

Problem Set I

MA4090: Combinatorics

Section I: Problems of enumeration

1. In how many ways can the letters of NEWYORK be arranged without containing NEW or YORK as substrings?
2. Let $S = \{1, 2, \dots, 10^8\}$. How many numbers in S contain the digit 1 or the digit 7 but not both?
3. Find the number of subsets S of $\{1, 2, 3, 4, 5, 6, 7\}$ such that the sum of the elements of S is divisible by 7.
4. Find the number of ways to partition mn people into m groups of n each.
5. How many n -letter strings can be formed using the letters A and B and not containing three consecutive A s?

Section II: Pigeon-hole problems

1. Suppose that there are 50 points inside a circle of radius 1. Show that two of these points must be at a distance less than one from each other. Can you do this with a much smaller number of points than 50?
2. Assuming friendship to be a symmetric relation, show that in any group of people, there must be two with the same number of friends in that group.

Section III: Extremal problems

1. What is the maximum number of vectors in \mathbb{R}^n that you can pick so that every pair of them make an obtuse angle?
2. What is the maximum number of elements in $\{1, 2, \dots, n\}$ that you can pick, so that no two numbers sum to a third among the picked elements?