Infrared Singularities beyond three loops

Sourav Pal

Collaborators: Neelima Agarwal, Lorenzo Magnea, Anurag Tripathi

Department of Physics, IIT Hyderabad

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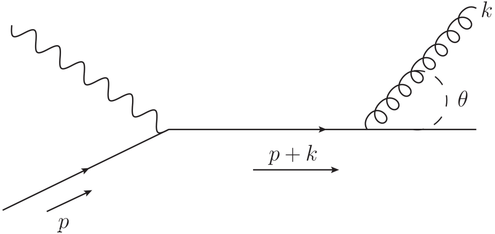
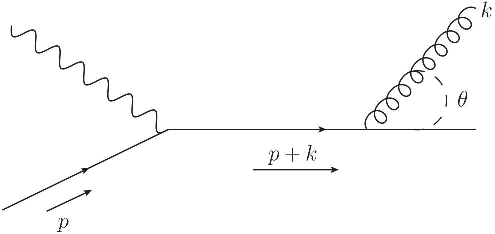
IR singularity



$$\frac{1}{(p+k)^2} = \frac{1}{2p.k} = \frac{1}{2\mid p\mid\mid k\mid(1-cos\theta)}$$

IR singularity



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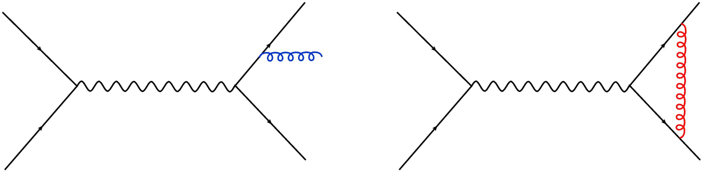
Two types of divergences

- Soft divergences $[k \to 0]$
- Collinear divergences $[\theta \to 0]$

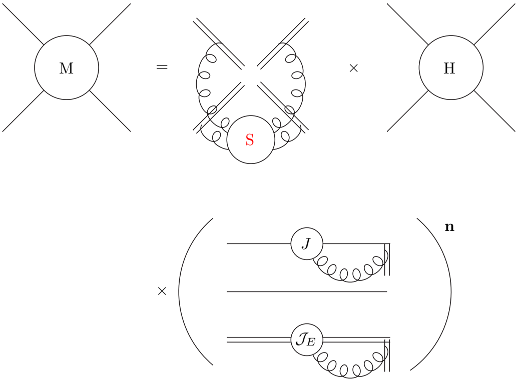
Why study infrared singularities



Dimensional regularization = finite! with large logs

- These logs have large values and can disturb the convergence of expansion in α. We need to do resummation.
- Knowing the IR singularities at all orders, resummation is easy.

Factorization of Multileg amplitudes



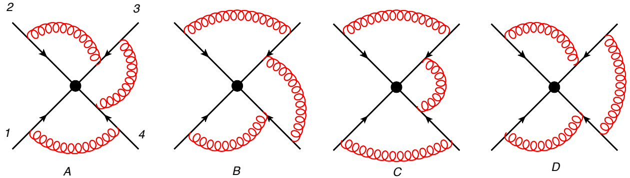
ref: Mueller (81), Sen (83), Botts Sterman (89), Kidonakis Oderda Sterman (98), Catani (98), Tejeda-Yeomans Sterman (02), Kosower (03), Aybat Dixon Sterman (06), Becher Neubert (09), Gardi Magnea (09)

Soft function

- Diagrammatic exponentiation

$$S_n = \exp(\mathcal{W}_n)$$

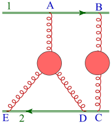
- In multiparton case, the concept of webs generalizes non-trivially.
  - A web in the multiparton case is a set of diagrams which differ only by the order of the gluon attachment on each Wilson line.



S=1 S=1 S=2 S=2

Cwebs

- A Cweb is a set of diagrams, built out of connected gluon correlators attached to Wilson lines, closed under shuffles of gluon attachments to each Wilson line. [Agarwal, Magnea, SP, Tripathi 2020]



$$C\,[\text{shuffle}]\,DE = \{CDE, DCE, DEC\}$$

- If a diagram is $D = F(D)C(D)$ a Cweb $\mathcal{W}$ is expressed as

$$\mathcal{W} = \sum_{D} F(D)\tilde{C}(D) = \sum_{D,D'} F(D)R_{DD'}C(D)$$

- Properties of R
  - Idempotence: $R^2 = R$, eigenvalues 1 or 0.
  - Zero-sum rows. [Gardi et. al. (2010)]
  - Conjecture: $\sum_D c(D)s(D) = 0$. [Gardi et. al. (2011)]

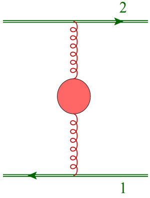
Challenges at four loops

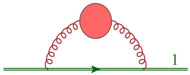
- Enumeration of Cwebs at four loops.
- 60 Cwebs at four loops.
- The largest dimension of the mixing matrix for the web is 36×36
- Results available having dimension 16×16 at three loops.

Replica method: In-house Mathematica code .

Enumeration using Cwebs

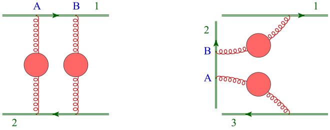
One loop Cweb



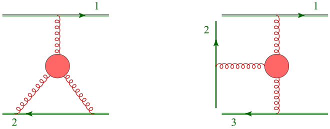


Enumeration using Cwebs

- 2 loop Cweb:
  - Add a propagator to 1-loop Cweb.



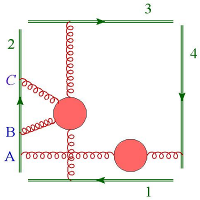
  - Connect a m point correlator to Wilson line and turn them into a $(m+1)$ point correlator



  - Connect a m point correlator to a n point correlator, if you have more than one correlator.
  - Discard double counted Cwebs.

Results

- $\mathbf{W}_4^{(1,0,1)}(1,1,1,3)$



Diagrams	Sequences	S-factors
C_1	$\{\{ABC\}\}$	1
C_2	$\{\{BAC\}\}$	0
C_3	$\{\{BCA\}\}$	1

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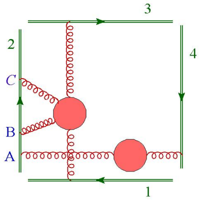

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- Mixing Matrix

$$R = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Results

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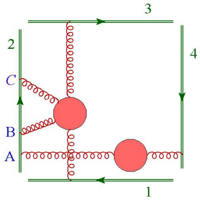

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- Exponentiated colour factors

$$\tilde{C}_1 = i f^{abg} f^{cdg} f^{ebh} \mathrm{T}_1^a \mathrm{T}_2^h \mathrm{T}_2^c \mathrm{T}_3^d \mathrm{T}_4^e ,$$

$$\tilde{C}_2 = -i f^{abg} f^{cdg} f^{cej} \mathrm{T}_1^a \mathrm{T}_2^b \mathrm{T}_2^j \mathrm{T}_3^d \mathrm{T}_4^e .$$

Results: Direct construction

Steps of direct construction:

- Consider a generic matrix.
- Apply row sum rule
- Apply column sum rule
- Apply Trace=Rank for idempotent matrix

Results:

- 2×2 Mixing matrix

$$R = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}.$$

- 3×3 Mixing matrix

$$R = \begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}.$$

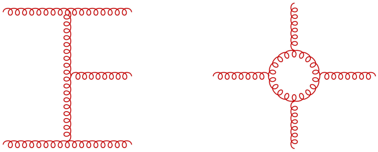
Results: Direct construction

- $p \times p$ mixing matrix, p is prime

$$R = \begin{pmatrix} \frac{1}{2} & 0\ 0\ \ldots\ 0 & -\frac{1}{2} \\ -\frac{1}{2} & 1\ 0 \ldots 0 & -\frac{1}{2} \\ & \ldots & \\ -\frac{1}{2} & 0\ 0\ \ldots\ 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0\ 0\ \ldots\ 0 & \frac{1}{2} \end{pmatrix}.$$

Conclusions

- Soft function exponentiate in terms of Cwebs.
- We have computed mixing matrices and exponentiated colour factors for 60 Cwebs using our in-house Mathematica code.
- General color structure at four loops



Agarwal, Magnea, SP, Tripathi, 2021

- Direct construction of 2×2, 3×3 and $p \times p$ mixing matrices are complete.
- All the mixing matrices obey row-sum, column sum rule and they are idempotent.

Thank You