LFV in LEFT
(Lepton flavour violation in Low Energy Effective Theory)

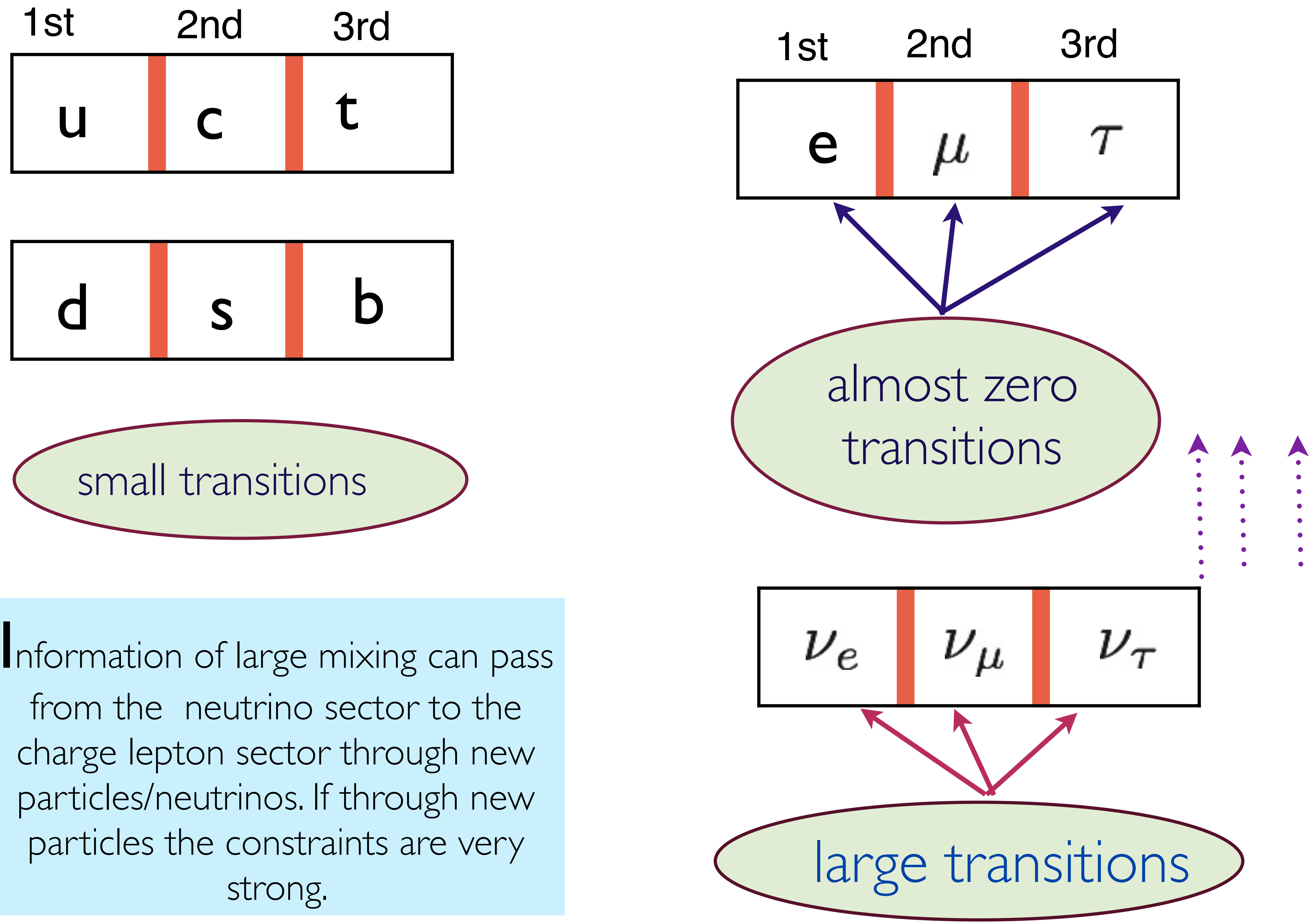
Sudhir K Vempati
Centre for High Energy Physics,
Indian Institute of Science
Bangalore , India

ANOMALIES 2021, ONLINE

November 10, 2021

With
Priyanka Lamba and Mathew Thomas, arXiv: 2111:XXXX

Inter-generational transitions are small for quarks and almost zero for charged leptons



Information of large mixing can pass from the neutrino sector to the charge lepton sector through new particles/neutrinos. If through new particles the constraints are very strong.

Signal for Physics Beyond Standard Model

$$m_{\nu 2} \sim \sqrt{\Delta m^2_{\odot}} \sim 0.05 \text{ eV}$$

$$m_{\nu 2} \sim \sqrt{\Delta m^2_{\odot}} \sim 0.008 \text{ eV}$$

Sub eV masses to the Neutrinos (assuming normal hierarchy)

Theoretically it means that the Standard Model has to be extended

New Particles and/or Additional Symmetry
or both

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c}{\Lambda} L\tilde{H} L\tilde{H}$$

neutrinos are complicated.

Majorana Neutrinos

$$\mathcal{L}_{SM} + Y\bar{\nu}_L\nu_R\tilde{H} + \frac{1}{2}M_R\bar{\nu}^c_R\nu_R$$

$$m_\nu = \frac{Y^2 < \tilde{H} >^2}{M_R} \qquad M_R \uparrow \qquad m_\nu \downarrow$$

seesaw mechanism

$Y \sim 1, M_R \sim M_{GUT}$

Dirac Neutrinos

$$\mathcal{L}_{SM} + Y\bar{\nu}_L\nu_R\tilde{H}$$

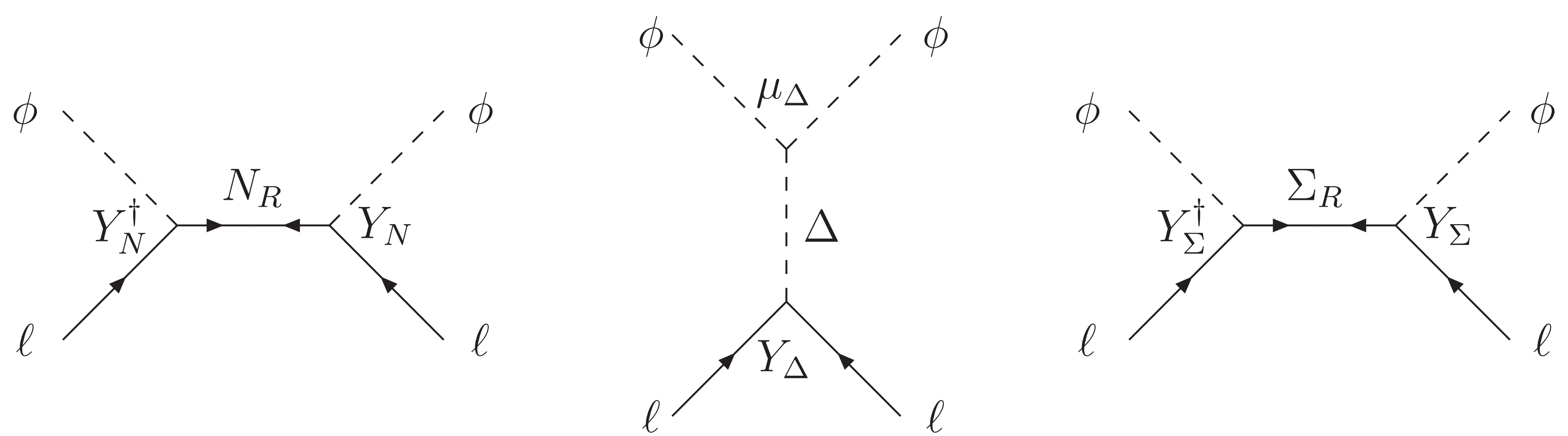
Lepton number has to be imposed

$$Y \sim 10^{-12}$$

Extremely small coupling looks highly unnatural

Majorana Neutrinos

Graphical representation of various seesaw models



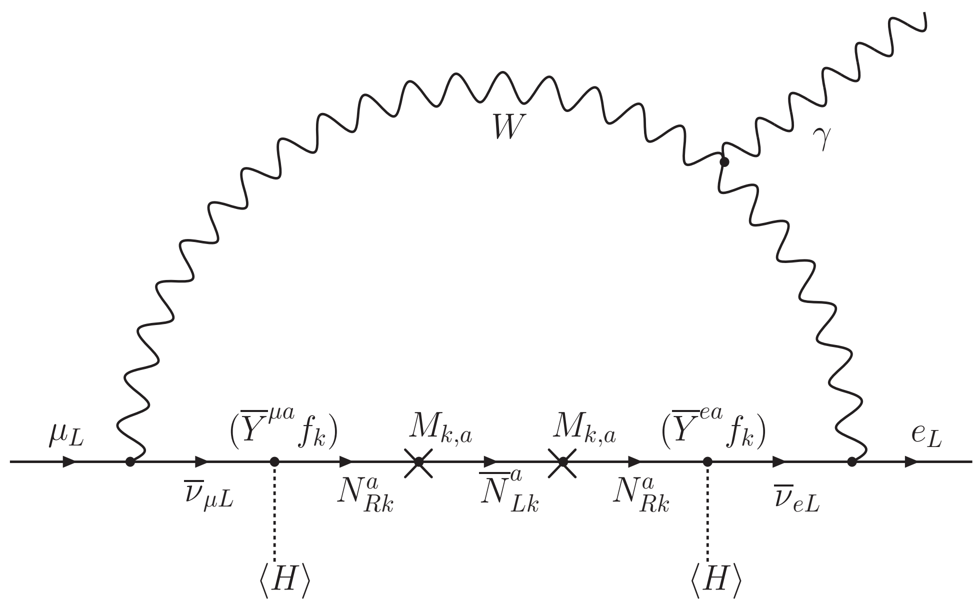
$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta} \left(\overline{\ell^c_{L\alpha}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\,\ell_{L\beta}\right) + \text{h.c.}$$

Weinberg Operator

$$\delta\mathcal{L}^{d=6} = c^{d=6}_{\alpha\beta} \left(\overline{\ell_{L\alpha}}\tilde{\phi}\right) i\partial\!\!\!/ \left(\tilde{\phi}^\dagger\ell_{L\beta}\right)$$

non-unitarity of mixing matrix

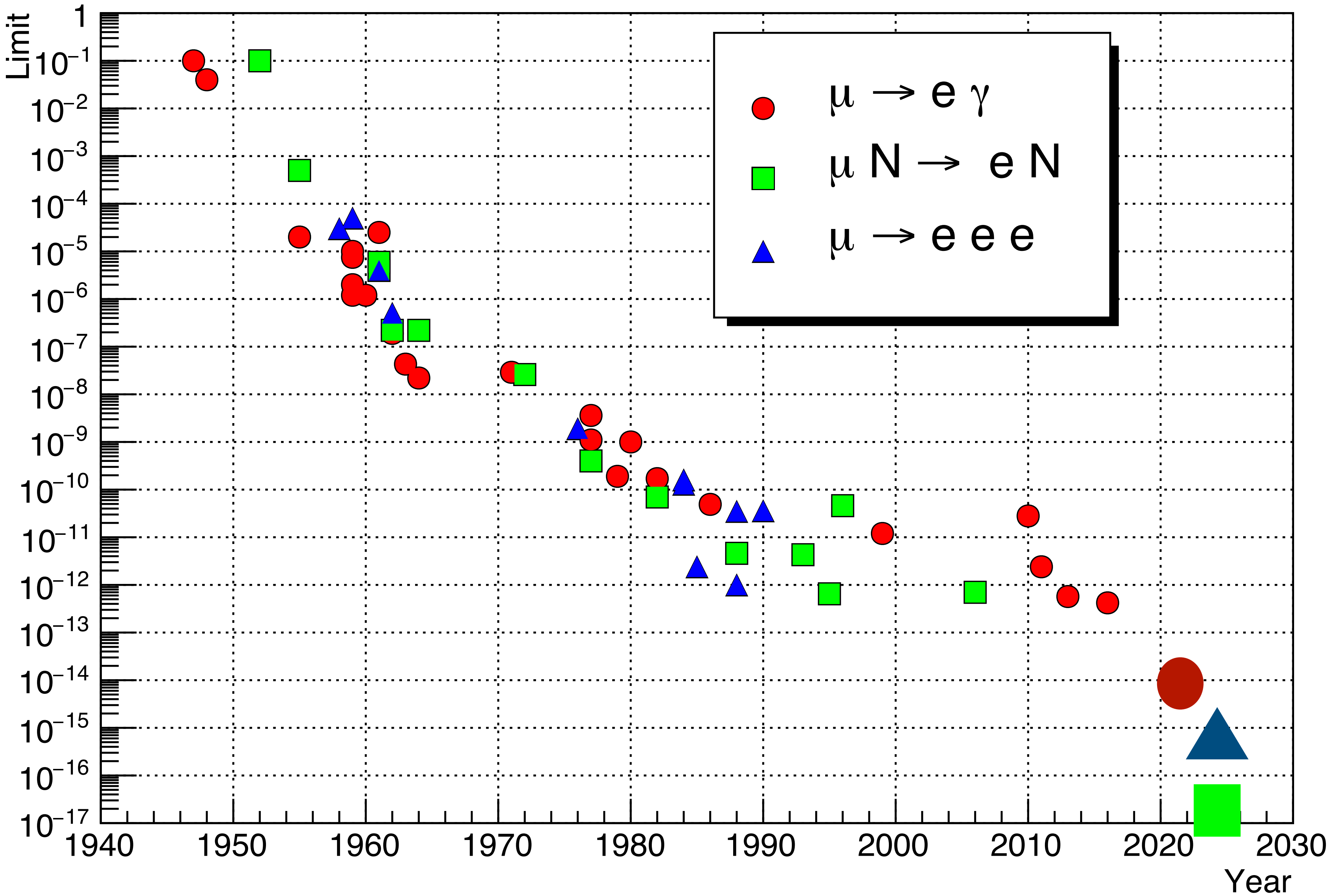
lepton number violating



TeV scale new physics
can lead to significant constraints.

Supersymmetry, extra dimensions,
Clockwork etc..

calibbi and signorelli, 1709.00294



Charged lepton flavour violating decays present limits

calibbi and signorelli, 1709.00294

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}\ ^\dagger$	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}\ ^\dagger$	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}\ ^\dagger$	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006
$\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}^*\ ^\dagger$	$< 3.6 \times 10^{-11}$	90%	SINDRUM II	1998
$\mu^+e^- \to \mu^-e^+$	$< 8.3 \times 10^{-11}$	90%	SINDRUM	1999
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	90%	BaBar	2010
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	90%	BaBar	2010
$\tau \to eee$	$< 2.7 \times 10^{-8}$	90%	Belle	2010
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	90%	Belle	2010
$\tau \to \pi^0 e$	$< 8.0 \times 10^{-8}$	90%	Belle	2007
$\tau \to \pi^0 \mu$	$< 1.1 \times 10^{-7}$	90%	BaBar	2007
$\tau \to \rho^0 e$	$< 1.8 \times 10^{-8}$	90%	Belle	2011
$\tau \to \rho^0 \mu$	$< 1.2 \times 10^{-8}$	90%	Belle	2011

calibbi and signorelli, 1709.00294

Reaction	Present limit	Expected Limit	Reference	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	5×10^{-14}	[316]	MEG II
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	10^{-16}	[46]	Mu3e
$\mu^-\mathrm{Al} \to e^-\mathrm{Al}$ †	$< 6.1 \times 10^{-13}$	10^{-17}	[321, 324]	Mu2e, COMET
$\mu^-\mathrm{Si/C} \to e^-\mathrm{Si/C}$ †	–	5×10^{-14}	[282]	DeeMe
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	5×10^{-9}	[339]	Belle II
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	10^{-9}	[339]	"
$\tau \to eee$	$< 2.7 \times 10^{-8}$	5×10^{-10}	[339]	"
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	5×10^{-10}	[339]	"
$\tau \to e$ had	$< 1.8 \times 10^{-8}$ ‡	3×10^{-10}	[339]	"
$\tau \to \mu$ had	$< 1.2 \times 10^{-8}$ ‡	3×10^{-10}	[339]	"
had $\to \mu e$	$< 4.7 \times 10^{-12}$ §	10^{-12}	[340]	NA62
$h \to e\mu$	$< 3.5 \times 10^{-4}$	3×10^{-5} ¶	[341]	HL-LHC
$h \to \tau\mu$	$< 2.5 \times 10^{-3}$	3×10^{-4} ¶	[341]	"
$h \to \tau e$	$< 6.1 \times 10^{-3}$	3×10^{-4} ¶	[341]	"

TABLE XII. – *Present and future limits for selected CLFV processes.* †*Rate normalised to the muon capture rate by the nucleus, see Eq. (99).* ‡*Best limits from* $\tau \to e\rho^0$ *and* $\tau \to \mu\rho^0$ *respectively.* §*Best limit from* K_L^0 *decay.* ¶*Reference [341] quotes the branching ratio for which one can make a 2σ or 5σ observation; we use the number of expected signal and background events in there to infer 95% C.L. sensitivities on the three channels, which turn out to be compatible with the scaling for the square root of the relative luminosity - 3000 fb*$^{-1}$ *assumed in [341] vs* 20 *[74] or 36 [75] fb*$^{-1}$.

calibbi and signorelli, 1709.00294

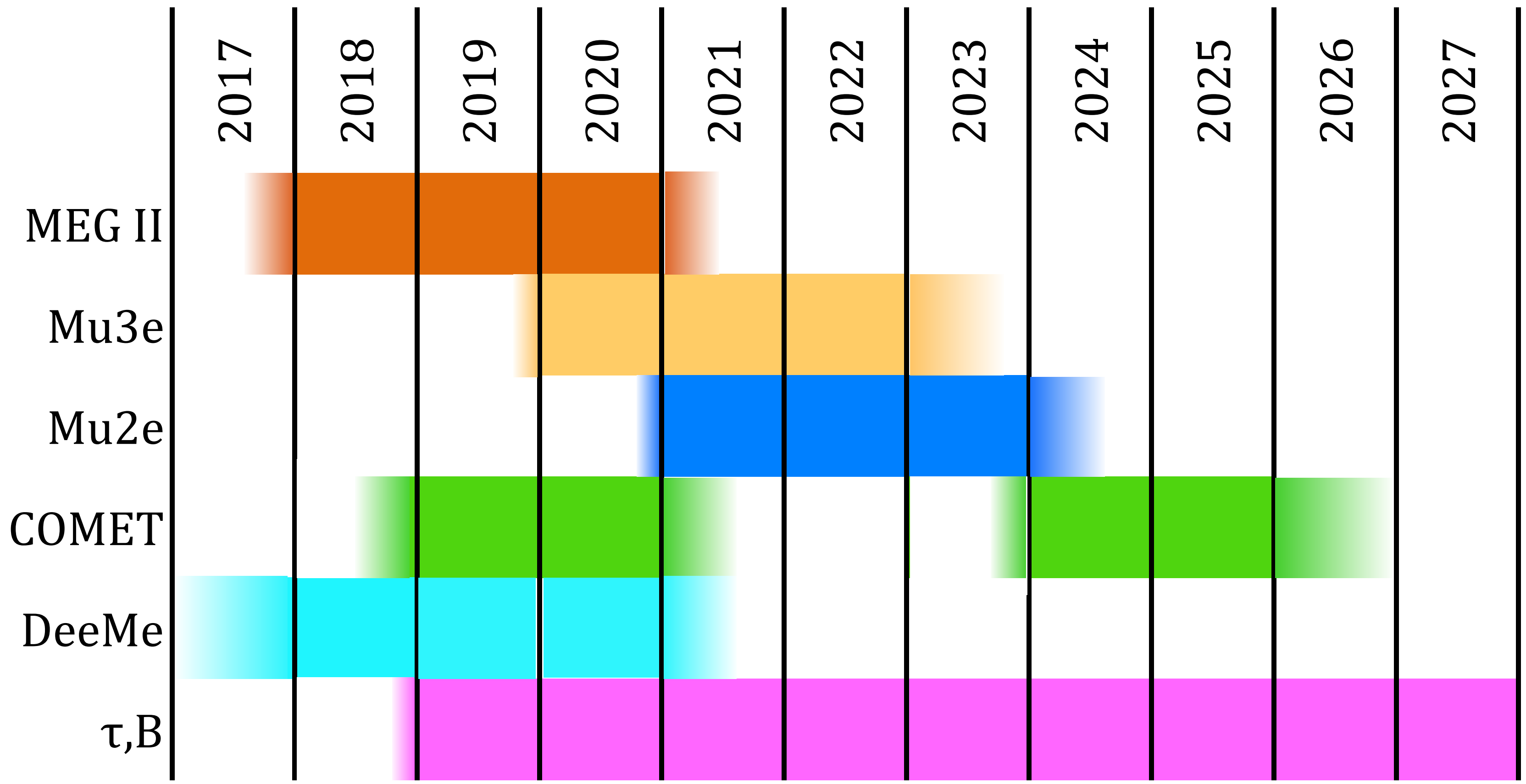


Figure 47. – Projected time lines for different projects searching for CLFV decays. MEG IIis expected to start data taking in 2018 after an engineering run in 2017; Mu3e magnet and detectors are expected at the end of 2019; Mu2e foresees three years of data taking starting in 2021; COMET Phase-I is expected to start commissioning and data taking in 2018 for two-three years, followed by a stop to develop and deploy the beamline and detectors for Phase-II; DeeMe is expected to start soon and take data with graphite and silicon carbide targets in sequence; Belle II is schedule to start data taking at end 2018.

LFV in meson decays

Petrov, Ambrosio, Crivellin, et.al

Channel	Br	Reference
$K^+ \to \pi^+ \mu^+ e^-$	$< 1.3 \times 10^{-11}$	E865, E777 [89]
$K^+ \to \pi^+ \mu^- e^+$	$< 5.2 \times 10^{-10}$	E865 [90]
$K_L \to \pi^0 \mu^\pm e^\mp$	$< 7.6 \times 10^{-11}$	KTeV [91]
$K_L \to \mu^\pm e^\mp$	$< 4.7 \times 10^{-12}$	E871 [92]

Decay	*Leptonic*	*Semileptonic*
K	$BR_2^{exp}(K_L^0 \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [16]	$BR_3^{exp}(K^+ \to \pi^+ \bar{\mu} e) < 1.3 \times 10^{-11}$
	-	$BR_3^{exp}(K^+ \to \pi^+ \bar{e} \mu) < 5.2 \times 10^{-10}$ [19]
D	$BR_2^{exp}(D^0 \to \mu^\pm e^\mp) < 1.3 \times 10^{-8}$ [17]	$BR_3^{exp}(D^+ \to \pi^+ \bar{\mu} e) < 3.6 \times 10^{-6}$
	-	$BR_3^{exp}(D^+ \to \pi^+ \bar{e} \mu) < 2.9 \times 10^{-6}$ [20]
D_s	-	$BR_3^{exp}(D_S^+ \to K^+ \bar{\mu} e) < 9.7 \times 10^{-6}$
	-	$BR_3^{exp}(D_S^+ \to K^+ \bar{e} \mu) < 1.4 \times 10^{-5}$ [20]
B	$BR_2^{exp}(B^0 \to \mu^\pm e^\mp) < 2.8 \times 10^{-9}$ [18]	$BR_3^{exp}(B^+ \to \pi^+ \mu^\pm e^\mp) < 1.7 \times 10^{-7}$ [21]
	-	$BR_3^{exp}(B^+ \to K^+ \mu^\pm e^\mp) < 9.1 \times 10^{-8}$ [22]
B_s	$BR_2^{exp}(B_S^0 \to \mu^\pm e^\mp) < 1.1 \times 10^{-8}$ [18]	–

Table 1: Experimental bounds on leptonic and semileptonic decays.

UV Model

BSM Physics Scale

Matching with SMEFT

Effective theory (SMEFT)

SM Scale

B-L conserving and lepton flavour violating

4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

TABLE IV. – *Complete list of the CLFV dimension-6 operators from [107]. The SM fields are denoted as in Eq. (3), and $B_{\mu\nu}$ and $W^I_{\mu\nu}$ ($I = 1,2,3$) are the $U(1)_Y$ and $SU(2)_L$ field strengths. Family indices are not shown, while $a,b = 1,2$ are $SU(2)_L$ indices, and τ_I are the Pauli matrices. Flavour indices of the fermions are not indicated.*

Crivellin et.al, 1312.0634

suppressed by two mass powers

Falkowski et. al, ’15-‘17

Signer and Pruna,’15-‘17
Davidson et. al, ’15-‘17
calibbi and signorelli, 1709.00294

	$\lvert C_a \rvert$ [$\Lambda = 1$ TeV]	Λ (TeV) [$\lvert C_a \rvert = 1$]	CLFV Process
$C_{e\gamma}^{\mu e}$	2.1×10^{-10}	6.8×10^{4}	$\mu \to e\gamma$
$C_{\ell e}^{\mu\mu\mu e, e\mu\mu\mu}$	1.8×10^{-4}	75	$\mu \to e\gamma$ [1-loop]
$C_{\ell e}^{\mu\tau\tau e, e\tau\tau\mu}$	1.0×10^{-5}	312	$\mu \to e\gamma$ [1-loop]
$C_{e\gamma}^{\mu e}$	4.0×10^{-9}	1.6×10^{4}	$\mu \to eee$
$C_{\ell\ell, ee}^{\mu eee}$	2.3×10^{-5}	207	$\mu \to eee$
$C_{\ell e}^{\mu eee, ee\mu e}$	3.3×10^{-5}	174	$\mu \to eee$
$C_{e\gamma}^{\mu e}$	5.2×10^{-9}	1.4×10^{4}	$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$
$C_{\ell q, \ell d, ed}^{e\mu}$	1.8×10^{-6}	745	$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$
$C_{eq}^{e\mu}$	9.2×10^{-7}	1.0×10^{3}	$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$
$C_{\ell u, eu}^{e\mu}$	2.0×10^{-6}	707	$\mu^- \mathrm{Au} \to e^- \mathrm{Au}$
$C_{e\gamma}^{\tau\mu}$	2.7×10^{-6}	610	$\tau \to \mu\gamma$
$C_{e\gamma}^{\tau e}$	2.4×10^{-6}	650	$\tau \to e\gamma$
$C_{\ell\ell, ee}^{\mu\tau\mu\mu}$	7.8×10^{-3}	11.3	$\tau \to \mu\mu\mu$
$C_{\ell e}^{\mu\tau\mu\mu, \mu\mu\mu\tau}$	1.1×10^{-2}	9.5	$\tau \to \mu\mu\mu$
$C_{\ell\ell, ee}^{e\tau ee}$	9.2×10^{-3}	10.4	$\tau \to eee$
$C_{\ell e}^{e\tau ee, eee\tau}$	1.3×10^{-2}	8.8	$\tau \to eee$

TABLE V. – *Bounds on the coefficients of some of the flavour-violating operators of e IV for* $\Lambda = 1$ TeV *, and corresponding bounds on* Λ *(in* TeV*) for* $\lvert C_a \rvert = 1$*. Superscripts refer to the flavour indices of the leptons appearing in the operators. Adapted from [107, 112, 114].*

UV Model

BSM Physics Scale

Matching with SMEFT

Effective theory (LEFT)

SM Scale

Matching with LEFT

Chiral PT matching

Mass of muon, tau etc.

$$\pi \rightarrow \mu + e$$

Jenkins et. al
1711.05270
1709.04486

LEFT Lagrangian relevant for LFV physics

Hadronic bilinear - leptonic bilinear

$(\bar{L}L)(\bar{L}L)$

$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Lw}\gamma_\mu u_{Lt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Lw}\gamma_\mu d_{Lt})$

$(\bar{L}L)(\bar{R}R)$

$\mathcal{O}_{eu}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Rw}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rw}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ue}^{V,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{e}_{Rw}\gamma_\mu e_{Rt})$
$\mathcal{O}_{de}^{V,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{e}_{Rw}\gamma_\mu e_{Rt})$

$(\bar{L}R)(\bar{L}R) + \text{h.c.}$

$\mathcal{O}_{eu}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Lw}u_{Rt})$
$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Lw}d_{Rt})$
$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{u}_{Lw}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{ed}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Lw}\sigma_{\mu\nu}d_{Rt})$

$(\bar{R}R)(\bar{R}R)$

$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rw}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{d}_{Rw}\gamma_\mu d_{Rt})$

$(\bar{L}R)(\bar{R}L) + \text{h.c.}$

$\mathcal{O}_{eu}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Rw}u_{Lt})$
$\mathcal{O}_{ed}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Rw}d_{Lt})$

Table 1. Semileptonic LEFT operators involving a charged-lepton bilinear and a quark bilinear.

Scalar	Vector	
$\mathcal{O}_{ijwt}^{SRR} = (\bar{\ell}_{Li}\ell_{Rj})(\bar{q}_{Lw}q_{Rt})$	$\mathcal{O}_{ijwt}^{V\ LL} = (\bar{\ell}_{Li}\gamma_\mu\ell_{Lj})(\bar{q}_{Lw}\gamma^\mu q_{Lt})$	$\mathcal{O}_{ijwt}^{V\ LR} = (\bar{\ell}_{Li}\gamma_\mu\ell_{Lj})(\bar{q}_{Rw}\gamma^\mu q_{Rt})$
$\mathcal{O}_{ijwt}^{SRL} = (\bar{\ell}_{Li}\ell_{Rj})(\bar{q}_{Rw}q_{Lt})$	$\mathcal{O}_{ijwt}^{V\ RR} = (\bar{\ell}_{Ri}\gamma_\mu\ell_{Rj})(\bar{q}_{Rw}\gamma^\mu q_{Rt})$	$\mathcal{O}_{ijwt}^{V\ RL} = (\bar{\ell}_{Ri}\gamma_\mu\ell_{Rj})(\bar{q}_{Lw}\gamma^\mu q_{Lt})$

TABLE I: LEFT semileptonic scalar and scalar operators. Notation: ℓ, q for lepton and quark. i,j=e, μ, τ and $w,\ t = u,\ d,\ s$. For $\pi^0 \to \mu^+\ e^-$ value of i=μ, j=e ,$(w,\ t)$=(u,u),(d,d)

Detailed analysis by Sacha Davidson et.al

Decay	*Leptonic*	*Semileptonic*
K	$BR_2^{exp}(K_L^0 \to \mu^\pm e^\mp) < 4.7 \times 10^{-12}$ [16]	$BR_3^{exp}(K^+ \to \pi^+ \bar{\mu} e) < 1.3 \times 10^{-11}$
	-	$BR_3^{exp}(K^+ \to \pi^+ \bar{e} \mu) < 5.2 \times 10^{-10}$ [19]
D	$BR_2^{exp}(D^0 \to \mu^\pm e^\mp) < 1.3 \times 10^{-8}$ [17]	$BR_3^{exp}(D^+ \to \pi^+ \bar{\mu} e) < 3.6 \times 10^{-6}$
	-	$BR_3^{exp}(D^+ \to \pi^+ \bar{e} \mu) < 2.9 \times 10^{-6}$ [20]
D_s	-	$BR_3^{exp}(D_S^+ \to K^+ \bar{\mu} e) < 9.7 \times 10^{-6}$
	-	$BR_3^{exp}(D_S^+ \to K^+ \bar{e} \mu) < 1.4 \times 10^{-5}$ [20]
B	$BR_2^{exp}(B^0 \to \mu^\pm e^\mp) < 2.8 \times 10^{-9}$ [18]	$BR_3^{exp}(B^+ \to \pi^+ \mu^\pm e^\mp) < 1.7 \times 10^{-7}$ [21]
	-	$BR_3^{exp}(B^+ \to K^+ \mu^\pm e^\mp) < 9.1 \times 10^{-8}$ [22]
B_s	$BR_2^{exp}(B_S^0 \to \mu^\pm e^\mp) < 1.1 \times 10^{-8}$ [18]	–

Table 1: Experimental bounds on leptonic and semileptonic decays.

Sacha davidson et.al
1807.10288
1008.0280

What about pions ?

$$\Gamma(\pi \to \mu + e) < 2.97 \times 10^{-18}$$

Earlier Chiral PT matching, see Criligiliano et.al, 0707.4464

Chiral Lagrangian

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^0_{\rm QCD} + \bar{q}\gamma^\mu\Big(v_\mu(x) + a_\mu(x)\Big)q - \bar{q}\Big(s(x) - i\gamma^5 p(x)\Big)q \\ &+ \bar{q}\sigma^{\mu\nu}t_{\mu\nu}(x)q - \frac{1}{16\pi^2}\theta(x)tr\Big(G_{\mu\nu}\tilde{G}^{\mu\nu}\Big) \ , \\ \mathcal{L}^0_{\rm QCD} &= -\frac{1}{2g_s^2}G_{\mu\nu}G^{\mu\nu} + \bar{q}i\gamma^\mu\Big(\partial_\mu - iG_\mu\Big)q \ ,\end{aligned}$$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^0_{\rm QCD} + \bar{q}_L\gamma^\mu l_\mu(x)q_L + \bar{q}_R\gamma^\mu r_\mu(x)q_R \\ &+ \bar{q}_L S(x)q_R + \bar{q}_R S^\dagger(x)q_L \\ &+ \bar{q}_L\sigma^{\mu\nu}t_{\mu\nu}(x)q_R + \bar{q}_L\sigma^{\mu\nu}t^\dagger_{\mu\nu}(x)q_R \ ,\end{aligned}$$

$$\begin{aligned}r_\mu(x) &= v_\mu(x) - a_\mu(x) \ , \\ l_\mu(x) &= v_\mu(x) + a_\mu(x) \ , \\ S(x) &= s(x) - ip(x) \ .\end{aligned}$$

$$\begin{aligned}\bar{q}_{Lw}q_{Rt} &= -2B_0\left\{\frac{1}{4}F_0^2U_{tw} + L_4\langle D_\mu U^\dagger D^\mu U\rangle U_{tw}\right. \\ &+ L_5(UD_\mu U^\dagger D^\mu U)_{tw} + 2L_6\langle U^\dagger\chi + \chi^\dagger U\rangle U_{tw} - 2L_7\langle U^\dagger\chi - \chi^\dagger U\rangle U_{tw} \\ &+ \left. 2L_8(U\,\chi^\dagger\,U)_{tw} + H_2\,\chi_{tw}\right\} + \mathcal{O}(p^6) \ ,\end{aligned}$$

$$U \equiv \exp\iota\frac{\sqrt{2}}{f_0}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{\eta_0}{\sqrt{3}} - \frac{\eta_8}{\sqrt{3/2}} \end{pmatrix}$$

Constraints on Scalar Operators

$$\langle \mu e | \Big(C^{SRL}_{\mu e w t} \mathcal{O}^{SRL}_{\mu e w t} + C^{SRR}_{\mu e w t} \mathcal{O}^{SRR}_{\mu e w t} \Big) | \pi^0 \rangle$$

$$\begin{aligned} \mathcal{O}^{SRR}_{\mu e w t} &= (\bar{\mu}_L e_R)(\bar{q}_{Lw} q_{Rt}) \\ &= (\bar{\mu}_L e_R)\Big[-2B_0 \frac{1}{4} F_0^2 U_{tw} \Big] + \mathcal{O}(p^4), \end{aligned}$$

$$\begin{aligned} \mathcal{O}^{SRL}_{\mu e w t} &= (\bar{\mu}_L e_R)(\bar{q}_{Rw} q_{Lt}) \\ &= (\bar{\mu}_L e_R)\Big[-2B_0 \frac{1}{4} F_0^2 U^\dagger_{tw} \Big] + \mathcal{O}(p^4), \end{aligned}$$

$$\begin{aligned} \mathcal{M}(\pi^0(p_1) \to \mu^+(p_2) e^-(p_3)) &= -\Big(\frac{i B_0 F_0}{2} \Big) (C^{SRR}_{\mu e u u} - C^{SRR}_{\mu e d d} - C^{SRL}_{\mu e u u} + C^{SRL}_{\mu e d d}) \\ &\times \; \bar{u}(p_e) \frac{1}{2} (1 - \gamma^5) v(p_\mu) \; . \end{aligned}$$

$$\begin{aligned} \Gamma(\pi^0 \to \mu^+ e^-) &= \Big| \Big(C^{SRR}_{\mu e u u} - C^{SRR}_{\mu e d d} - C^{SRL}_{\mu e u u} + C^{SRL}_{\mu e d d} \Big) \Big|^2 (6.007 \times 10^{-6})\, \mathrm{GeV}^5 \\ &\leq \; 2.97 \times 10^{-18} \end{aligned}$$

$$C \lesssim 10^{-6} - 10^{-7} \mathrm{GeV}^{-2}$$

	In GeV		In GeV
m_π^0	134.97×10^{-3}	F_0	92.3×10^{-3} [14]
m_μ	105.65×10^{-3}	m_e	0.51×10^{-3}
m_u	2.16×10^{-3}	m_d	4.67×10^{-3}
$\Gamma(\pi^0 \to \mu^+ e^-)$	2.97×10^{-18}	$B_0 = \frac{m^2_{\pi^0}}{m_u + m_d}$	2.667

TABLE II: Input values in GeV. m_u, m_d values are MS bar masses at 2 GeV. All values are taken from PDG.

Constraints on Vector Operators

$$\langle e\mu|\left(C^{V\ LL}_{\mu ewt}\mathcal{O}^{V\ LL}_{\mu ewt} + C^{V\ LR}_{\mu ewt}\mathcal{O}^{V\ LR}_{\mu ewt} + C^{V\ RR}_{\mu ewt}\mathcal{O}^{V\ RR}_{\mu ewt} + C^{V\ RL}_{\mu ewt}\mathcal{O}^{V\ RL}_{\mu ewt}\right)|\pi^0\rangle$$

$$\begin{aligned}
\bar{q}_L\gamma^\mu q_L \ =\ & \frac{i}{2}F_0^2\langle D_\mu UU^\dagger\rangle + 4iL_1\langle D_\nu U^\dagger D^\nu U\rangle\langle D_\mu UU^\dagger\rangle + 4iL_2\langle D^\mu U^\dagger D^\nu U\rangle\langle D_\nu UU^\dagger\rangle \\
+\ & 2iL_3\langle\left(U^\dagger D^\mu U - D^\mu U^\dagger U\right) D_\nu U^\dagger D^\nu U\rangle + 2iL_4\langle D_\mu UU^\dagger\rangle\langle U^\dagger\chi + \chi^\dagger U\rangle \\
+\ & iL_5\langle\left(U^\dagger D^\mu U - D^\mu U^\dagger U\right)(U^\dagger\chi + \chi^\dagger U)\rangle \\
+\ & L_9\left[-\langle F_R^{\mu\nu}D_\nu UU^\dagger\rangle - \langle UD_\nu U^\dagger F_R^{\mu\nu}\rangle + \langle D_\nu UF_L^{\mu\nu}U^\dagger\rangle + \langle UF_L^{\mu\nu}D_\nu U^\dagger\rangle\right] \\
-\ & iL_9\langle D^\nu(D_\mu UD_\nu U^\dagger - D_\nu UD_\mu U^\dagger\rangle + 2L_{10}\langle D_\nu(UF_L^{\mu\nu}U^\dagger)\rangle \\
+\ & 4H_1\langle D_\nu F_R^{\mu\nu}\rangle + \epsilon\ \text{terms} + \mathcal{O}(p^6)\ ,
\end{aligned}$$

$$\begin{aligned}
\langle e\mu|\left(C^{V\ LL}_{\mu ewt}\mathcal{O}^{V\ LL}_{\mu ewt} + C^{V\ LR}_{\mu ewt}\mathcal{O}^{V\ LR}_{\mu ewt} + C^{V\ RR}_{\mu ewt}\mathcal{O}^{V\ RR}_{\mu ewt} + C^{V\ RL}_{\mu ewt}\mathcal{O}^{V\ RL}_{\mu ewt}\right)|\pi^0\rangle \ &= \\
\langle e\mu|\left(C^{V\ LR}_{\mu eu_iu_i} - C^{V\ LL}_{\mu eu_iu_i}\right)(\bar{\mu}_L\gamma^\mu e_L)\frac{F_0}{2}\partial_\mu\pi^0|\pi^0\rangle \ &+ \\
\langle e\mu|\left(C^{V\ RR}_{\mu eu_iu_i} - C^{V\ RL}_{\mu eu_iu_i}\right)(\bar{\mu}_R\gamma^\mu e_R)\frac{F_0}{2}\partial_\mu\pi^0|\pi^0\rangle &
\end{aligned}$$

Define

$$A_{LL} = C^{V\ LR}_{\mu eu_iu_i} - C^{V\ LL}_{\mu eu_iu_i}$$

$$A_{RR} = C^{V\ RR}_{\mu eu_iu_i} - C^{V\ RL}_{\mu eu_iu_i}$$

$$\begin{aligned}
\mathcal{M}(\pi^0(p^{\pi^0}) \to \mu^+(p_\mu)e^-(p_e)) \ =\ & \frac{F_0}{2}\Big(A_{LL}(\bar{u}(p_e)\gamma^\mu\frac{(1-\gamma^5)}{2}v(p_\mu))p_\mu^{\pi_0} \\
+\ & A_{RR}(\bar{u}(p_e)\gamma^\mu\frac{(1+\gamma^5)}{2}v(p_\mu))p_\mu^{\pi_0}\Big)\ .
\end{aligned}$$

$$\Gamma(\pi^0 \to \mu^+e^-) \ =\ (|A_{LL}|^2 + |A_{RR}|^2) \times (9.57 \times 10^{-9})\ (\text{GeV})^5$$

$$(|A_{LL}|^2 + |A_{RR}|^2)\ \leq\ 3.104 \times 10^{-10}\ (\text{GeV})^{-4}$$

Summary

In addition to purely leptonic LFV processes,
LFV in meson decays also provides
Significant constraints.

We studied tree level matching of Chiral PT to LEFT
operators and derived constraints
On them.

A Full Global analysis in the operator basis would be
An interesting study.