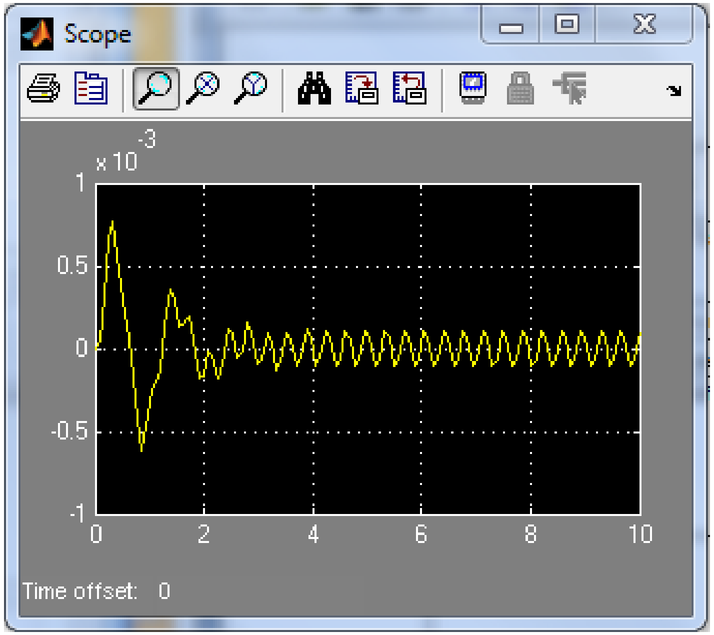
# SIMULINK MODEL :



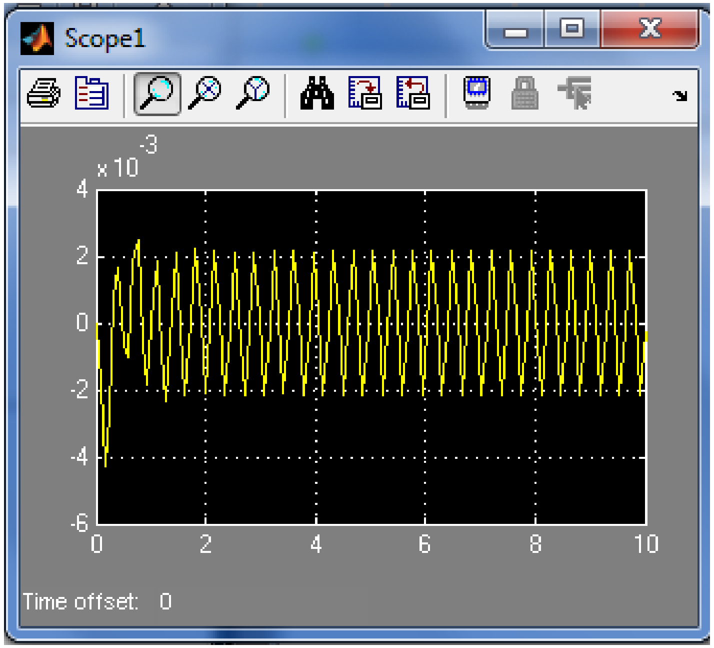
Approximating the road as sinusoidal with amplitude of 10 mm and wavelength as 5m,assuming velocity as 50km/hr ,these conditions provide the inputs  Y1 and Y2 for simulation,

$$Y1 = A \sin(wt)$$

$$Y2 = A \sin(wt - \Theta)$$

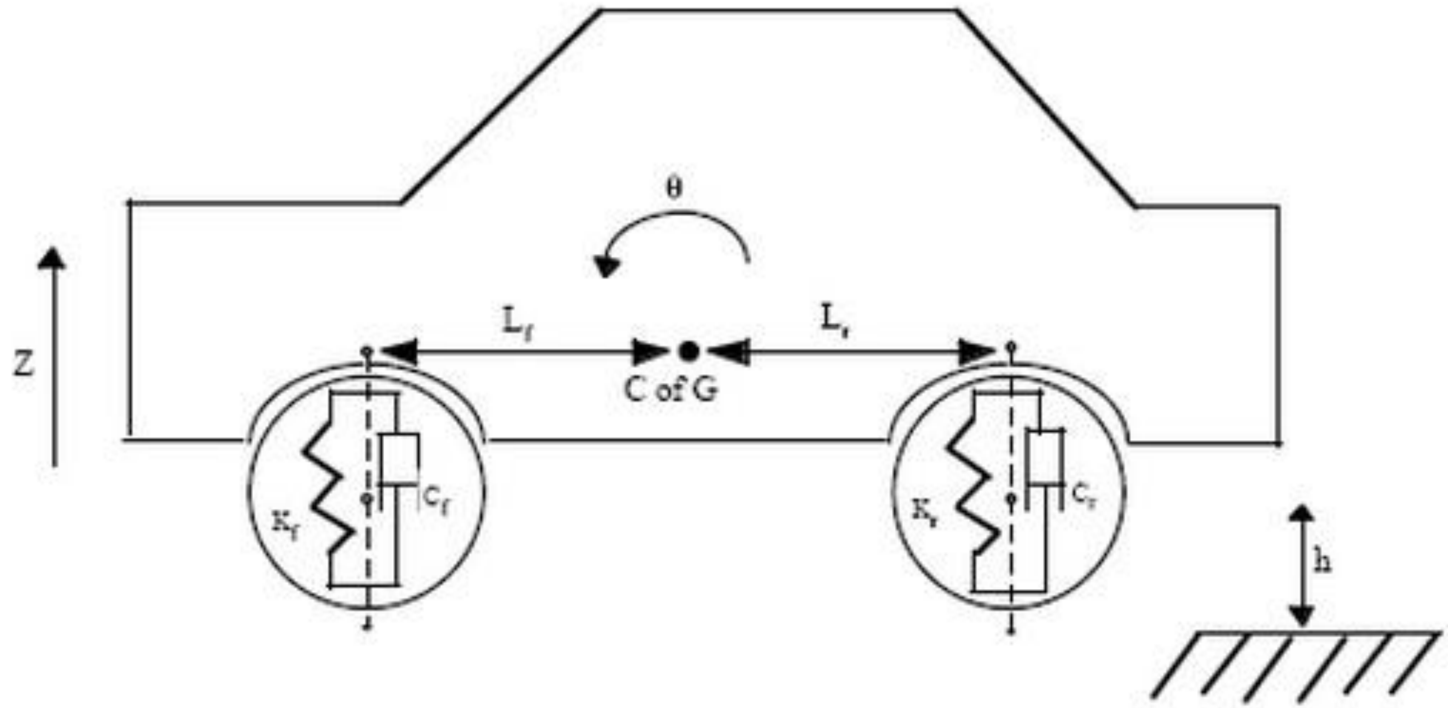



Bounce response

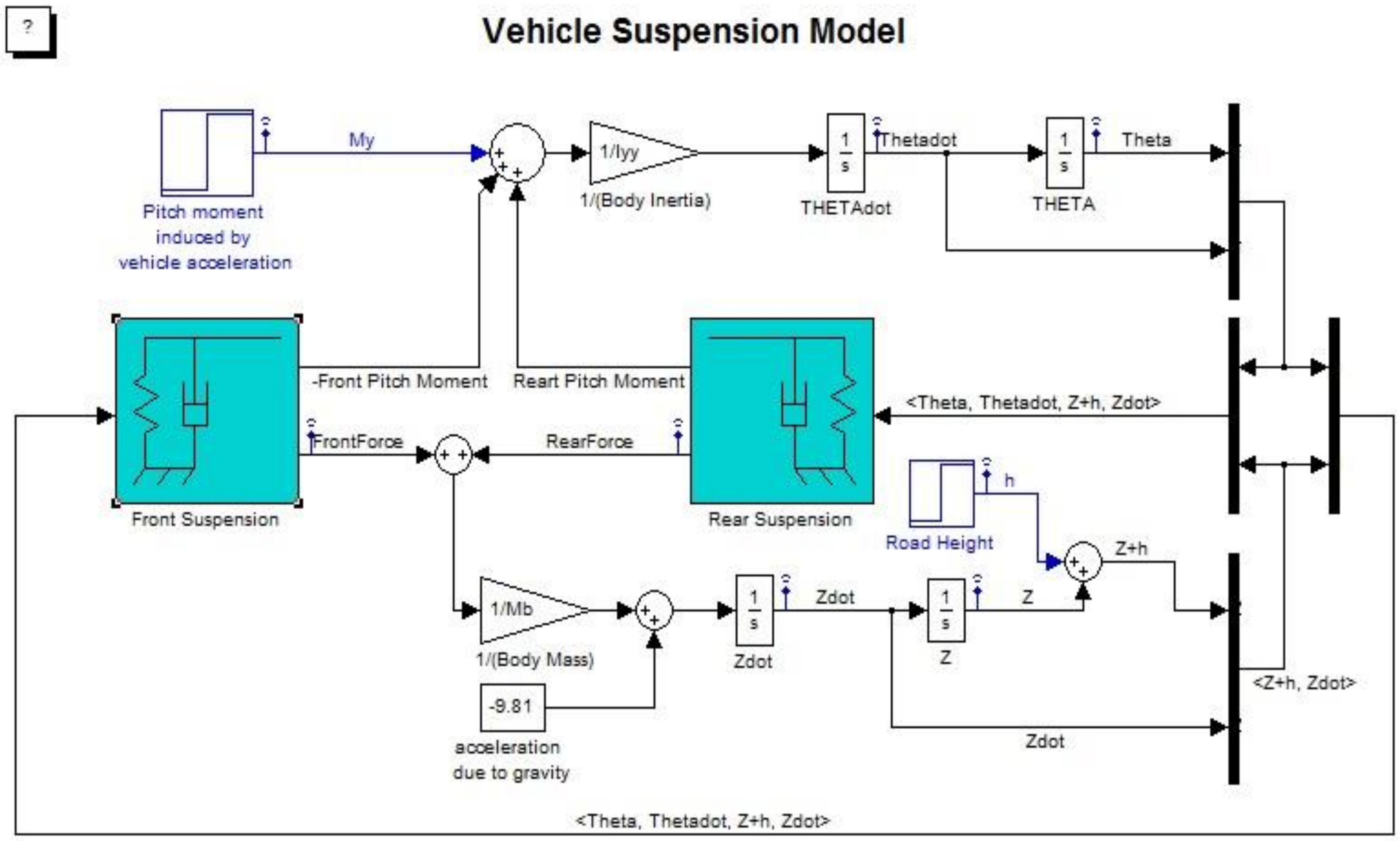



Pitch response

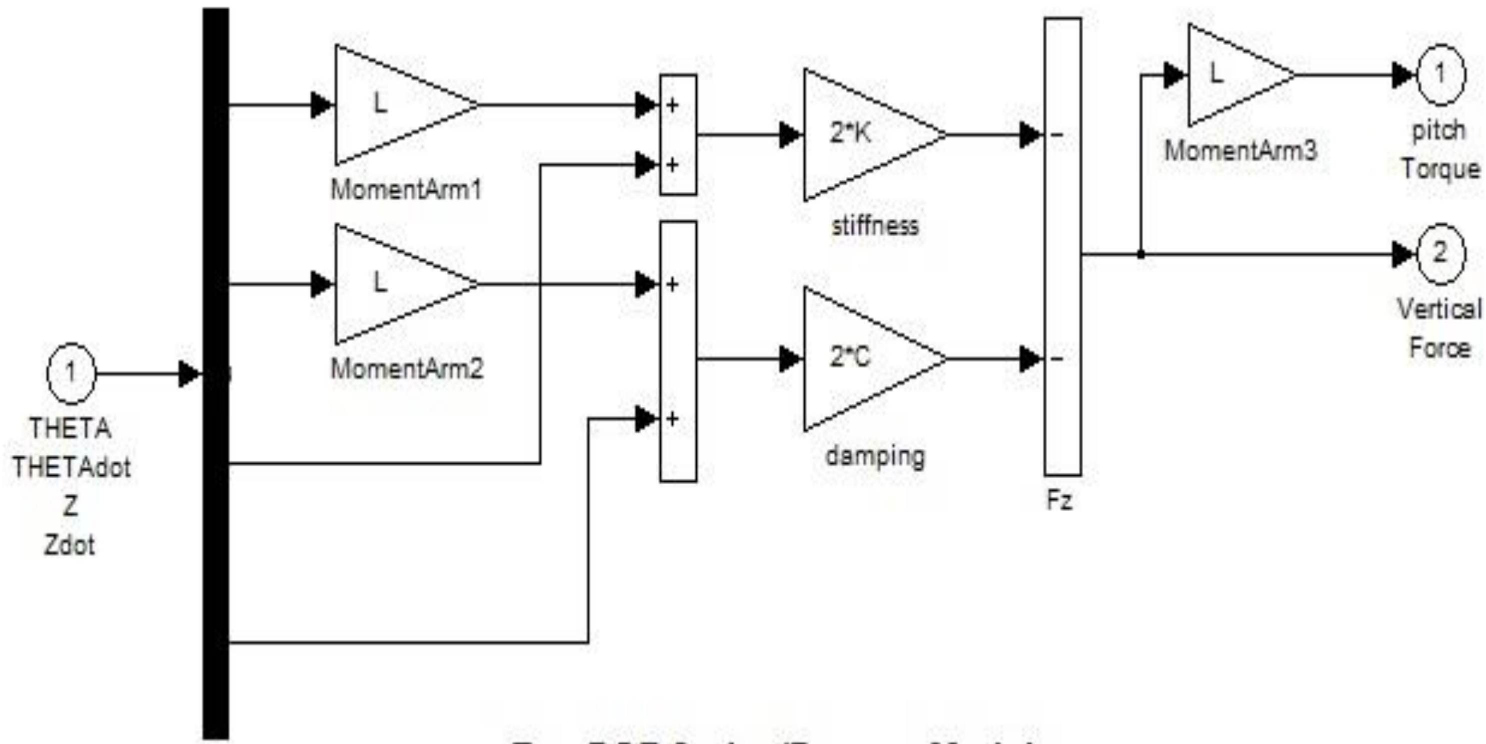
# HALF CAR MATLAB MODEL



# BLOCK DIAGRAM



# UNMASKED BLOCK DIAGRAM




Two DOF Spring/Damper Model

# MATHEMATICAL EQUATIONS

$F_{front} = 2K_f(L_f\,\theta - z) + 2C_f(L_f\,\dot{\theta} - \dot{z})$

$\theta =$ Pitch Angle

$z =$ Bounce

$L_f =$ Horizontal distance from gravity centre

$M_{front} = -L_{front}F_{front}$

$M_{front} =$ Pitch moment due to the front suspension

$F_{rear} = -2K_r(L_f\,\theta + z) - 2C_r(L_f\,\dot{\theta} + \dot{z})$

$M_{rear} = L_{rear}F_{rear}$

$m_b\ddot{z} = F_{front} + F_{rear} - m_b g$

$I_{yy}\,\ddot{\theta} = M_{front} + M_{rear} + M_y$

$m_b =$ Body mass

$M_y =$ Pitch moment induced by vehicle acceleration

$I_{yy} =$ Body moment inertia

# PARAMETERS USED

$L_f = 0.9$ m

$M_b = 1200$ kg

$L_r = 1.2$ m

$I_{YY} = 2100$ kg-m^2

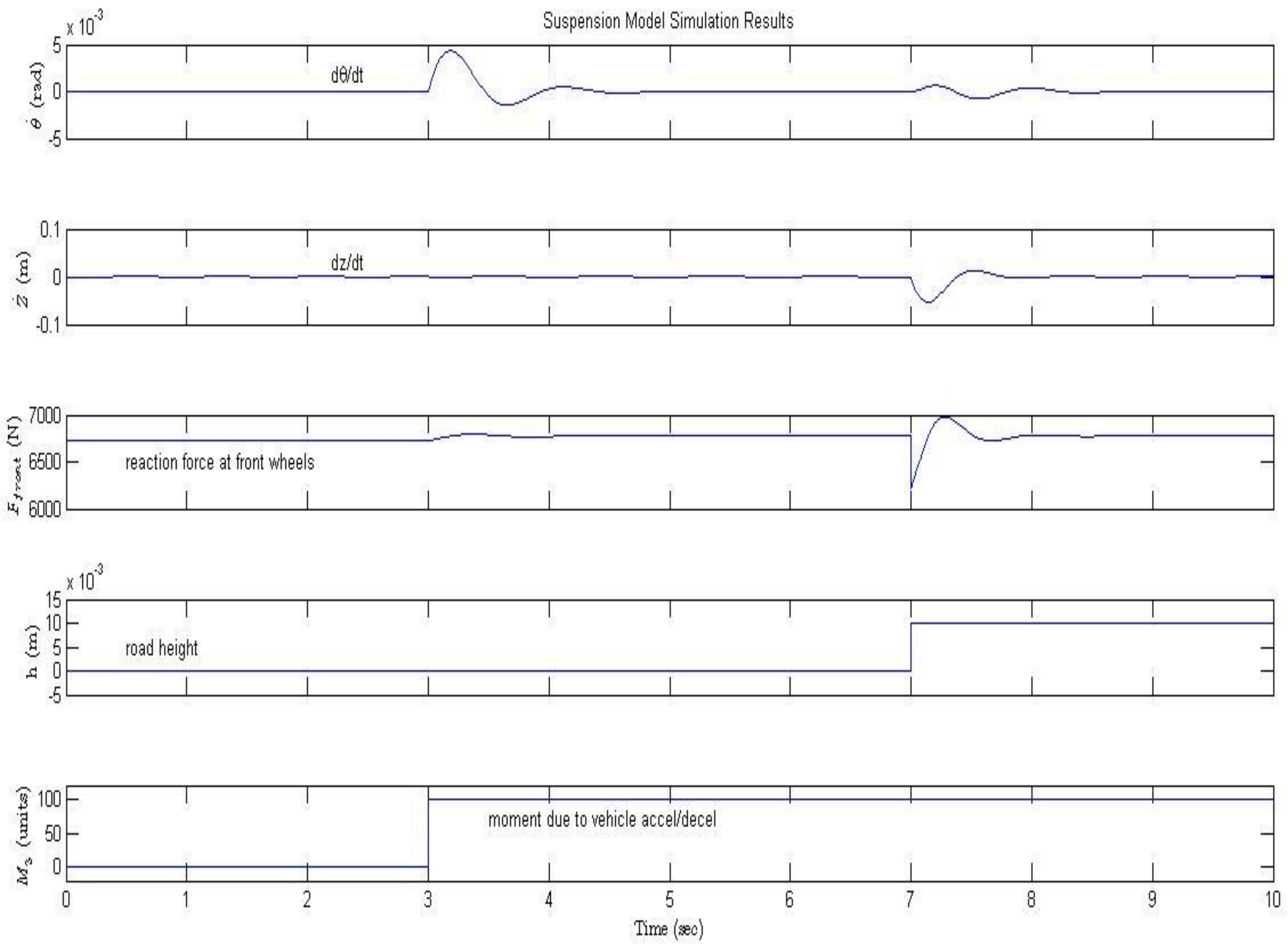
$K_f = 28000$ N/m

$K_r = 21000$ N/m

$C_f = 2500$ N sec/m

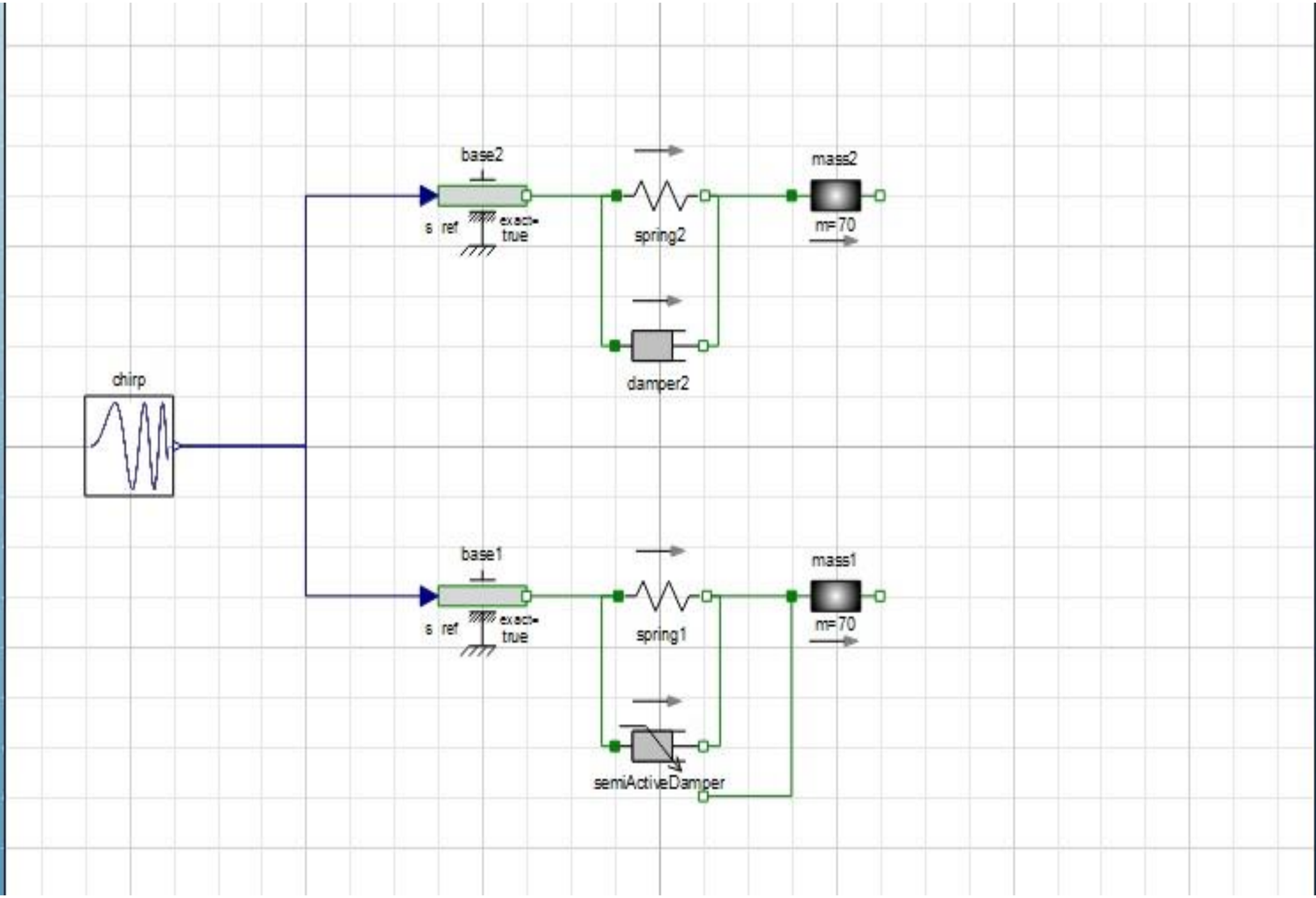
$C_r = 2000$ N sec/m

# RESULTS

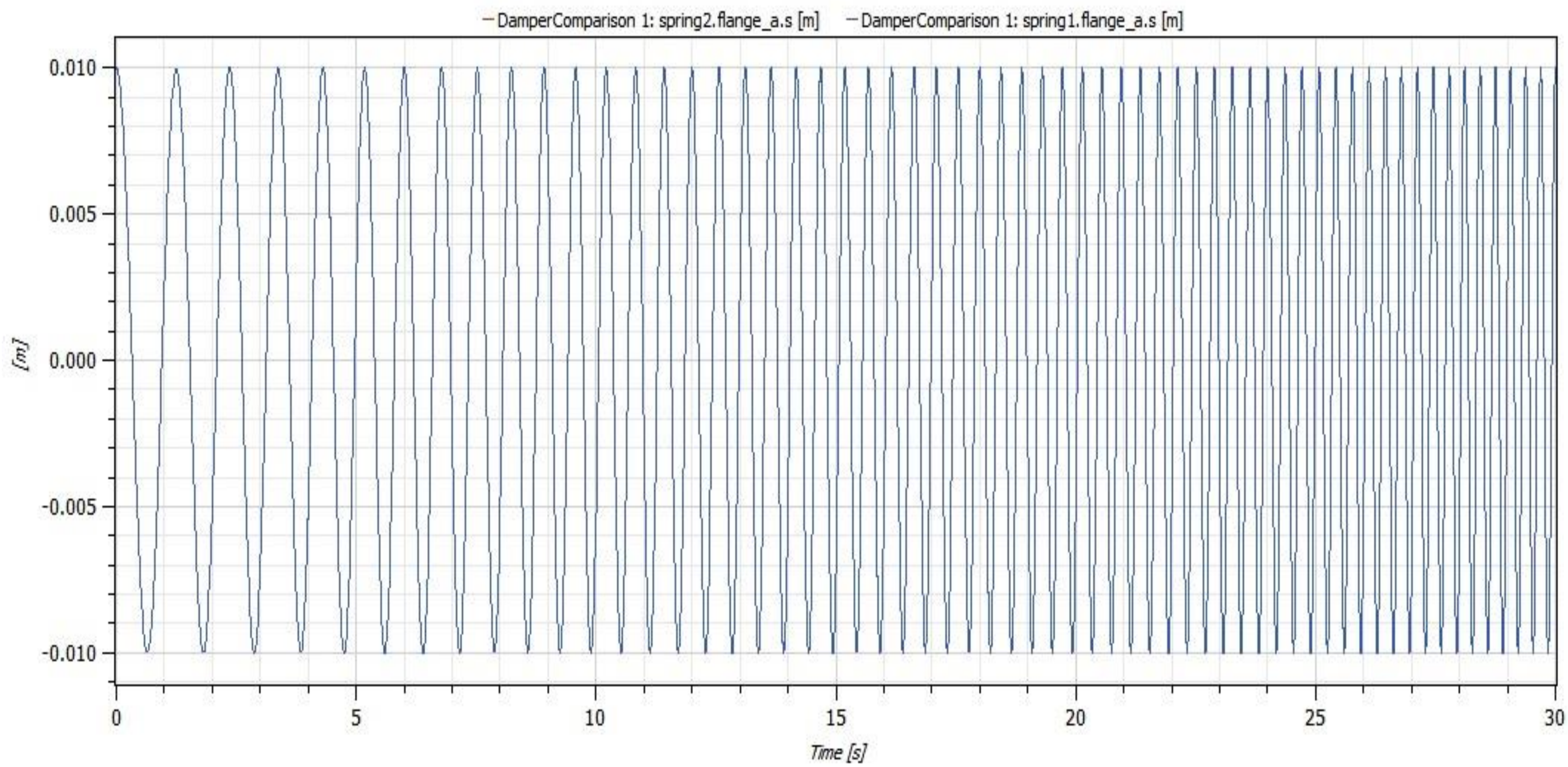


Suspension Model Simulation Results
x 10^-3
5
0
-5
θ̇ (rad)
dθ/dt
0.1
0
-0.1
Ż (m)
dz/dt
7000
6500
6000
F_front (N)
reaction force at front wheels
x 10^-3
15
10
5
0
-5
h (m)
road height
100
50
0
M_3 (units)
moment due to vehicle accel/decel
0
1
2
3
4
5
6
7
8
9
10
Time (sec)

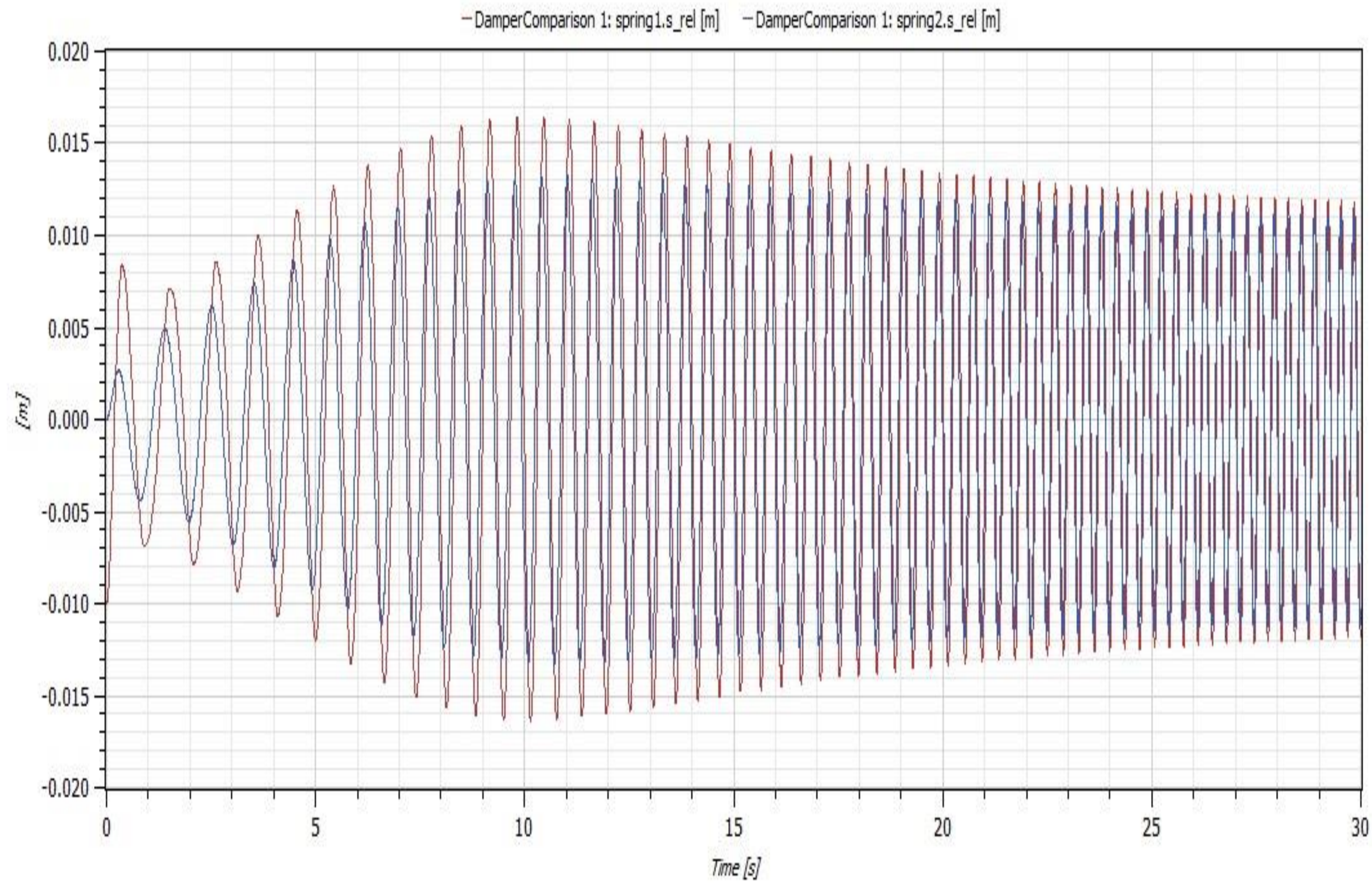
# SYSTEM MODELER



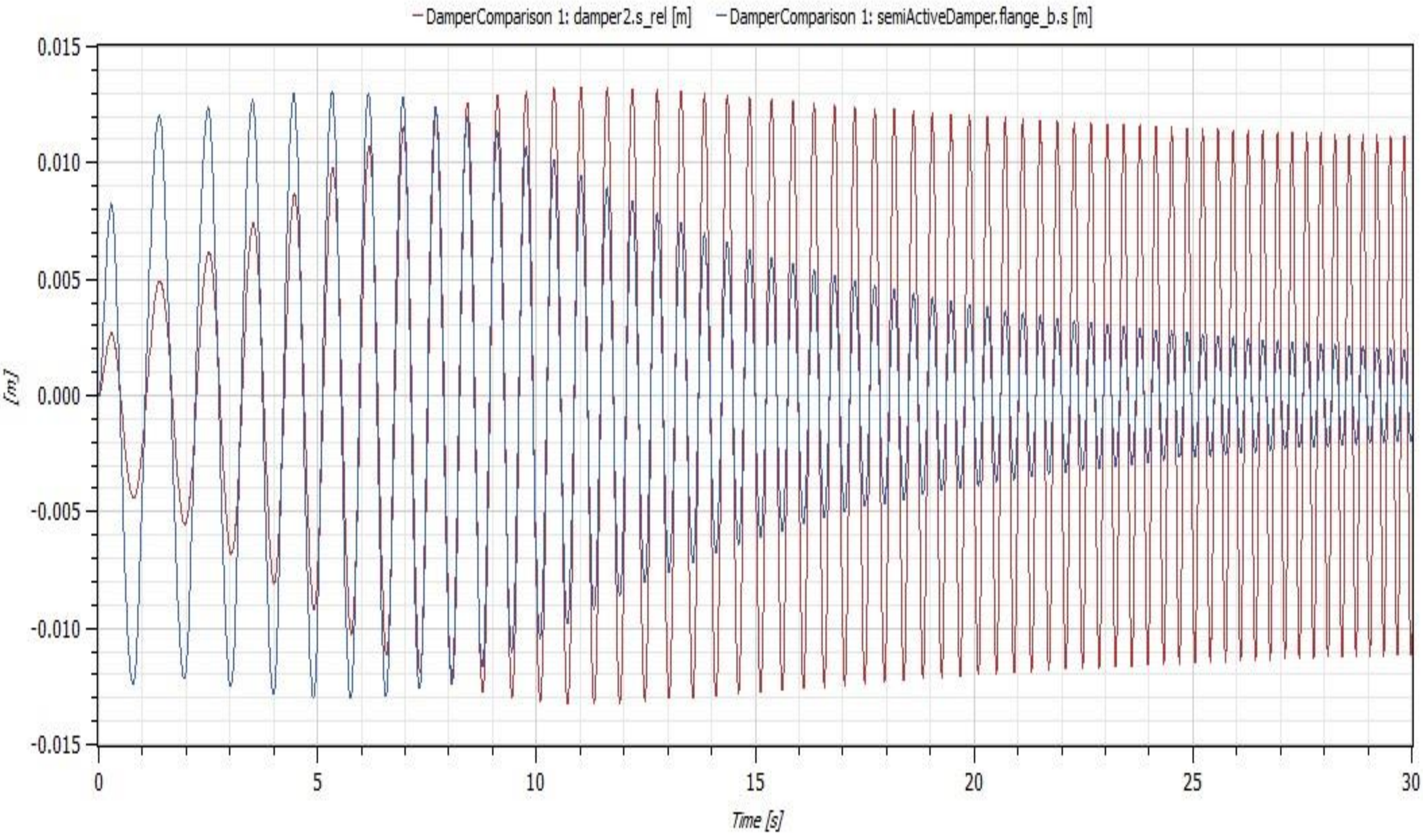
# COMPARISON RESULTS



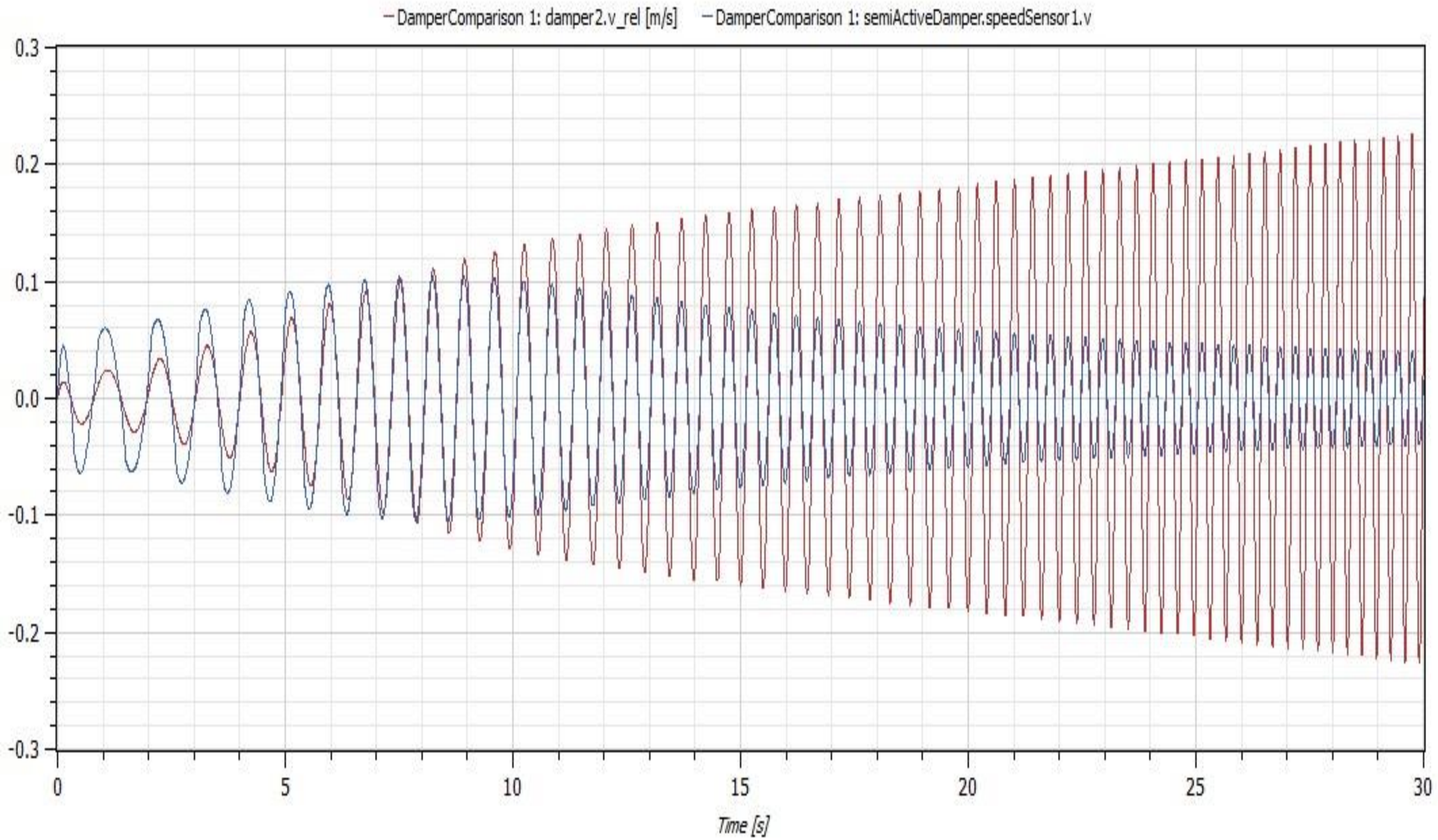
# S_RELATIVE(SPRING)



# S_RELATIVE(DAMPER)

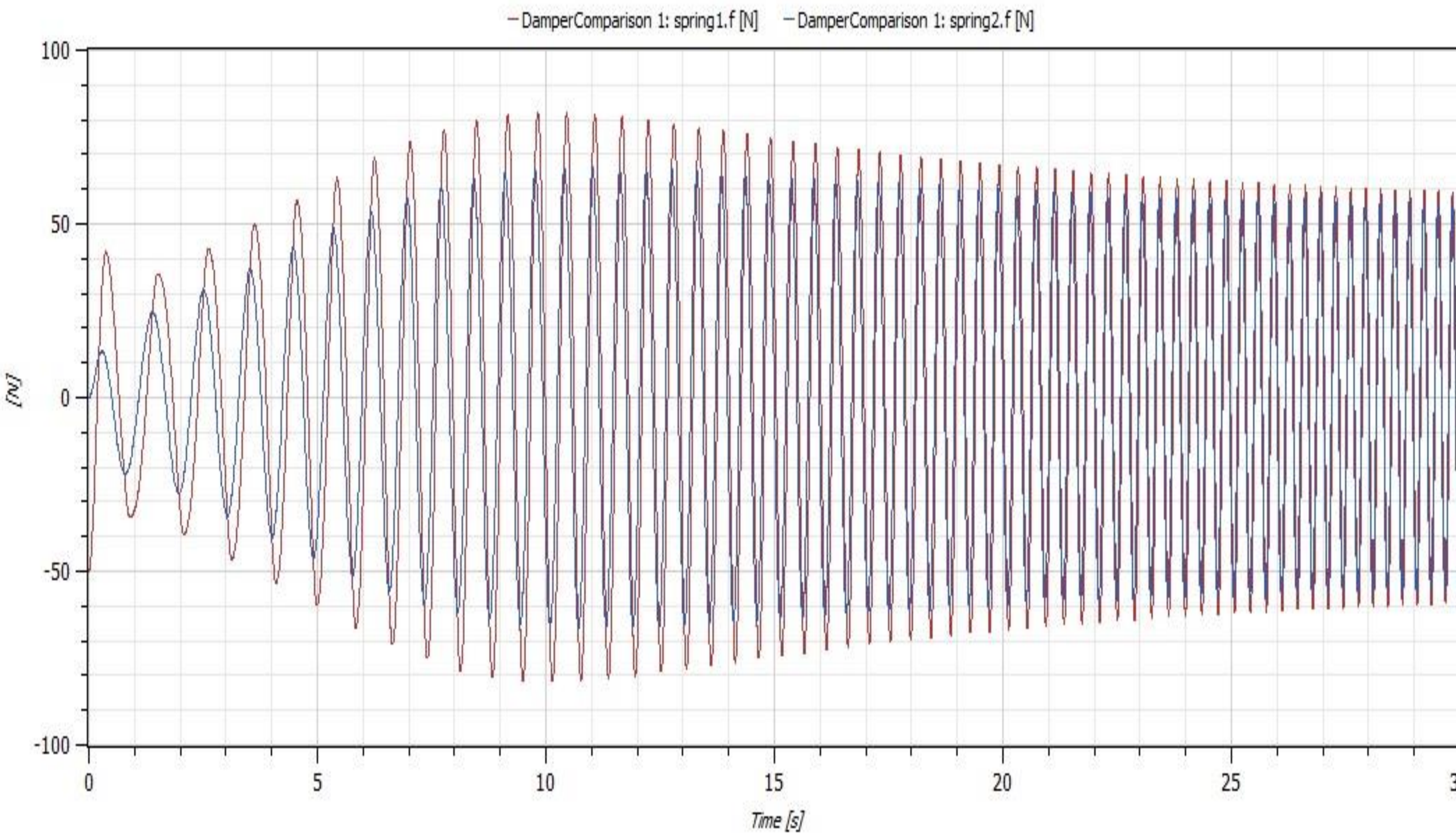


# V_RELATIVE



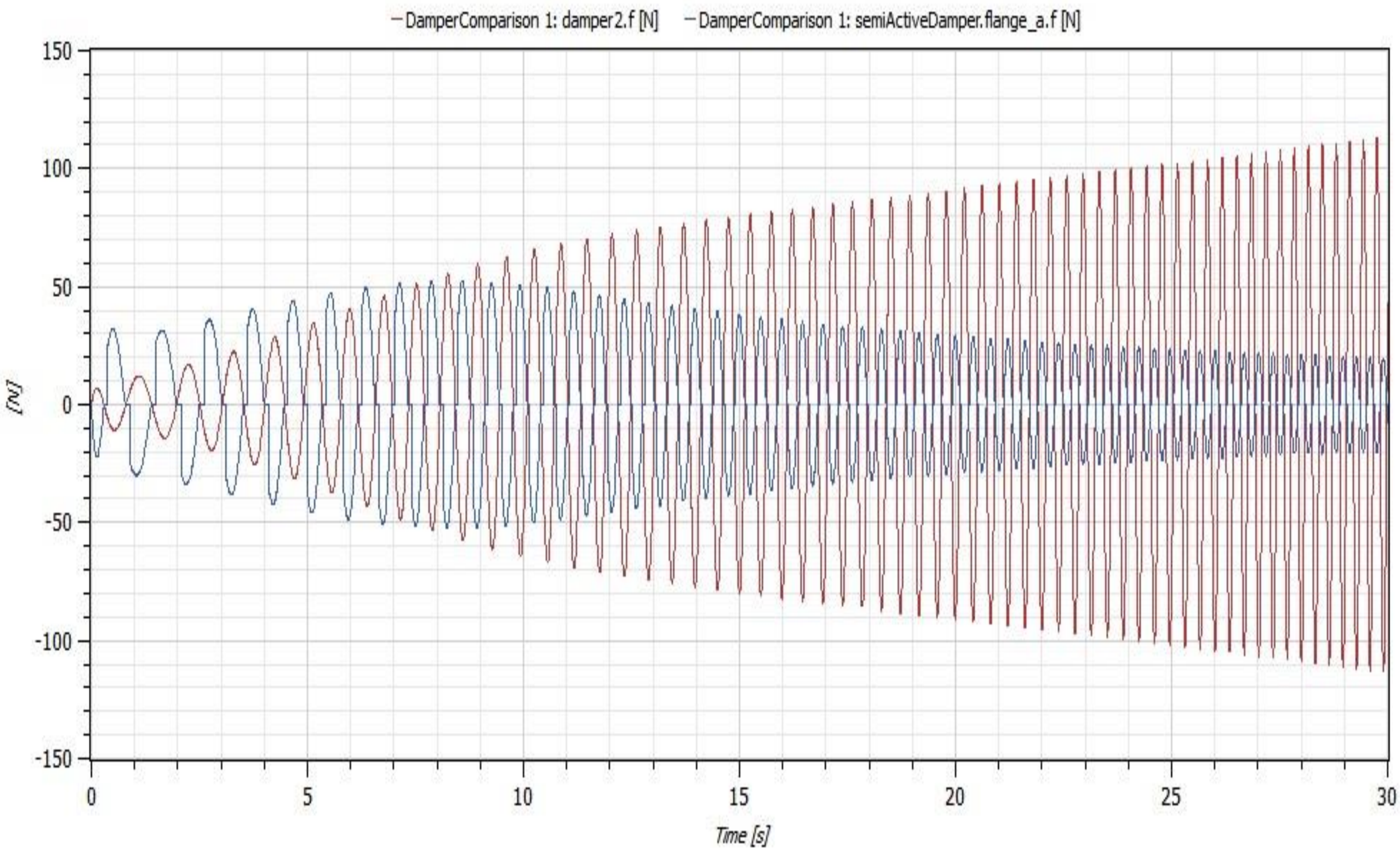
DamperComparison 1: damper2.v_rel [m/s]
DamperComparison 1: semiActiveDamper.speedSensor1.v
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
5
10
15
20
25
30
Time [s]

# FORCE



DamperComparison 1: spring1.f [N]
DamperComparison 1: spring2.f [N]
100
50
0
-50
-100
[N]
0
5
10
15
20
25
30
Time [s]

# FORCE(DAMPER)



# STABILITY

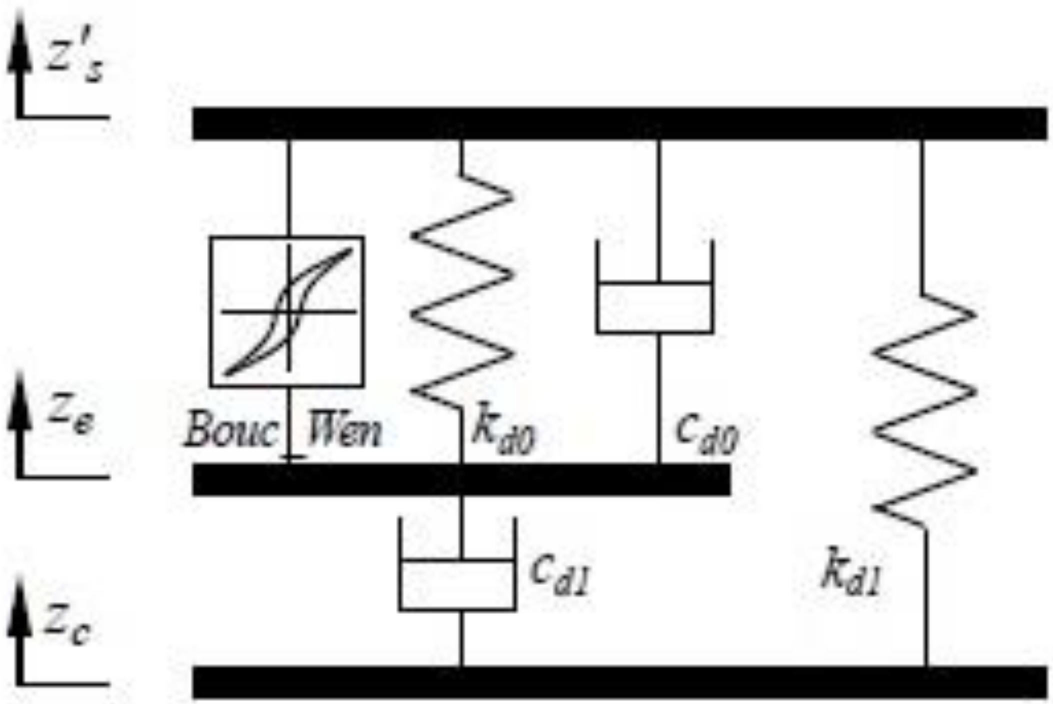



Fig.2 Improved Bouc-Wen model

# OBSERVATION

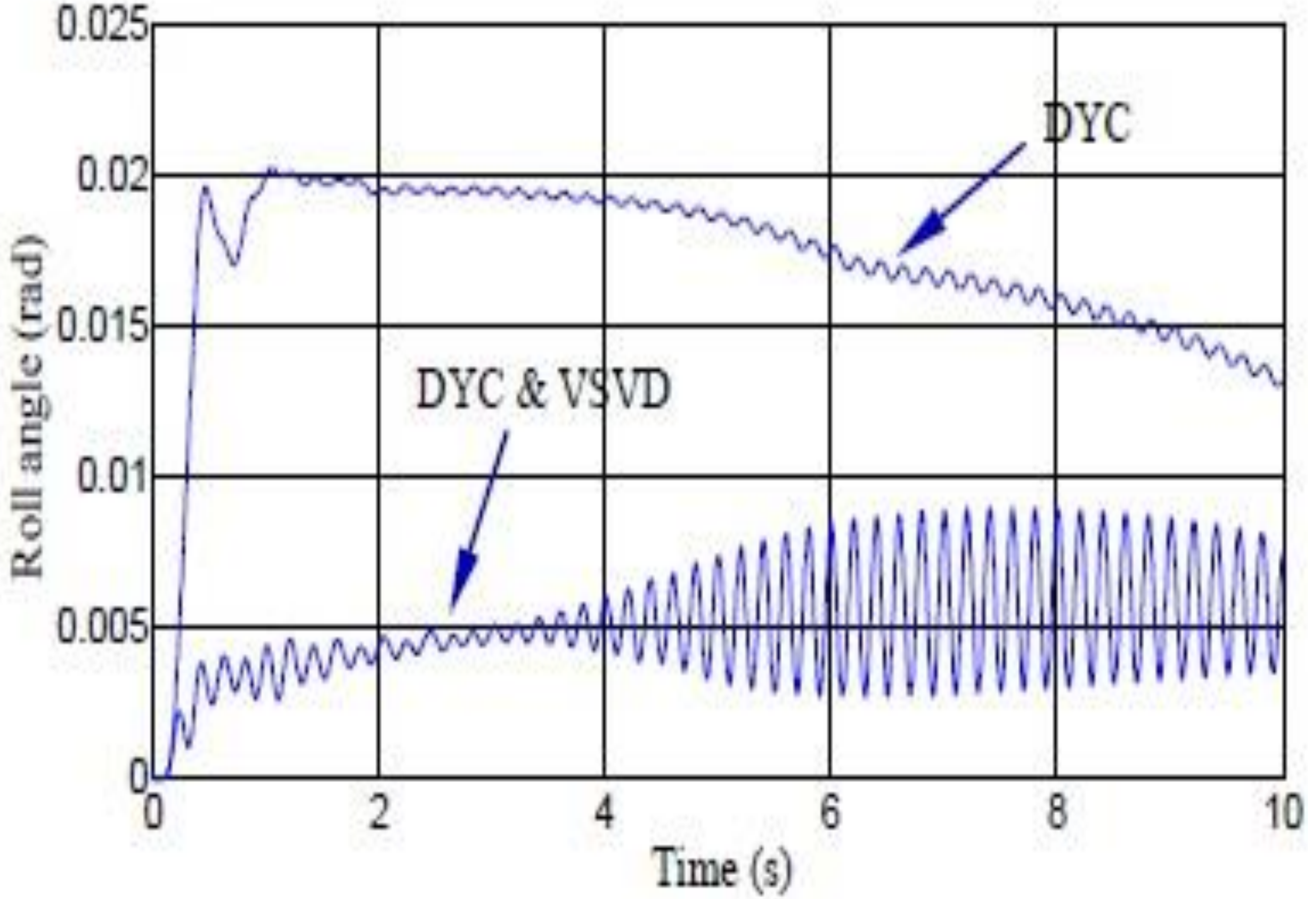
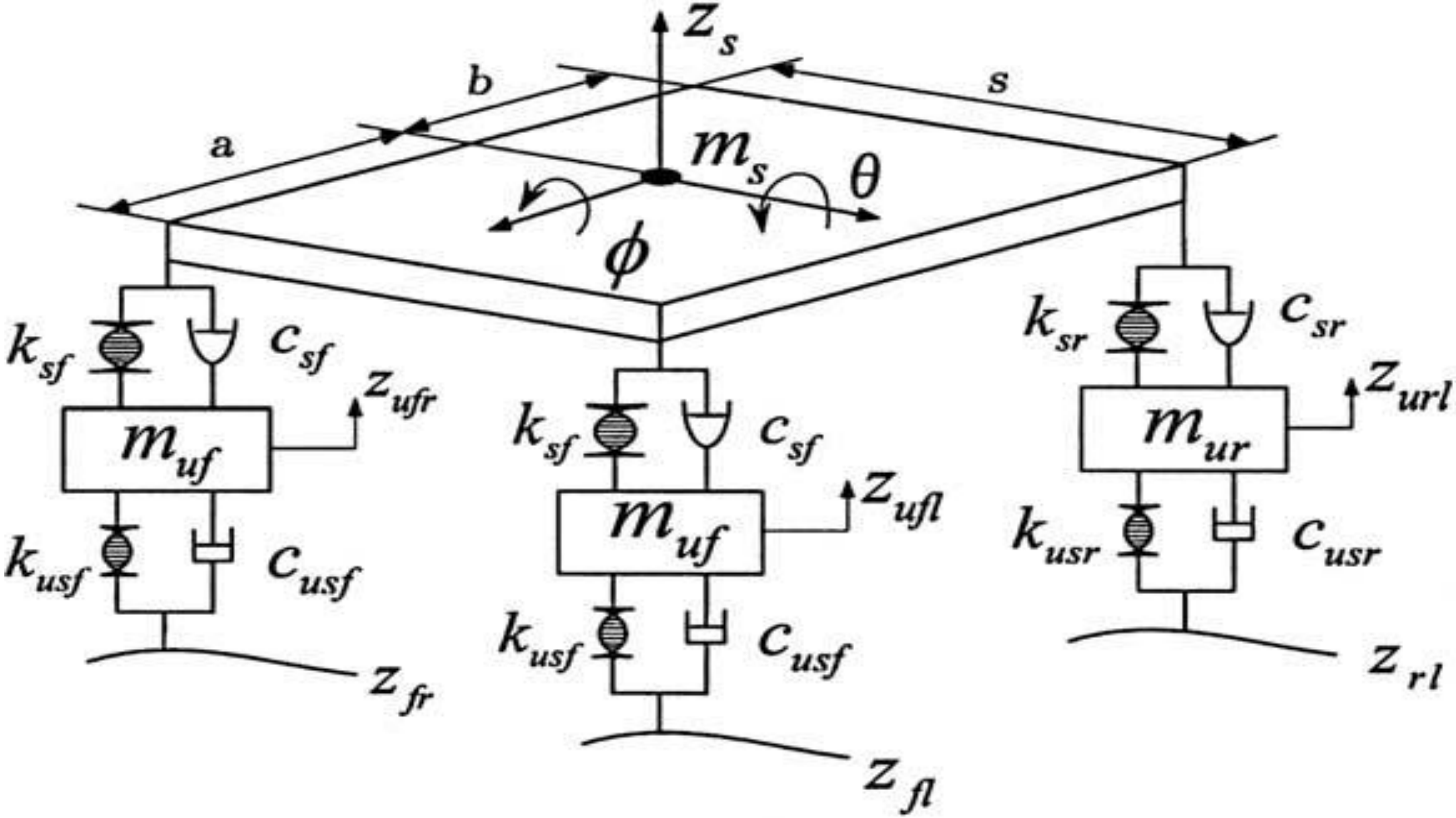



Fig.17 Roll angle

# Non linear Full car model



# MATHEMATICAL FORMULATIONS

Sprung masses

$$m_s \ddot{z}_s = -F_{sfl} - F_{cfl} - F_{sfr} - F_{cfr} - F_{srl} - F_{srr} - m_s g$$

$$I_\phi \ddot{\phi} = (-F_{sfl} - F_{cfl} + F_{sfr} + F_{cfr} - F_{srl} - F_{crl} + F_{srr} + F_{crr})\tfrac{s}{2}\cos\phi$$

$$I_\theta \ddot{\theta} = (F_{sfl} + F_{cfl} + F_{sfr} + F_{cfr})a\cos\theta - (F_{srl} + F_{crl} + F_{srr} + F_{crr})b\cos\theta$$

Front unsprung masses

$$m_{uf} \ddot{z}_{ufl} = F_{sfl} + F_{cfl} - F_{usfl} - F_{ucfl} - m_{uf} g$$

$$m_{uf} \ddot{z}_{ufr} = F_{sfr} + F_{cfr} - F_{usfr} - F_{ucfr} - m_{uf} g$$

Rear unsprung masses

$$m_{ur} \ddot{z}_{url} = F_{srl} + F_{crl} - F_{usrl} - F_{ucrl} - m_{ur} g$$

$$m_{ur} \ddot{z}_{urr} = F_{srr} + F_{crr} - F_{usrr} - F_{ucrr} - m_{ur} g$$

Numerical values of the system parameters

| Parameter | Value |
|---|---|
| Sprung mass, $m_s$ | 1500 kg |
| Roll axis moment of inertia, $I_\phi$ | 460 kg m |
| Pitch axis moment of inertia, $I_\theta$ | 2160 kg m |
| Front unsprung mass, $m_{uf}$ | 59 kg |
| Rear unsprung mass, $m_{ur}$ | 59 kg |
| Front suspension spring stiffness, $k_{sf}$ | 35,000 N/m |
| Rear suspension spring stiffness, $k_{sr}$ | 38,000 N/m |
| Nonlinear coefficient of suspension spring, $n_{sf}$, $n_{sr}$ | 1.5 |
| Damping coefficient of front suspension, $c_{suf}$ | 1000 N/m/s |
| Damping coefficient of front suspension, $c_{sdf}$ | 720 N/m/s |
| Damping coefficient of rear suspension, $c_{sur}$ | 1000 N/m/s |
| Damping coefficient of rear suspension, $c_{cdr}$ | 720 N/m/s |
| Tire spring stiffness, $k_{usf}$, $k_{usr}$ | 190,000 N/m |
| Damping coefficient of tire, $c_{usf}$, $c_{usr}$ | 10 N/m/s |
| Nonlinear coefficient of tire spring, $n_{usf}$, $n_{usr}$ | 1.25 |
| Length between the front of vehicle and the center of gravity of sprung mass, $a$ | 1.4 m |
| Length between the rear of vehicle and the center of gravity of sprung mass, $b$ | 1.7 m |
| Width of sprung mass, $s$ | 3 m |

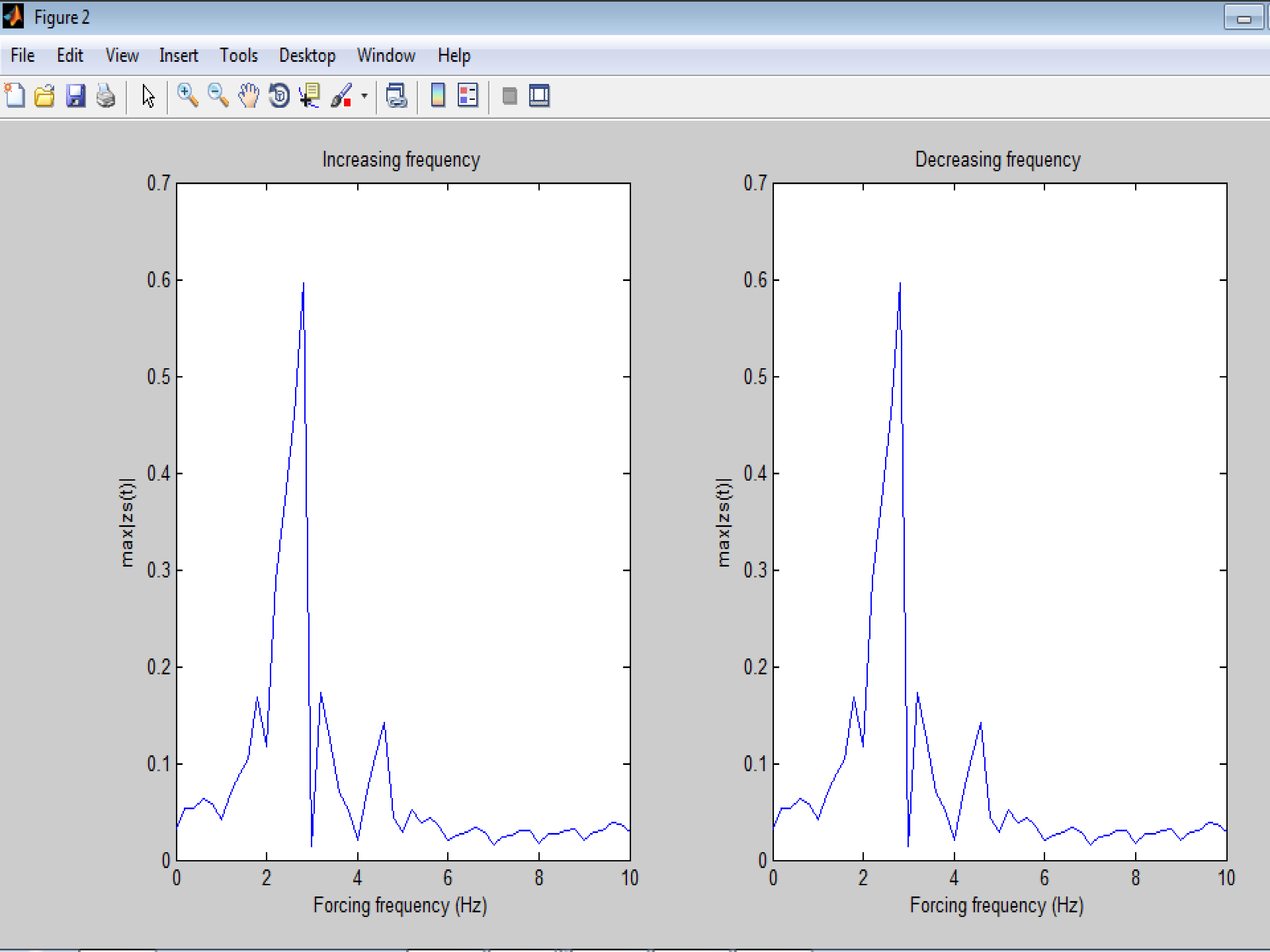


Figure 2
File Edit View Insert Tools Desktop Window Help
Increasing frequency
Decreasing frequency
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
max|zs(t)|
0 2 4 6 8 10
Forcing frequency (Hz)

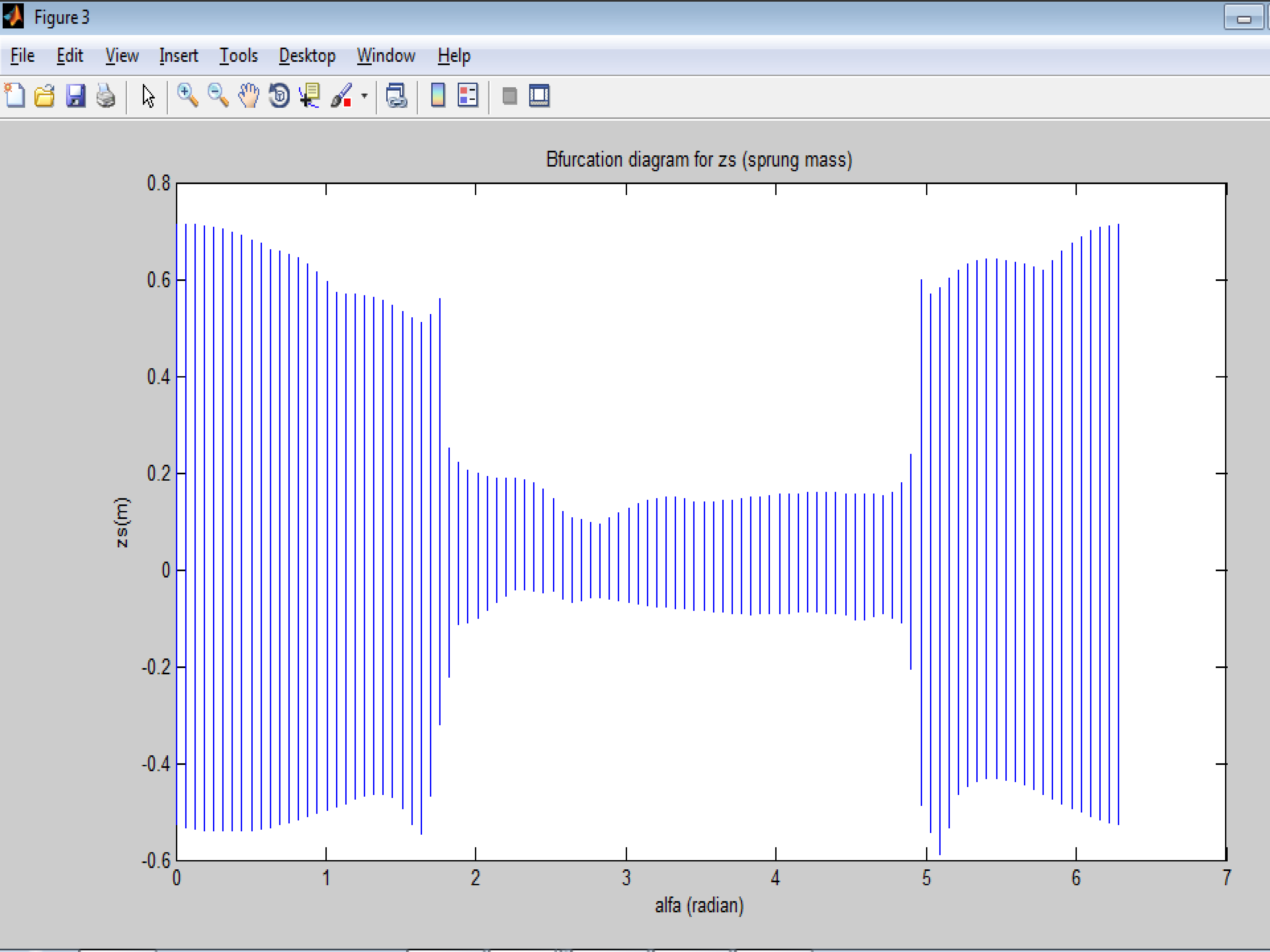


Figure 3
File Edit View Insert Tools Desktop Window Help
Bfurcation diagram for zs (sprung mass)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
zs(m)
0
1
2
3
4
5
6
7
alfa (radian)

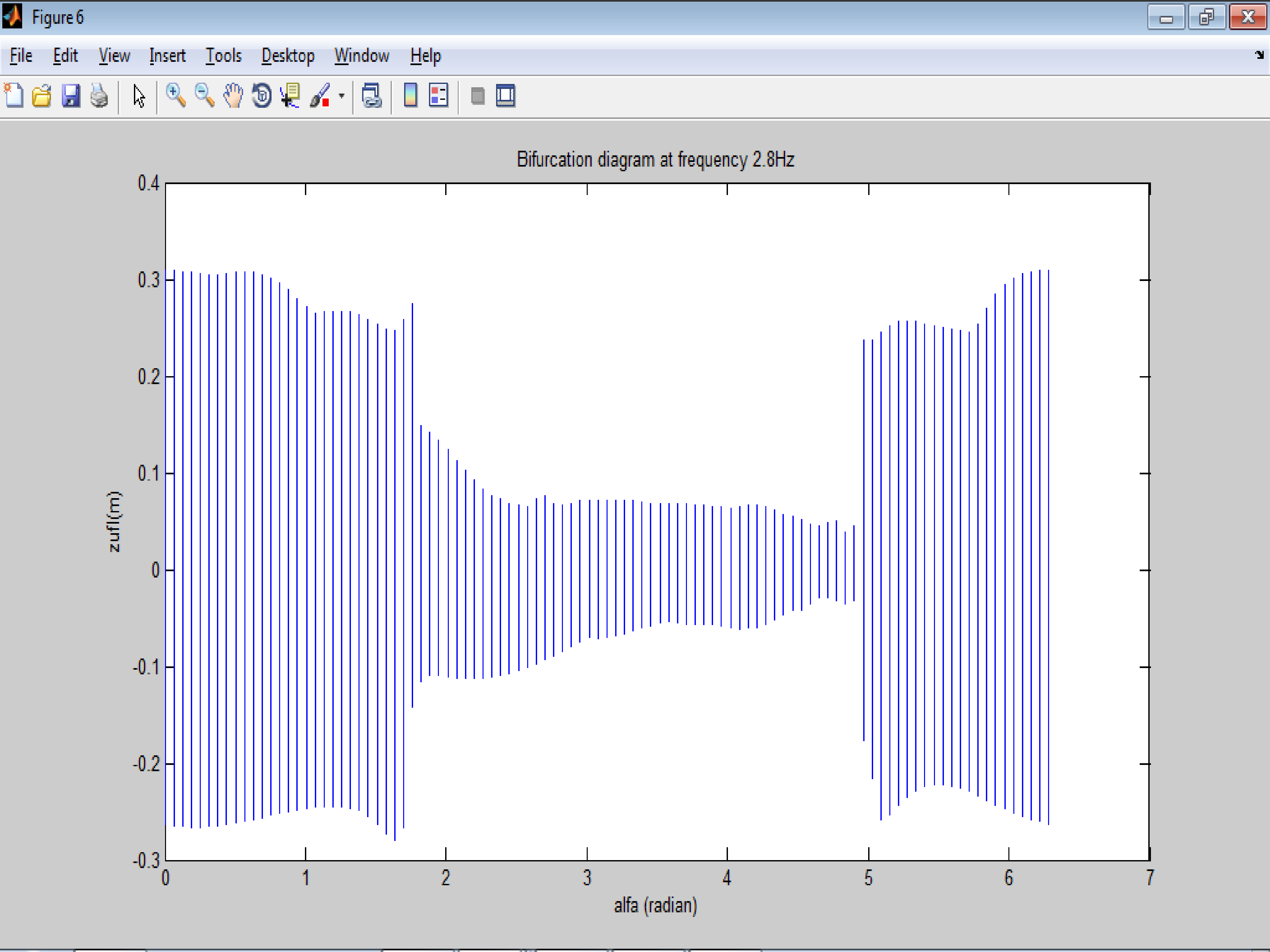


Figure 6
File Edit View Insert Tools Desktop Window Help
Bifurcation diagram at frequency 2.8Hz
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
zufl(m)
0
1
2
3
4
5
6
7
alfa (radian)

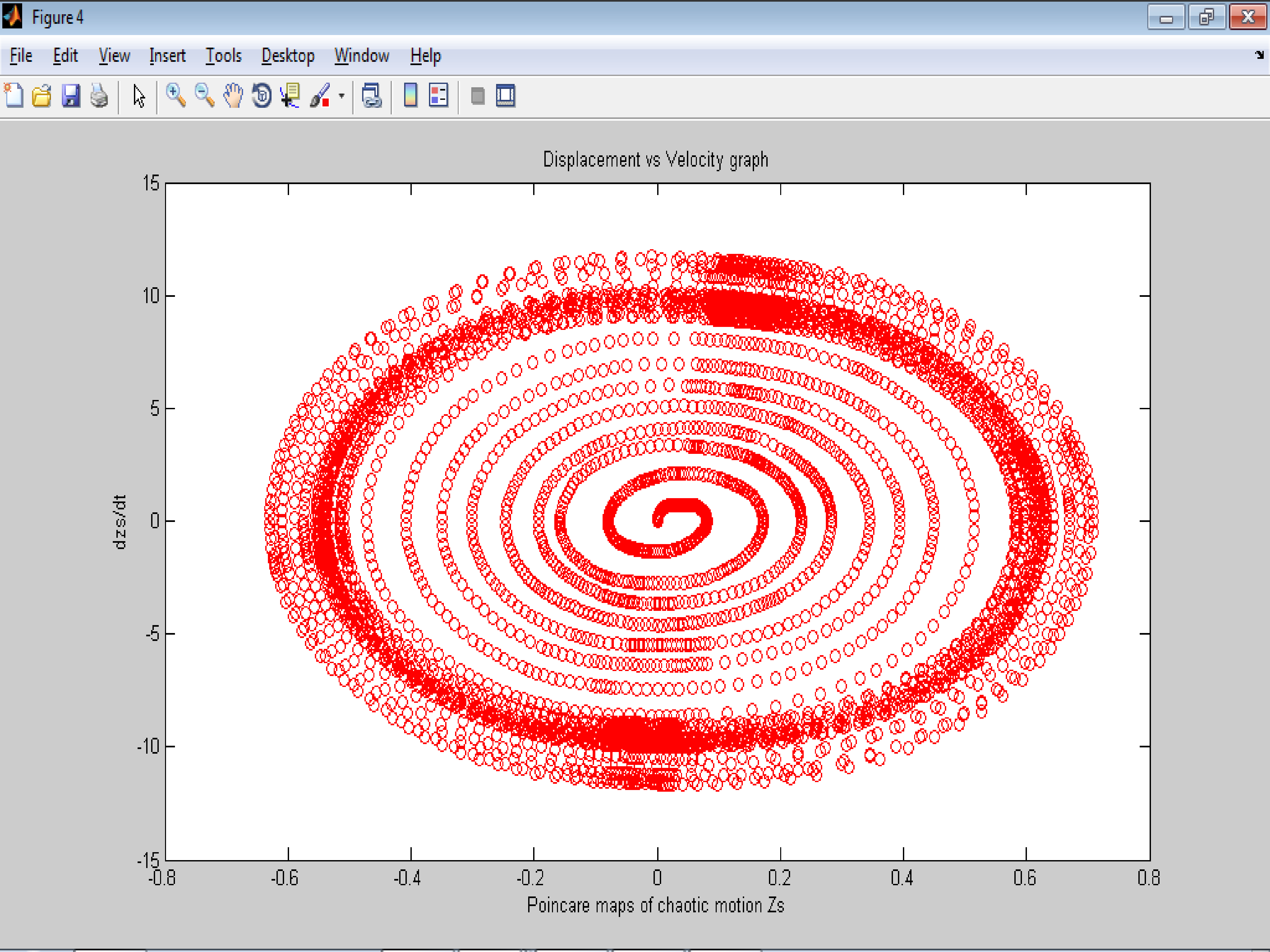


Figure 4
File Edit View Insert Tools Desktop Window Help
Displacement vs Velocity graph
15
10
5
0
-5
-10
-15
dzs/dt
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Poincare maps of chaotic motion Zs

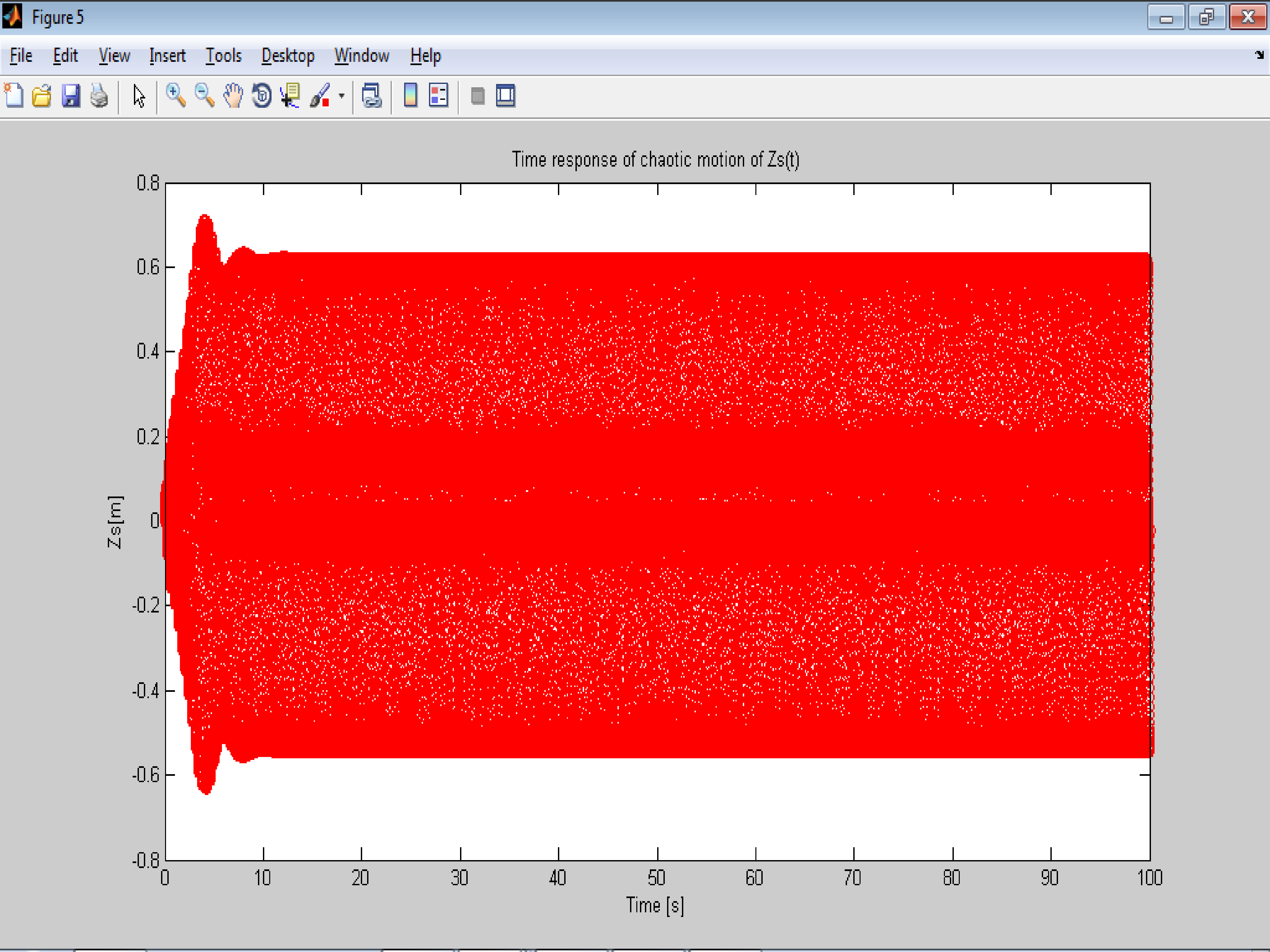


Figure 5
File Edit View Insert Tools Desktop Window Help
Time response of chaotic motion of Zs(t)
Zs[m]
Time [s]
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0 10 20 30 40 50 60 70 80 90 100

# CONCLUSION

- The various observations made during the project clearly indicate that the preference priority level for suspensions is

1:- Active suspension

2:- Semi-active suspension

3:- Passive suspension

- The results also indicated that a stability system is also required for better overall performance of the vehicle.

# FUTURE WORK

- Control system should include the use of **delta delay** function.
- More emphasis should be made on developing a cheap Active suspension system
- Also considerations should be given to the stability effect of the suspension system on the vehicle.

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