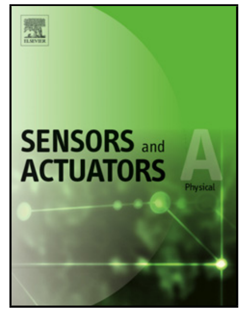
# Accepted Manuscript




Title: Achieving Wideband Micromechanical System using Coupled Non-Uniform Beams Array

Author: Akarapu Ashok P Manoj Kumar Sajal Sagar Singh P Raju Prem Pal Ashok Kumar Pandey

PII: S0924-4247(17)31857-5
DOI: https://doi.org/doi:10.1016/j.sna.2018.02.008
Reference: SNA 10630

To appear in: *Sensors and Actuators A*

Received date: 19-10-2017
Revised date: 2-2-2018
Accepted date: 7-2-2018

# Achieving Wideband Micromechanical System using Coupled Non-Uniform Beams Array

Akarapu Ashok[a], P Manoj Kumar[a], Sajal Sagar Singh[a], P Raju[a], Prem Pal[b], Ashok Kumar Pandey[a,*]

[a]*Department of Mechanical and Aerospace Engineering, IIT Hyderabad, Kandi, 502285, India.*
[b]*Department of Physics, IIT Hyderabad, Kandi, 502285, India.*



**Abstract**

Uniform cantilever microbeam has been at the helm of affair since the advent of micromechanical system (MEMS) technology to develop sensitive MEMS based sensors and actuators. Since then, several improved designs were incorporated to improve their sensitivity and bandwidth. In the work described in this paper, we focus on improving the frequency bandwidth by utilizing a unique characteristics of non-uniform beams. To do the study, we first fabricated single non-uniform diverging and converging beams and characterize them to find their resonance frequency variation with respect to uniform beams under ambient and vacuum conditions. Subsequently, we took two mechanical coupled beams with different combinations of uniform and non-uniform beams. We measured their first in-phase and out-of-phase modes. We found that a combination of diverging and converging beam can tune the difference between these frequencies from 278 Hz to 8.8 kHz with respect to the frequency difference of 316 Hz for a combination of two uniform beams. A frequency tuning of about 2685% signifies the importance of non-uniform beams. After showing the coupling effect of arrays of mechanically coupled three, four and five uniform beams, we numerically demonstrated the tuning for a specific combination of uniform beam, diverging beam and converging beam in five beams array. The obtained results can be applied to increase the frequency band of various MEMS resonators based on the combinations of uniform and non-uniform coupled beams.

*Keywords:* Non-Uniform beams, Resonance frequency, Coupled beams, Quality factor.

## 1. Introduction

Frequency tuning has been studied in controlling the operating bandwidth of microelectromechanical system (MEMS) and nanoelectromechanical systems (NEMS) through various mechanisms [1, 2, 3, 4, 5, 6, 7, 8, 9]. In MEMS and NEMS devices, such effect can be brought in by utilizing different types of coupling mechanisms such as linear and nonlinear mechanical coupling [10, 11, 12], electromechanical coupling [13, 14, 15], thermoelastic coupling [16], etc. However, these coupling mechanisms have been studied using either

*Corresponding author
*Email address:* ashok@iith.ac.in (Ashok Kumar Pandey)

*Preprint submitted to Elsevier* *February 7, 2018*

single or an array of uniform beams. The main objective of the paper is to bring out the influence of non-uniform beams in tuning the bandwidth of coupled micromechanical structures in arrays. Although, the focus of the paper is mainly to study the phenomena in the paper, such concepts can be used for designing a wide band piezoelectric micromechanical energy harvesters [17], and other sensors and actuators.

Numerous research groups have proposed and demonstrated coupled MEMS resonator systems for different purposes such as sensing and filtering. Spletzer et al. [1] demonstrated the vibration localization as an extremely sensitive method over the resonance frequency shift method in nearly identical coupled MEMS resonators to detect the added mass of target analyte. Furthermore, Spletzer et al. [2] proposed mechanically coupled array of 15 uniform microcantilevers for ultrasensitive mass detection and identification using vibration localization technique. Zhao et al. [3] also reported a three degree-of- freedom (3DoF) MEMS resonant sensing device comprising of three weakly coupled resonators for enhanced sensitivity compared to the present state of the art resonant force sensor. Further, using a pair of rectangular clamped-clamped beams, Wood et al. [4] demonstrated the mode-localization effect exhibited in electrostatically-coupled MEMS resonators for enhancement in sensitivity. Hajhashemi et al. [5] proposed a micromechanical filter design made of two micro-resonators that are coupled electrostatically using middle electrode for independent tuning of its centre frequency and bandwidth. Pourkamali and Ayazi [6, 7] presented coupling of micromechanical resonators for execution of high-order narrow bandwidth MEMS bandpass filters using active and passive coupling elements under the influence of electrostatic coupling. Thiruvenkatanathan et al. [8] proposed the use of vibration mode localization phenomenon as a highly sensitive method of detecting small parametric variations in the structural properties of electrically coupled micromechanical resonators. Further, Thiruvenkatanathan et al. [9] reported the experimental evidence of the veering theory and mode localization in micromechanical resonator arrays coupled by weak electrostatic coupling. Matheny *et al.* [10] studied the influence of nonlinear modal coupling due to inter and intramodal nonlinearities of a fixed-fixed nanomechanical beam. Conley *et al.* [11] demonstrated the nonlinear coupling between planer and non-planer modes in nanomechanical beam. Cadeddu *et al.* [12] investigated the influence of coupling of linear modes of planer and out of plane modes by driving one of the mode harder using PZT. Our group has also studied the coupling effect of linear in-plane and out-of-plane modes under the influence of electrostatic loading applied along in-plane direction [13]. Buks and Roukes [14] have studied electrostatic coupling in an array of micromechanical fixed beams. Kambali *et al.* [15] have performed experimental and theoretical studies of in-plane and out-of-plane modes of micromechanical beams array. Recently, Wang *et al.* [18, 19] have utilized mechanically coupled beams array near the fixed end to increase the performance of a sensor. A careful examination of all the above studies reveal that coupling of different types can be brought into the design spectrum of nano- and micro-mechanical sensors and actuators. We have also noticed that most of the above studies employ fixed-fixed or cantilever beam with uniform section along its length under the influence of electrostatic excitation. In such system, the frequency tuning is achieved through electrostatic coupling by changing its dc bias which is mostly softening in nature. Moreover, the electrostatic coupling is limited by pull-in voltage. However, non-uniform beams with converging width can increase the frequency and those with diverging width reduce the frequency with respect to uniform beam [25, 26]. Moreover, when coupled with electrostatic forcing, the pull-in voltage of converging beam increases drastically [25]. Hence, it can provide extra region for frequency tuning. Most significant changes can be observed in an array of two beams in which the in-phase and out-of-phase frequencies can be tuned through the combination of uniform and non-uniform beams which is the subject matter of research in the current paper. The same advantage can be obtained in array of n-beams with different combinations of uniform and non-uniform beams. To include non-uniformity in the beam width, we select linear and nonlinear variation of width along beam length. Linear variation can be obtained by varying beam width linearly along its length. Since, we get closed form solution for non-uniform beam as described in references [25, 26] in case of quartic varying beam, we use quartic varying beam rather than quadratic or cubic varying beam to include nonlinear variation in width along the beam length.

As we know that two identically uniform beams coupled through overhang support can be modelled as spring-mass system [18, 19]. If $K$ and $M$ are effective stiffness and effective mass of both identical beams and $K_c$ is the coupled spring constant provided by overhang coupling, then in-phase and out-of-phase frequencies can be obtained using $f_{\text{in}} = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$ and $f_{\text{out}} = \frac{1}{2\pi}\sqrt{\frac{K+2K_c}{M}}$ [18, 19]. Therefore, coupling of these modes is controlled using $K_c$. Now, if two non-identical beams with different spring constants $K_1$ and $K_2$ are coupled through overhang coupling of stiffness $K_c$, then the corresponding in-phase and out-of-phase frequencies can be obtained using $f_{\text{in,out}} = \frac{1}{2\pi}\sqrt{\frac{K_c+\left[(K_1+K_2)\pm\sqrt{(K_1-K_2)^2+4K_c^2}\right]/2}{M}}$ for same effective mass, i.e., $M_1 \approx M_2 = M$. If $M_1$ and $M_2$ are different then the corresponding frequencies can be modified as $f_{\text{in,out}} = \frac{1}{2\pi}\sqrt{\frac{K_c\beta+K_c+(K_1+K_2)\pm\sqrt{\beta^2 {K_2}^2+2\,\beta^2 K_2 K_c-2\,\beta\,K_2 K_1-2\,\beta\,K_2 K_c+{K_c}^2\beta^2-2\,\beta\,K_c K_1+2\,{K_c}^2\beta+{K_1}^2+2\,K_c K_1+{K_c}^2}}{2M_1}}$, where, $\beta = M_1/M_2$. The expression of above frequencies show that $f_{\text{in}}$ and $f_{\text{out}}$ not only depend on coupling stiffness but these can also be tunned by varying effective spring and/or mass constants of individual beam elements. Such variation in spring and mass constants of two coupled beams can be induced by taking a combination of beams with different types of non-uniformity. Motivated by the outcome of non-uniform beams, we experimentally study the influence of non-uniform beam on frequency characteristics of mechanically coupled beams. In this paper, we show how a combination of different types of beams can tune these frequencies effectively.

## 2. Fabrication and Measurements

To perform this study, we first fabricated single uniform and non-uniform beams of converging and diverging type with linear and quartic variation in width by following the process steps described in Figure 1(a). The non-uniform micromechanical beams with linear and quartic varying width (converging as well as diverging) corresponding to different $\alpha$ values (-0. 6 to 0.6) were generated based on $b(x) = b_0(1 + \alpha\frac{x}{l})^n$, where $n = 1$ for linearly varying beam and $n = 4$ for quartic varying beam, $\alpha$ is negative for converging beam and positive for diverging beam. First for a given $\alpha$ value with fixed length and width of the beam, $b(x)$ values corresponding to different $x$ values at the interval of 5 were determined using aforementioned formulas. Then, using $b(x)$ values corresponding to different $x$ values, beams with linear and quartic varying widths for different $\alpha$ values were generated in the Clewin4 layout software. After that using e-beam lithography the micromechanical beams designed in Clewin4 layout software were transferred onto a glass plate. The pattern on the glass plate (Mask) is transferred onto the wafer surface by lithography followed by wet bulk micromachining in 25 wt% TMAH at $75^oC$ to finally fabricate over hanging $SiO_2$ structures [27]. All the beams are of silicon dioxide material. Figure 1(b) shows images of uniform beam, linearly converging and diverging beams, and quartic converging beam. Figure 1(c) also shows a typical mechanically coupled converging and diverging beams. The length, thickness and width at the fixed end of single non-uniform beams are taken as 200 $\mu$m and $0.95 - 0.96$ $\mu$m, and 40 $\mu$m, respectively. Each beam is separated from the bottom substrate by a depth of about $90 - 103$ $\mu$m. Similarly, the dimensions of coupled beams are same as that of single beams, but they are coupled through an overhang with a width of 16 $\mu$m at fixed end and a length of 25 $\mu$m along beam length. To obtain the experimental results of single non-uniform beams, we fabricated the linearly varying beams with tapering ratio, $\alpha$, of -0.6, -0.5, -0.4, -0.3, -0.2, -0.1, 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. For the quartic varying beams, we fabricated beams with tapering ratio, $\alpha$, of -0.6, -0.4, -0.2, 0.2, 0.4, and 0.6. The length of all beams is 200 $\mu$m except quartic converging beam with $\alpha = -0.6$ for which the length is measured as 167 $\mu$m.

To measure the resonance frequencies and quality factors of cantilever beams with different taperings, we performed experiments using laser scanning vibrometer. Figure 1(d) shows the outline of experimental setup which consists of laser source, optical detectors, internal function generator, analyzer, and piezoactuators. The fabricated structure (i.e., cantilever) is mounted on peizoactuator which is excited with pseudorandom excitation signal. Subsequently, frequency response of oscillating structure is measured from reflected laser light on the detector. Signal from the detector is measured using analyzer which gives frequency response curve as shown in Figure 1(e). Resonance frequency, $f_n$, is measured by finding the peak frequency of response curve. The damping ratio, $\xi$, or quality factor, $Q$, can be found either by half-width method or by fitting the dynamic amplification factor due to base excitation. Expression of dynamic amplification as the ratio of displacement amplitude, $X$, and excitation amplitude, $Z$, can be written in terms of frequency ratio $r = f/f_n$ and quality factor $Q$ as [29]

$$\left|\frac{X}{Z}\right| = \frac{r^2}{\sqrt{(1-r^2)^2 + (r/Q)^2}}. \tag{1}$$

Figure 1(e) shows the fitting of dynamic amplification factor from Eqn.(1) with experimental results. Figure 1(f) shows mode shapes of converging and diverging beams corresponding to first four modes. In this

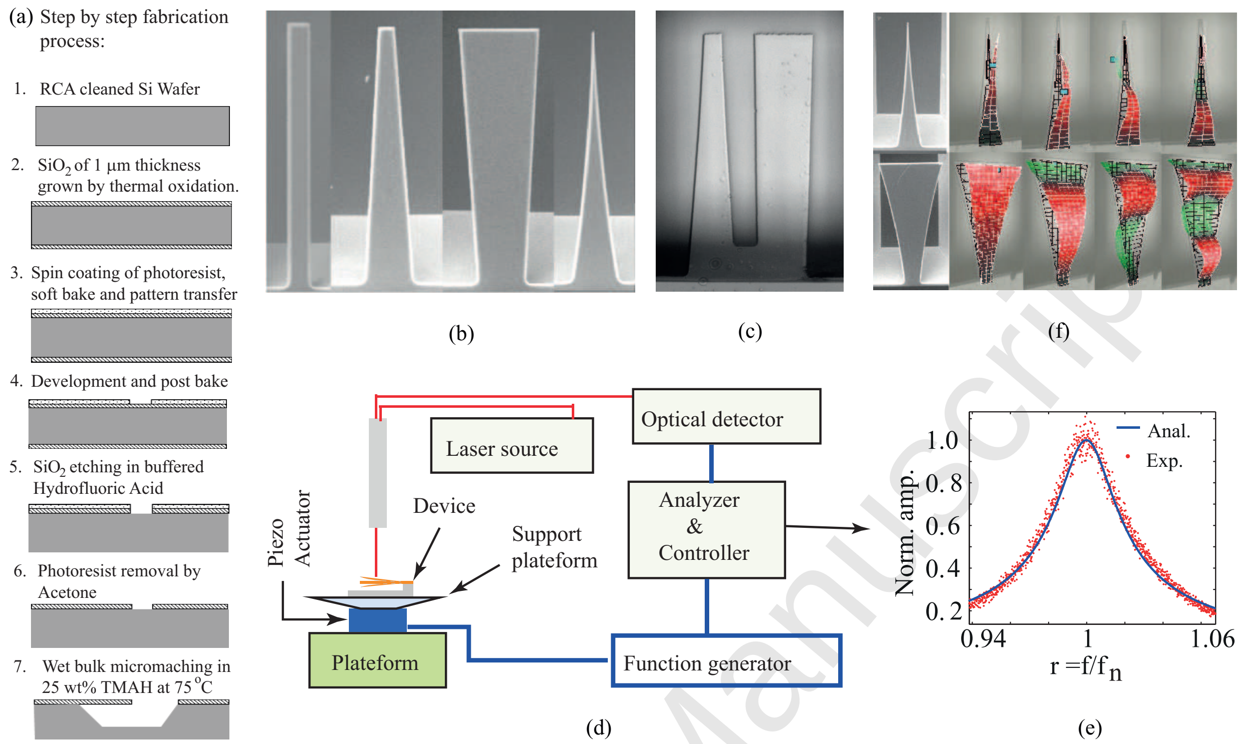



Figure 1: (a) Step by step fabrication process for fabricating uniform and non-uniform beams; Images of (b) uniform beam, linearly converging beam, linearly diverging beam, quartic converging beam and (c) a coupled two beams array. (d) A schematic representation of experimental procedure to characterize the beams using laser vibrometer to get (e) frequency response curve. (f) Four transverse modes of a converging and diverging beams measured using laser vibrometer.



paper, we restrict our analysis to first mode of single converging and diverging beams. After experimentally characterizing non-uniform beams of different types, we measured the variation of frequency and quality factor with non-uniform parameter under ambient and vacuum conditions. Subsequently, we characterized mechanically coupled two beams array consisting of different combinations of non-uniform and uniform beams to study their influence on the coupling of first in-phase mode and out-of-phase mode frequencies. It was found that frequency difference between in-phase and out-of-phase can be reduced by using combination of only converging beams, and can be increased by using combinations of a converging beam and a diverging or uniform beam. Similar effect can also be seen in an array of three, four and five beams consisting of uniform and nonuniform beams.

## 3. Results and Discussions

To extract the material properties of silicon dioxide beam, we compared the theoretically computed frequency of uniform and non-uniform beams corresponding to their first mode. If the beam is having a length $L$, width $b$, thickness $t$, elastic modulus, $E$ and the density, $\rho$, we use standard formula of frequency expression of cantilever beam as [29]

$$f_n = \frac{K_n}{2\pi L^2}\sqrt{\frac{EI_0}{\rho A_0}} \tag{2}$$

where, $I_0 = b_0 t^3/12$ is the area moment of inertia and $A_0 = b_0 t$ is cross-sectional area and $b_0$ is width at the fixed end. $K_n$ is the frequency parameter which is different for uniform, linearly tapered and quartic tapered beams. For uniform cantilever beam, $K_n$ =3.516, 22.0345, 61.6972, and 120.90 for first, second, third and fourth modes, respectively. Since, the density of silicon oxide, $\rho$, is 2200 kg/m$^3$ and its elastic modulus is found in the range of $43 - 77$ GPa [28], we computed the elastic modulus from experimental results of for each sets of uniform and non-uniform beams. To describe influence of individual non-uniform beams and coupled system of uniform and non-uniform beams, we fabricated different structures in different dies. In first die, we fabricated uniform beam and single non-uniform beams. In other die, we fabricated single uniform beam and arrays of non-uniform beams. For a uniform cantilever beam of length $L = 200$ $\mu$m, width $b = b_0 = 20$ $\mu$m and $t = 0.95$ $\mu$m, first four measured frequencies in die 1 are found as 20.56 kHz, 129.48 kHz, 362.38 kHz, and 710.31 kHz, respectively. On comparing theoretical frequency with experimental values for the first mode of beam in die 1, we found the elastic modulus of $SiO_2$ as $E = 63.2$ GPa. Using $E = 63.2$ GPa, we get theoretical frequencies for second, third, and fourth modes as 128.87 kHz, 360.83 kHz, and 707.08 kHz, respectively, with the percentage error of less than 0.5%. Hence, we use $E = 63.2$ GPa for finding frequencies of single non-uniform beams with different tapering ratios. Similarly, elastic modulus of second die is found as $E = 66.26$ GPa. Therefore, we use $E = 66.26$ GPa in analyzing the response of systems of coupled beams. To quantify the order of coupling effect of two coupled beams, we take $E = 66.26$ GPa, $\rho = 2200$kg/m$^3$, beam length $L = 200\mu$m from the fixed support and beam thickness $t = 0.966$ $\mu$m of two coupled uniform beams fabricated in die 2. Taking the measured values of in-phase and out-of-phase frequencies as 23.93 kHz and 24.19 kHz of two coupled uniform beams and using theoretical model of in-phase and out-of-phase frequencies as $f_{\text{in}} = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$ and $f_{\text{out}} = \frac{1}{2\pi}\sqrt{\frac{K+2K_c}{M}}$ [18, 19], we find $K_c \approx 1.01 \times 10^{-3}$ N/m and $K \approx 90.58 \times 10^{-3}$ N/m for $M_{\text{eff}} = 33M/140 \approx 4 \times 10^{-12}$ kg.

*3.1. Single Nonuniform Beams*

In this section, we compared the measured first mode frequency values of non-uniform beams with theoretical values to confirm the effects of tapering on frequency tuning. Subsequently, we discuss its effect on damping under ambient and vacuum condition. Taking $\rho = 2200$ kg/m$^3$, and $E = 63.2$ GPa, we compute the frequencies of non-uniform beams using Eqn. (2). $K_n$ for linearly converging beam with $\alpha = -0.1$, $-0.2$, $-0.3$, $-0.4$, $-0.5$, $-0.6$ are taken as 3.642, 3.771, 3.922, 4.098, 4.312, and 4.578 [25, 26]. For linearly diverging beam of tapering ratio 0.2, 0.4, 0.6, $K_n$ values are given by 3.32, 3.16, and 3.03, respectively. For quartic converging and diverging beam of tapering parameters $\alpha = -0.6$, $-0.4$, $-0.2$, 0.2, 0.4, 0.6, $K_c$ values are taken as 9.23, 6.30, 4.59, 2.8, 2.3, and 1.93. For more details on how to computing the values of

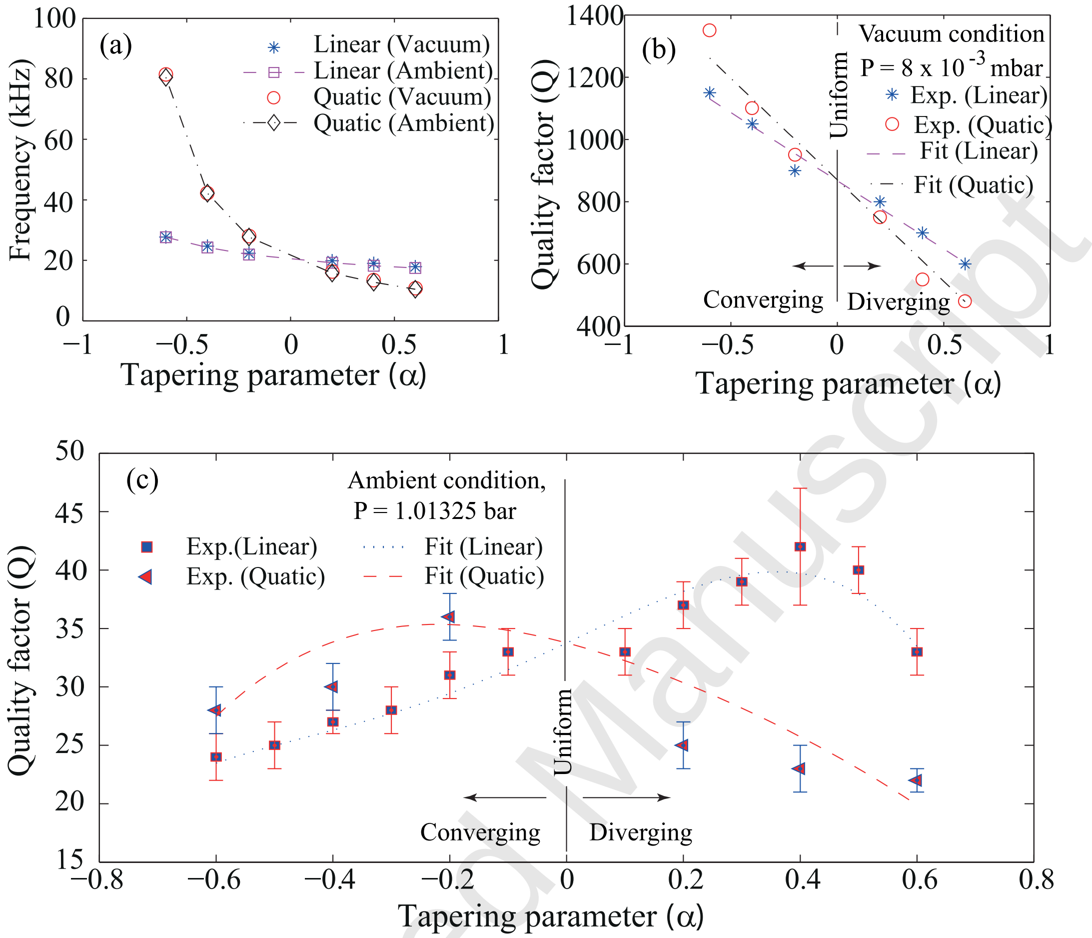



Figure 2: (a) Variation of measured first mode frequencies with different tapering ratios (-0.6 to 0.6) of linearly and quartic varying beams under ambient condition (1.01325 bar) and vacuum condition ($8 \times 10^{-3}$ mbar). Variation of quality factor, $Q$, corresponding to first mode with different tapering ratios (-0.6 to 0.6) of linearly and quartic varying beams under (b) vacuum condition ($8 \times 10^{-3}$ mbar) and (c) ambient condition (1.01325 bar).



$K_c$ for different tapering ratios, readers are referred to paper by Sajal et al.[25, 26]. Figure 2(a) shows the measured frequencies of non-uniform beams of different tapering parameters which follow the same trend as the theoretical frequency reported by Sajal et al. [25, 26]. Based on the comparison, we found that frequency can be increased significantly by quartic converging beams than linearly converging beam of same tapering parameter. For diverging beams, the frequency reduces marginally in both cases. But in case of quartic varying beams, the change in frequency is more as compared to linearly varying beams. Hence, a combination of non-uniform and uniform beams can be analyzed to tune the frequencies of coupled beam arrays.

To see the influence of tapering on damping in vacuum and ambient conditions, we measured quality factor of single non-uniform beams. Figure 2(b) shows that the quality factor increases linearly as tapering

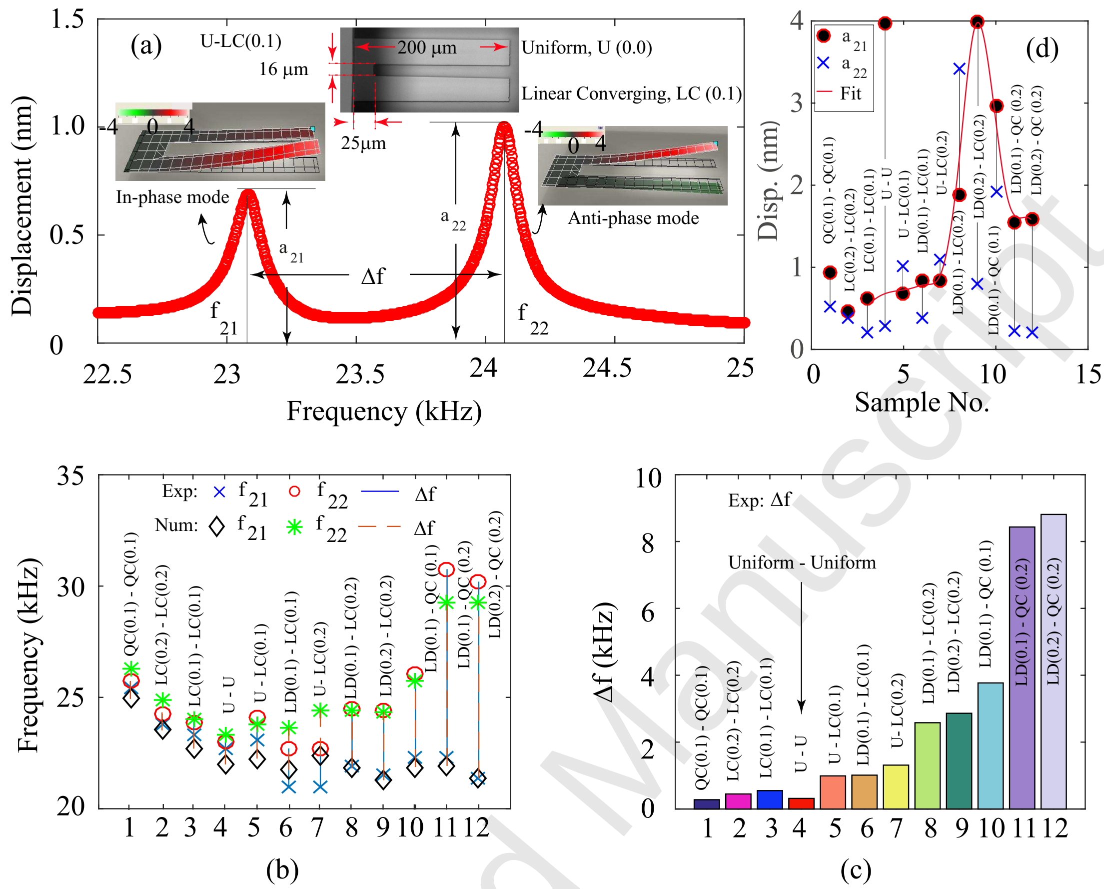



Figure 3: (a) Experimental frequency response of a coupled uniform and linearly converging beam with tapering parameter of 0.1 showing the in-phase, $f_{21}$, and out-of-phase, $f_{22}$, frequencies and modes. Variation of (b) $f_{21}$ and $f_{22}$, and (c) $\delta f = f_{22} - f_{21}$ for 12 different sets of two coupled beams. The shapes '×' and '∘' represent experimental values of in-phase and out-of-phase frequencies. The corresponding numerical values are represented by '◇' and '∗'. (d) Variation of measured value of displacement amplitudes $a_{21}$ and $a_{12}$ marked as 'filled circle' and 'x' corresponding to in-phase, $f_{21}$, and out-of-phase, $f_{22}$ frequencies for 12 different sets of two coupled beams.

ratio decreases from 0.6 (diverging) to -0.6 (converging) under vacuum condition, i.e., $8 \times 10^{-3}$mbar, in linearly as well as quartic varying beams. It is also found that the quality factor increases from 600 to 1150 in case of linearly varying beam and 480 to 1350 in case of quartic varying beam under vacuum conditions. On linear curve fit of measured data, we found that curves intersect at the condition of uniform beam, i.e., $\alpha = 0$. The source of damping under vacuum condition may be due to pressure independent sources such as support loss, thermoelastic damping, etc. Figure 2(c) shows variation of quality factor (Q) with tapering ratios of linearly and quartic varying cantilever beams under ambient condition. Curve fitting the experimental results, we found that quality factor attains maximum value of about 35 corresponding to $\alpha = -0.2$ in case of quartic varying beam and about 40 at $\alpha = 0.4$ in case of linearly varying beam. Such variation leads to the crossing of two curves when beam becomes uniform. This kind of behavior may be due to relative interplay of squeeze-film damping and drag force damping similar to the case presented by Verbridge et al. [20] and Vishwakarma et al. [21] for the uniform beam of different width. However, a further theoretical analysis is needed to correctly understand the damping behavior of non-uniform beams under ambient and vacuum conditions. Figure 2(c) shows measured first mode frequency of linear and quartic varying beams under ambient and vacuum conditions. An overall analysis reveals that non-uniform beams under ambient and vacuum conditions can be used to tune the frequency as well quality factors, effectively.

### *3.2. Coupled Systems of Uniform and Nonuniform Beams*

To analyze the influence of non-uniform effect on frequency tuning of coupled beams, we first consider two coupled beams with different combinations of uniform and/or non-uniform beams. Figure 3(a) shows frequency response of a coupled uniform beam and linearly converging beam with tapering ratio of 0.1 under vacuum condition. Due to coupling at the overhang support, two beams oscillate in in-phase mode at frequency corresponding to $f_{21} = 23.07$ kHz and out-of-phase mode at frequency of $f_{22} = 24.07$ kHz giving a difference of $\Delta f = 1$ kHz. To observe the influence of non-uniformity, we fabricated different sets of coupled beams and measured their frequencies corresponding to in-phase mode ($f_{21}$), out-of-phase mode ($f_{22}$) and the difference $\Delta f = f_{22} - f_{21}$. Considering 'U' as uniform beam, LC($\alpha$) and LD($\alpha$) as linearly converging and diverging beams with tapering parameter $\alpha$, and QC($\alpha$) and QD($\alpha$) as quartic converging and diverging beams with tapering parameter $\alpha$, 12 sets of two coupled beams can be marked as QC(0.1)-QC(0.1), LC(0.2)-LC(0.2), LC(0.1)-LC(0.1), U-U, U-LC(0.1), LD(0.1)-LC(0.1), U-LC(0.2), LD(0.1)-LC(0.2), LD(0.2)-LC(0.2), LD(0.1)-LC(0.1), LD(0.1)-QC(0.2), LD(0.2)-QC(0.2). In Figure 3(b), we show the distribution of $f_{21}$ marked as 'x' and $f_{22}$ marked as 'o'for 12 sets of coupled beams with different $\alpha$. Figure 3(c) shows variation of frequency difference $\Delta f$ for the same 12 sets of coupled beams. On analyzing the results, we found that a set of converging beams (QC(0.1)-QC(0.1)) bring the two frequencies closer giving a difference of 278 Hz, while a set of converging and diverging beams (LD(0.2)-QC(0.2)) make them apart by a difference of 8.8 kHz. It is also noted that coupled uniform beams (U-U) have frequencies at $f_{21} = 22.697$ kHz and $f_{22} = 23.013$ kHz which give a difference of $\Delta f = f_{22} - f_{21} = 316$ Hz. Thus, for given sets of coupled beams, a frequency reduction as low as 12% and a frequency increment as high as 2685% are obtained by utilizing a combination of non-uniform beams. To further increase the frequency band of coupled beams, number of elements in an array can be increased. Additionally, corresponding to in-phase mode ($f_{21}$) and out-of-phase mode ($f_{22}$), response amplitudes $a_{21}$ and $a_{22}$ for 12 different sets of two coupled beams are shown in Fig.3(d). It is found that the vibration amplitude, $a_{21}$, corresponding to in-phase mode ($f_{21}$) shows maximum response in the case of LD(0.2)-LC(0.2) which consists of a linearly diverging beam and a linearly converging beam both with a tapering ratio of 0.2. Thus, a combination of diverging and converging beams helps in achieving an optimum amplitude other than giving a wide frequency difference between in-phase and out-of-phase frequencies. However, a further analysis is required to understand the

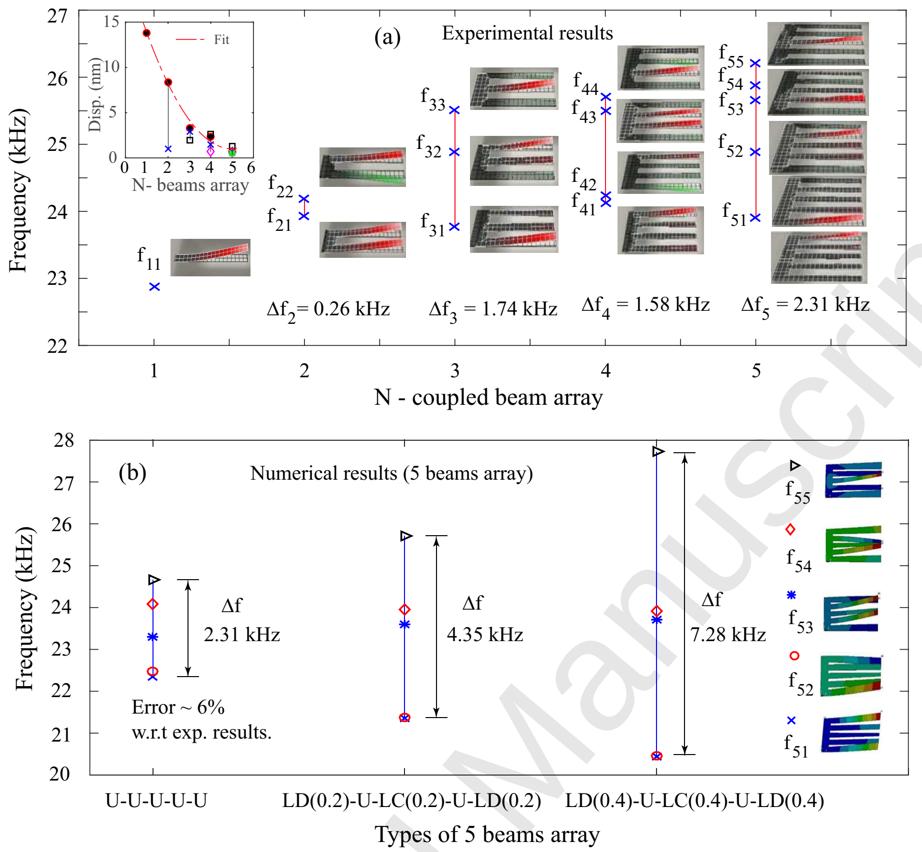



Figure 4: (a) Variation of measured frequencies $fn1$ to $f_{nn}$ and the corresponding frequency difference/bandwidth $\Delta f_n = f_{nn} - f_{n1}$ for n-coupled uniform beams arrays, where n=1,2,3,4,5. The inset of the figure shows the variation of response amplitudes, marked as 'filled circle' and 'x' corresponding to in-phase, $f_{21}$, and out-of-phase, $f_{22}$, frequencies, as the number of beams in an array increases. (b) Variation of computed frequencies $f51$ to $f_{55}$ and frequency difference $\Delta f = f_{55} - f_{51}$ for 5 coupled uniform beams (U-U-U-U-U) and two combinations of uniform and non-uniform beams such as LD(0.2)-U-LC(0.2)-U-LD(0.2) and LD(0.4)-U-LC(0.4)-U-LD(0.4).

influence of tapering on vibrational amplitude of two coupled beams with different combination of uniform and non-uniform beams.

To show the effect of varying number of elements in an array, we fabricated a single uniform beam and coupled uniform beams of 2, 3, 4 and 5 elements. Figure 4(a) shows the variation of measured frequency band consisting of frequencies of many modes as the number of elements in an array of coupled uniform beams. It also shows the modes of coupled frequencies in an array. For $M_{\text{eff}} =\approx 4 \times 10^{-12}$ kg, $K_c$ and $K$ are $1.01 \times 10^{-3}$ N/m and $90.58 \times 10^{-3}$ N/m for 2 uniform beams, $4.52 \times 10^{-3}$ N/m and $89.40 \times 10^{-3}$ N/m for 3 uniform beams, $3.65 \times 10^{-3}$ N/m and $92.10 \times 10^{-3}$ N/m for 4 uniform beams, and $5.06 \times 10^{-3}$ N/m and $90.40 \times 10^{-3}$ N/m for 5 uniform beams. The corresponding ratio of $K_c/K$ as 0.011, 0.050, 0.0396, 0.056 for 2, 3, 4, and 5 coupled beams arrays, respectively. It shows that as the number of uniform beams increases from 2 to 5, the frequency difference is also increased by around 788%. However, the response amplitudes corresponding to different frequencies of n-beam array reduces exponentially as the number of beams, $n$, increases as shown in the inset of Fig. 4(a). To improve the response amplitude of n-beam array, a suitable combination of uniform and non-uniform beams can be explored. To show the effect of non-uniform beams in tuning the frequency of five beams array, we performed numerical analysis. To validate the numerical model with measured results, we first model two coupled beams consisting of uniform and non-uniform beams for 12 different sets and compare the results with measured values as shown in Fig. 3(b). The comparison of results clearly shows that the numerical results in computing in-phase and out-of-phase frequencies follow the similar trend as that of measured values for 12 different sets of uniform and non-uniform beams with average percentage error of about 2.4% and maximum percentage error of about 7%. Moreover, on comparing the numerical result with measured value for a coupled 5 uniform beams, the numerical result gives a frequency difference of 2.31 kHz with a percentage error of around 6%. Considering such level of accuracy of numerical method, we used it to analyze the influence of non-uniform beams on frequency tuning of 5 coupled beams array in Fig. 4(b). Based on the numerical analysis, it is found that the difference of 2.31 kHz for a set of 5 coupled uniform beams (U-U-U-U-U) increases to 4.35 (88%) and 7.28 (215%) for sets of uniform and non-uniform beams, LD(0.2)-U-LC(0.2)-U-LD(0.2) and LD(0.4)-U-LC(0.4)-U-LD(0.4), respectively, as shown in Fig. 4(b). Such variation can further be tuned by including non-uniform beams of higher tapering ratio. Thus, using the experimental and numerical analysis, we have demonstrated an exciting application of non-uniform beams in effectively tuning the frequencies of coupled beams array for MEMS resonators.

## 4. Conclusions

In short, we first analyzed the variation of first transverse mode frequency of a single non-uniform beam with linearly and quartic varying widths based on experimental technique. After quantifying the variation in which frequency reduces for diverging beams and increases for converging beams, we utilize such characteristics in tuning the in-phase and out-of-phase modes of coupled two beams with uniform/non-uniform widths. Based on the analysis, we found that a set of two converging beams bring these two modes closer than that of two coupled uniform beams. Contrarily, a combination of uniform/converging beam and diverging beam makes them apart. Following this method, we were able to reduce the difference between two modes by 12% for a combination of quartic converging beams with tapering parameters of 0.1 (QC(0.1)-QC(0.1)) and increase the difference by 2685% for a combination of linearly diverging and quartic converging beams with tapering parameters 0.2 (LD(0.2)-QC(0.2)). Moreover, we have also observed that the frequency span over which these frequencies appear can further be increased significantly by selecting an appropriate combinations of uniform and non-uniform beams in a five coupled beams array. Taken together, the techniques presented in this paper can be utilized to increase bandwidth of most of the MEMS resonators.

### Acknowledgements

This work was partially supported by Grant no. CSIR 22(0696)/15/EMR-II.

### Reference

[1] M. Spletzer, A. Raman, A.Q. Wu, X. Xu, R. Reifenberger, "Ultrasensitive mass sensing using mode localization in coupled microcantilevers", *Appl. Phys. Lett.*, **88**(25) 254102 (3 pp), (2006).

[2] M. Spletzer, A. Raman, H. Sumali, J.P. Sullivan, "Highly sensitive mass detection and identification using vibration localization in coupled microcantilever arrays", *Appl. Phys. Lett.*, **92**(11), 114102 (3 pp), (2008).

[3] C. Zhao, G.S. Wood, J. Xie, H. Chang, S.H. Pu, M. Kraft, "A force sensor based on three weakly coupled resonators with ultrahigh sensitivity", *Sens. Actuators A: Phys.*, **232**, 151-162, (2015).

[4] G. S. Wood, C. Zhao, I. Sari, M. Kraft, S. H. Pu, *Sensor based on the mode-localization effect in electrostatically-coupled MEMS resonators fabricated using an SOI process*, in: 2015 IEEE International Conference on Sensors, IEEE, pp. 14, 2015.

[5] M.S. Hajhashemi, A. Amini, B. Bahreyni, "A micromechanical bandpass filter with adjustable bandwidth and bidirectional control of centre frequency", *Sens. Actuators A: Phys.*, **187**, 10-15, (2012).

[6] S. Pourkamali, F. Ayazi, "Electrically coupled MEMS bandpass filters: Part I. With coupling element", *Sens. Actuators A: Phys.* **122**(2), 307-316, (2005).

[7] S. Pourkamali, F. Ayazi, "Electrically coupled MEMS bandpass filters: Part II. Without coupling element", *Sens. Actuators A: Phys.*, **122**(2), 317-325, (2005).

[8] P. Thiruvenkatanathan, J. Yan, J. Woodhouse, A.A. Seshia, "Enhancing parametric sensitivity in electrically coupled MEMS resonators", *J. Microelectromech. Syst.* **18**(5) 1077-1086 (2009).

[9] P. Thiruvenkatanathan, J. Woodhouse, J. Yan, A. A. Seshia, "Manipulating vibration energy confinement in electrically coupled microelectromechanical resonator arrays", *J. Microelectromech. Syst.*, **20**(1) 157-164 (2011).

[10] M. H. Matheny, L. G. Villanueva, R. B. Karabalin, J. E. Sader, and M. L. Roukes, "Nonlinear Mode-Coupling in Nanomechanical Systems", *Nano. Lett.*, **13**, 1622-1626, (2013).

[11] William G. Conley, Arvind Raman, Charles M. Krousgrill, and Saeed Mohammadi, "Nonlinear and Nonplanar Dynamics of Suspended Nanotube and Nanowire Resonators", *Nano. Lett.*, **8**(6), 1590-1595, (2008).

[12] D. Cadeddu, F. R. Braakman, G. Tütüncüoglu, F. Matteini, D. Rüfer, A. Fontcuberta i Morral, and M. Poggio, "Time-Resolved Nonlinear Coupling between Orthogonal Flexural Modes of a Pristine GaAs Nanowire", *Nano. Lett.*, **16**,926-931, (2016).

[13] A.K. Pandey, "Effect of coupled modes on pull-in voltage and frequency tuning of a NEMS device", *J. Micromech. and Microeng.*, **23**(8), 085015, (2013).

[14] E. Buks and M. L. Roukes, "Electrically Tunable Collective Response in a Coupled Micromechanical Array", *J. Microelectromech. Syst.*, **11**(6),802-807, (2002).

[15] P. N. Kambali, G. Swain, A. K. Pandey, E. Buks, and O. Gottlieb, "Coupling and tuning of modal frequencies in direct current biased microelectromechanical systems arrays" *Appl. Phys. Lett.* **107**, 063104, (2015).

[16] A.K. Pandey, O. Gottlieb, O. Shtempluck, and E. Buks, "Performance of an AuPd micromechanical resonator as a temperature sensor", *Appl. Phys. Lett.*, **96**, 203105, (2010).

[17] D. K. Sahoo, and A. K. Pandey, "Performance of non-uniform cantilever based piezoelectric energy harvester", *ISSS J Micro Smart Syst*, (2018), https://doi.org/10.1007/s41683-018-0018-2 .

[18] D. F. Wang, X. Li, X. Yang, T. Ikehara and R. Maeda, "Enhancing amplitude changes by mode localization in trio cantilevers with mass perturbation", *Microsyst. Technol.*, **18**, 1923-1929, (2012).

[19] D. F. Wang, K. Chatani, T. Ikehara and R. Maeda, "Mode localization analysis and characterization in a 5-beam array of coupled nearly identical micromechanical resonators for ultra-sensitive mass detection and analyte identification", *J. Micromech. Microeng.*, **25**, 095017-1-8, (2015).

[20] S. S. Verbridge, R. Ilic, H. G. Craighead, and J. M. Parpia, "Size and frequency dependent gas damping of nanomechanical resonators", *Appl. Phys. Lett.* **93**, 013101 (2008).

[21] S. D. Vishwakarma, A. K. Pandey, J. M. Parpia, S. S. Verbridge, H. G. Craighead and R. Pratap, "Size modulated transition in the fluid?structure interaction losses in nano mechanical beam resonators", *J. Appl. Phys.*, **119**, 194303 (2016).

[22] E. A. Corbin, K. Park, and W. P. King, "Room-temperature temperature sensitivity and resolution of doped-silicon microcantilevers", *J. Appl. Phys.*, **94**, 243503, (2009).

[23] M. Spletzer, A. Raman, and R. Reifenberger, "Elastometric sensing using higher flexural eigenmodes of microcantilevers", *Appl. Phys. Lett.* **91**, 184103 (2007).

[24] D. Ramos, J. Tamayo, J. Mertens, and M. Calleja, "Effect of the adsorbate stiffness on the resonance response of microcantilevers", *Appl. Phys. Lett.* **89**, 224104 (2006).

[25] S.S. Singh, P. Pal, and A. K. Pandey, "Pull-in analysis of non-uniform microcantilever beams under large deflection", *J. of Appl. Phys.* **118**, 204303 (2015).

[26] S.S. Singh, P. Pal, and A. K. Pandey, "Mass sensitivity of non-uniform microcantilever beam", *J. of Vib. Acoust.* **138(6)**, 064502 (2016).

[27] A. Ashok and P. Pal, "Investigation of anodic silicon dioxide thin films for microelectromechanical systems applications", *Micro and Nano Lett.*, **9(12)**, 830-834 (2014).

[28] http://www.memsnet.org/material/silicondioxidesio2film/.

[29] S.S.Rao, Mechanical Vibrations, Pearson Education India (2003).

These are the main highlights of the paper,

1) A systematic experiment is performed to find frequency and quality factor of non-uniform beam with linearly and quartic varying width.
2) A systematic experimental studies is performed on two mechanically coupled beams with different combinations of uniform and non-uniform beams.
3) Experimental and numerical studies are performed to see the influence of frequency tuning in a combination of five uniform/non-uniform beams.
4) All the above studies indicate that the frequency band width can be effectively controlled using non-uniform beams.

**Dr. Akarapu Ashok** received the B.Sc degree from Osmania University, Hyderabad, India, in 2003; theM.Sc degree in Physics from Osmania University, Hyderabad, India, in 2007; the M.Tech degreefrom department of Metallurgical & Materials Engineering, NIT Rourkela, Rourkela, India, in 2011;and the Ph.D. degree from department of Physics, IIT Hyderabad, Hyderabad, India, in 2015. He iscurrently working as a Post-doctoral fellow in the department of Mechanical & AerospaceEngineering, IIT Hyderabad, Hyderabad, India. His research interest include thin films,electrodeposition, anodic oxidation, wet bulkmicromachining for MEMS and solar cells, micromechanical beams for MEMS.

**PadalaManoj Kumar** is currently working as a CAE Analyst in Mercedes-Benz R&D India. He completed his BTech from RajivGandhi University of Knowledge Technologies,Basar, and completed his MTechfrom Indian Institute of Technology Hyderabad in Mechanics and Design stream. As part of his Masters' thesis he has worked on linear frequency analysis of mechanically coupled micro beams. His current areas of interests are in linear and nonlinear dynamics, acoustics and vibrations, and finite elements methods.

**Sajal Sagar Singh** completed his Bachelor's degree in the Department of Mechanical and Aerospace Engineering at Indian Institute of Technology (IIT) Hyderabad, India in 2015 and his Master's degree in the same department in 2016. Currently, he is a doctoral student in the Electrical Engineering and Computer Science department at University of Michigan Ann Arbor,USA. He is associated with the center of Wireless and IntegratedMicroSensing and Systems(WIMS2). His research interest lies in the field of inertial sensors, linear and nonlinear dynamics, sensors and actuators, micro-fabrication techniques etc. He has published more than ten research papers in peer-reviewed international journals of high impact factor.

**P.Raju (Raju Peta)**is working as Technical Superintendent in Department of Mechanical and Aerospace Engineering, IIT Hyderabad, India. He has been working in IITs (Indian Institute of Technology) since last 9 years. He completed his diploma in Mechanical Engineering from Government Polytechnic college of Hyderabad in 2003. He is also pursuing graduation through AMIE (Associate Member of Institution of Engineers, India). His research interest lies in manufacturing, material characterization of metals and composite materials, and vibration of macro and micro components.

**Dr. Prem Pal** did his M.Tech. in Solid State Material (SSM) in December 1999 and Ph.D. in the area ofMEMS in December 2004, both from Indian Institute of Technology Delhi (IIT Delhi), India. TheFoundation for Innovation and Technology Transfer (FITT), IIT Delhi awarded his thesis for "Best IndustryRelevant Ph.D. Project -2005". From July 2005 to June 2006, he worked as Postdoctoral Researcher atYonsei Microsystems Laboratory (YML), School of Mechanical Engineering, Yonsei University Seoul,South Korea. He worked in Sato laboratory, Department of Micro-Nano Systems Engineering, NagoyaUniversity, Nagoya, Japan: from July 2006 to March 2008 as COE Scientist and from March 2008 to March2010 as JSPS Fellow. He joined the Department of Physics, Indian Institute of Technology Hyderabad inApril 2010 as Assistant Professor. Presently, he is working as Associate Professor. He has published morethan 50 papers in peer reviewed international journals of high repute. His research interests include MEMStechnology, MEMS based sensors, Silicon Micromachining, Thin films for MEMS.

**Dr. Ashok Kumar Pandey** is currently an Associate Professor in the Department of Mechanical and Aerospace Engineering, Indian Institute of Technology Hyderabad. He completed his Bachelor of Engineering in Mechanical Engineering, Bhilai Institute of Technology, Durg (Ravi Shankar Shukla University, Raipur) in 2001. He completed his Masters and PhD from Indian Institute of Science, Bangalore in 2004 and 2008, respectively. He did his postdoctoral research from Technion-Isreal Institute of Technology, Haifa, Israel from 2008 to 2010. He is also a recipient of best teaching award from IIT Hyderabad and Hetenyi Award from Society of Experimental Mechanics, USA. His interest lies in linear and nonlinear vibration, MEMS, Vehicle Dynamics. He has published over 30 Journal papers and presented over 50 conference and Invited talks at various places.