MID SEMESTER EXAMINATION, IIT HYDERABAD

ME5010 – Mathematical Methods for Engineers **Date: 26/09/2014**

Maximum Marks – 30 **Time duration: 2 hours**

Instructions:

i) Mobile is not allowed in the exam.
ii) Only calculator is allowed.

1. a) Evaluate $\vec{C}=\vec{A}\times\vec{B}$ using both the normal technique and the tensorial notations, where $\vec{C}=C_i\hat{e}_i$, $\vec{B}=B_i\hat{e}_i$ and $\vec{A}=A_i\hat{e}_i$, $i=1,2,3$. Marks: 2
   b) Write the Stokes theorem, Divergence theorem and the Green's theorem in tensorial form. Marks: 2
   c) Obtain the values of ϵ_{216534} and $\epsilon_{ijk}\epsilon^{ijk}$ Marks: 2

2. If $(\hat{e}_1,\hat{e}_2,\hat{e}_3)$ and $(\hat{e}_r,\hat{e}_\theta,\hat{e}_\varphi)$ are the unit vectors in the Cartesian co-ordinate (x,y,z) and the spherical co-ordinate (r,θ,φ) then find
   a) the relation between the unit vectors in both the co-ordinate systems, Marks: 2
   b) the differential arc length dS in spherical co-ordinate systems. Marks: 1
   c) and the volume element dV in spherical co-ordinate systems. Marks: 1

3. a) Briefly define the maxima and minima conditions for the function of one variable, two variables and multivariable. Marks: 4
   b) Derive the Euler-Lagrange equation by extremizing the functional of given variables. Marks: 4

4. a) For a generalized Integral equation of form Marks: 3

$$\phi(x)u(x)=f(x)+\lambda\int_{\alpha(x)}^{\beta(x)}K(x,t)g(u(t))\ dt$$

   where, u(x) is an unknown function and λ is a known parameter. Based on different values of $\phi(x)$, $f(x)$, $g\big(u(x)\big)$, $\alpha(x)$ and$\beta(x)$, mention their values for
   i) Linear and nonlinear Integral equation,
   ii)Homogeneous and non-homogenous Integral equation
   iii)Volterra and Fredholm Integral equations of first and second kind.

   b) Obtain the integral equation from the initial value problem $u''(t)+u(t)=\cos(t)$, subjected to initial conditions $u(0)=0$; $u'(0)=0$. Marks: 4

   c) After classifying the following integral equation, find the values of λ for which it has a unique solution

   $u(x)=f(x)+\lambda\int_0^{\pi}\sin(x-t)\,u(t)\ dt$ Marks: 5

********* Good Luck************