QUIZ – 1, IIT HYDERABAD

ME5010 – Mathematical Methods for Engineers **Date: 13/08/2014**

Maximum Marks – 30 **Time duration: 1 hours**

Instructions:

i) Mobile is not allowed in the exam.
ii) Only calculator is allowed.

1. For a co-ordinate transformation from (x1, x2, x3) to (q1, q2, q3), write the relationship between (dx1, dx2, dx3) and (dq1, dq2, dq3) using the matrix notation, Einstein/Index notation, and Gibbs/vector notation, respectively. (Total Marks: 2)

2. Find the order of the following tensors written in its index form: (Total Marks: 4)
   (a) A, (b) A_{ij}, (c) A_i^{ij}, (d) A_{ii}, (e) $\frac{\partial}{\partial x_i}$, (f) $\frac{\partial^2}{\partial {x_i}^2}$, (g) A_{ijk} (h) ∇

3. If the components A_i of a vector A can be written in terms of its unit vectors e_i , $\vec{A} = A_i e_i$ then write the components of
   (a) ${}^2T$, (b) ${}^3T$ (c) T_i^{ij}, $(i, j = 1,2,3)$ (Total Marks: 3)

4. Using the index notation convention, write the index form of the following operations: (Total Marks: 8)

   (a) ${}^2T = \vec{A} \otimes \vec{B}$ (b) $\vec{B} = {}^2T.\vec{A}$ (c) ${}^2T : \vec{A}$

4. Do the following using index notation

   a) If $y_i = a_{ij}x_j$ and $x_i = a_{ij}z_j$ then write an index form of y in terms of z. (Total Marks: 3)
   b) Prove that $\vec{A} \times \left(\vec{B} \times \vec{C}\right) = \vec{B}\left(\vec{A}.\vec{C}\right) - \vec{C}\left(\vec{A}.\vec{B}\right)$ (Total Marks: 5)
   c) Prove that $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla.\vec{A}\right) - \nabla^2\vec{A}$ (Total Marks: 5)

********* Good Luck**********

Solution

(1) $(x_1, x_2, x_3) \longleftrightarrow (\xi_1, \xi_2, \xi_3)$

$(dx_1, dx_2, dx_3) \longleftrightarrow (d\xi_1, d\xi_2, d\xi_3)$

$$dx_1 = \frac{\partial x_1}{\partial \xi_1} d\xi_1 + \frac{\partial x_1}{\partial \xi_2} d\xi_2 + \frac{\partial x_1}{\partial \xi_3} d\xi_3$$

Matrix notation:
$$\begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial \xi_1} & \frac{\partial x_1}{\partial \xi_2} & \frac{\partial x_1}{\partial \xi_3} \\ \frac{\partial x_2}{\partial \xi_1} & \frac{\partial x_2}{\partial \xi_2} & \frac{\partial x_2}{\partial \xi_3} \\ \frac{\partial x_3}{\partial \xi_3} & \frac{\partial x_3}{\partial \xi_3} & \frac{\partial x_3}{\partial \xi_3} \end{bmatrix} \begin{bmatrix} d\xi_1 \\ d\xi_2 \\ d\xi_3 \end{bmatrix} \quad \text{— (2)}$$

Einstein / Index notation: $dx_i = \frac{\partial x_i}{\partial \xi_j} d\xi_j$

Gibbs / vector notation: $d\vec{x} = J\, d\vec{\xi}$

(2) a) 0 (b) 2 ; (c) 1 (d) 0 (e) 1 (f) 0 (g) 3 (h) 1] — (4)

(3) $\vec{A} = A_i e_i$; e_i = unit vector.

a) ${}^2T = e_i e_j T_{ij}$ (b) ${}^3T = e_i e_j e_k T_{ijk}$

c) $T_i^{ij} = e_j T_i^{ij} = (T_1^{1j} + T_2^{2j} + T_3^{3j}) e_j$] — (3)

(4) a) ${}^2T = \vec{A} \otimes \vec{B}$

$\Rightarrow T_{ij} = A_i B_j$

(2)

(b) $\vec{B} = {}^2T \cdot \vec{A}$

$B_i = e_i e_j T_{ij} \cdot e_k A_k$

$= e_i \underbrace{(e_j \cdot e_k)}_{\delta_{jk}} T_{ij} A_k$

$= e_i \delta_{jk} T_{ij} A_k$

$\boxed{B_i = e_i T_{ij} A_j}$

2+2 = (4)

Straight answer = 2

(c) ${}^2T : \vec{A}$

$e_i e_j T_{ij} : e_k A_k$

$(e_i e_j : e_k) T_{ij} A_k$

doesn't not exist

hence, operation not possible

(2)

(5) a) $y_i = a_{ij} x_j$; $x_i = a_{ij} z_j$ or $x_i = b_{ij} z_j$

y in terms of z

$\Rightarrow y_i = a_{ij} x_j = a_{ij} a_{jk} z_k = a_{ij} a_{jk} z_k$

or $y_i = a_{ij} x_j = a_{ij} b_{jk} z_k = a_{ij} b_{jk} z_k$

(3)

(b) $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{A} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})$

or H.S : $e_{ijk} A_j (\vec{B} \times \vec{C})_k = e_{ijk} A_j (e_{klm} B_l C_m) = e_{ijk} e_{klm} A_j B_l C_m$

$= e_{kij} e_{klm} A_j B_l C_m = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j B_l C_m$

$= A_j B_i C_j - A_j B_j C_i = B_i (\vec{A} \cdot \vec{C}) - C_i (\vec{A} \cdot \vec{B})$

(5)

$\Rightarrow \vec{A} \times (\vec{B} \times \vec{C}) = \vec{B} (\vec{A} \cdot \vec{C}) - \vec{C} (\vec{A} \cdot \vec{B})$

********* Good Luck**********

c) $\nabla \times (\nabla \times \vec{A}) = \nabla(\underbrace{\nabla \cdot \vec{A}}_{(A_j)_{,j}}) - \nabla^2 \vec{A}$ $\quad \nabla \cdot (\nabla \cdot \vec{A})$

L.H.S $= e_{ijk} (\nabla \times \vec{A})_{k,j} = e_{ijk} (e_{klm} A_{m,l})_{,j} = e_{ijk} e_{klm} (A_{m,l})_{,j}$

$= e_{kij} e_{klm} (A_{m,l})_{,j}$

$= (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) (A_{m,l})_{,j}$

$= (A_{j,i})_{,j} - (A_{i,j})_{,j}$

$= (A_{j,j})_{,i} - (A_{i,j})_{,j}$

5

$$\boxed{\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot A) - \nabla^2 \vec{A}}$$

********* Good Luck**********